MECHANICS AND HEAT

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MECHANICS AND HEAT

AN ELEMENTARY COURSE OF APPLIED PHYSICS

BY

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PREFACE

An attempt is made in this volume to provide a practical statement of the principles of Mechanics and Heat, likely to awaken interest in the application of these sciences, for pupils who as yet have not come in contact with the practice of engineering and the allied constructive arts.

The new science syllabus prescribed by the Civil Service Commissioners for candidates for Second Division Clerkships includes (1) the fundamental principles of Mechanics and (2) Heat as two of the sections, the former being obligatory for all candidates who offer science. The syllabus is distinguished by the prominence given to practical applications of scientific principles; and the present volume has been written in the same spirit.

While the book contains all that is required of candidates who take the subjects referred to in Civil Service Examinations for Second Division Clerkships, its contents are not limited by the syllabus of that examination. The course of work should be found suitable also for use in the Upper Forms of those Secondary Schools where it is customary to introduce boys to practical applications of the physical principles studied.

This application of principles is explained in the following chapters, partly by a large number of fully worked-out problems of the kind which present themselves in everyday workshop life, and partly by carefully designed laboratory experiments. The subjects of the experimental course have been brought together on pp. xi to xiii, and an examination of the nature of the experiments proposed will show that points of practical interest and importance have been emphasised consistently.

Additional exercises, for home or class work, will be found at the end of each chapter. A number of these questions have been extracted from examination papers of the Board of Education, and for permission to do this, thanks are due to the Controller of His Majesty's Stationery Office.

The opportunity is taken of making acknowledgment to Prof. R. A. Gregory and to Mr. A. T. Simmons for useful hints in the preparation of the book, and for reading the proof sheets. Thanks are also due to Mr. J. Norrie who has read the proofs. The kindness of many firms in permitting the use of copyright illustrations is gratefully acknowledged.

The Tables on the properties of steam are taken, with permission, from *The Steam Engine and Gas and Oil Engines*, by Prof. John Perry, F.R.S. (Macmillan); the Logarithmic Tables are from Mr. F. Castle's *Machine Construction and Drawing* (Macmillan).

JOHN DUNCAN.

WEST HAM, *April*, 1913.

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PART I.

MECHANICS

MECHANICS AND HEAT.

CHAPTER I.

INTRODUCTORY.

MEASURING INSTRUMENTS.

Straight Edge.—The engineer's *straight edge* consists of a long strip of metal with one edge bevelled, this edge being such that a straight line drawn from two points situated near the ends of the edge will lie wholly in the edge. If a straight edge has to be originated, it is necessary to make three at the same time, then by a continual process of comparing one with the others and removing the faulty parts by scraping, the edges of all three may be brought nearly true.

Surface Plate.—The *surface plate* consists of a rigid plate of cast iron, having three feet on its under side, in order always to distribute the supporting forces in the same manner and so prevent the plate warping. The upper surface of the plate is brought, as far as possible, all to lie in one plane. This is done by constructing three plates at a time. The upper surfaces of the plates having been carefully planed, are compared one with the others. A little oil and colouring matter rubbed

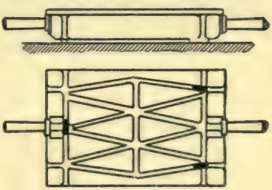


FIG. 1.—Surface plate.

on the surface show those spots which are in error, and these are removed by scraping. The operation is finished when any two of the three, on being brought together, show contact over a large number of bearing spots evenly distributed. These spots will then all lie in one plane. Any one of the plates may now be used for the reproduction of other plane surfaces.

External and Internal Gauges.—Sir Joseph Whitworth, by constructing bars having plane ends perpendicular to their axes, subdivided the standard yard into inches, etc. Engineers' steel rules, subdivided with considerable accuracy into inches, tenths, etc., are one of the results of his work, and can be used for producing objects having required dimensions. For standards of reference in the shops, *cylindrical external and internal gauges* are used. These are shown in Fig. 2, and consist of a collar with a hole, and a plug. Both hole and plug are brought nearly to size by machining, then hardened, ground and lapped down to size by hand. Gauges such as this are turned out true to $\frac{1}{10000}$th inch.

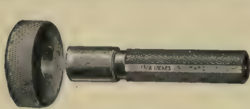


Fig. 2.—Standard cylindrical gauges.

Standard gauges are not suitable for the reproduction of dimensions to a given degree of accuracy, as **calipers** have to be used in transferring the dimension from the gauge to the work. The calipers are adjusted as nearly as the workman can tell by touch to the standard gauge and then are applied to the work. In this process too much is left to the skill and discretion of the workman, and no limits of accuracy can be stated and worked to. To secure good results, **limit gauges** are necessary. Two of these are shown in Figs. 3 and 4. The **internal** limit gauge is for measuring inside cylindrical holes. One end is made slightly larger in diameter than the other, the difference being determined by the limits of accuracy

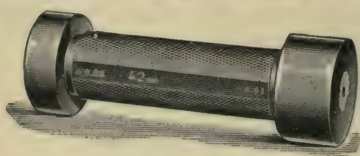


Fig. 3.—Internal limit gauge.

required in the work under execution. The smaller end must "go in" to the finished hole, and the larger end must "not go in." Consequently, the finished hole is larger in diameter than the small end of the gauge and smaller in diameter than the large end, and so the work is kept within the desired limits of accuracy. For example, a hole to be 1 inch approximately in diameter would be bored, using an internal limit gauge having diameters 1·006″ and 0·994″ respectively. The total variation in the diameter of the finished hole cannot exceed 0·012″.

FIG. 4.—External limit gauge.

The **external** limit gauge is used for turning cylindrical pieces down to size, and is used in a similar manner. As nothing is left to the discretion of the workman, interchangeable parts can easily be produced by the use of these gauges.

Standard screw-gauges are also useful for reference, and help to produce accurate work. Two of these, one external and one internal, are shown in Fig. 5.

FIG. 5.—Standard screw-gauges.

For the more accurate measurement of dimensions than can be secured by the use of calipers, **micrometers** are used. In Fig. 6 a micrometer is shown having its outer parts shown transparent. The instrument consists of a very finely cut screw, which may be rotated by turning the outer milled thimble. This screw works in a split nut, fitted with an adjusting nut to

take up wear, and terminates, as shown, between the jaws of the instrument, in a cylindrical portion having its end brought plane and square to the axis of the screw. The screw is protected from dust and grit by the outer thimble casing. The object to be measured is placed in the jaw of the instrument, and the thimble turned until the object is gently nipped. The dimension is then read from two scales, one engraved along the barrel longitudinally, and the other circularly round the edge of the thimble. In the instrument shown, the screw has 40 threads to an inch, and one inch on the longitudinal scale is divided into tenths of an inch, each tenth being subdivided into four parts; each part will therefore be $\frac{1}{40}''$ or $0{\cdot}025''$ long. One revolution of the thimble will consequently advance the screw one part on the longitudinal scale, or a distance of $0{\cdot}025''$ The circular scale on the thimble has 25 divisions round the complete circumference, consequently revolving the thimble through one division will advance the screw

$$\tfrac{1}{25} \times 0{\cdot}025'' = 0{\cdot}001''.$$

The instrument can therefore be used for taking dimensions to $\frac{1}{1000}$ inch. To read the scales, suppose, as in Fig. 6, that the longitudinal scale shows three parts beyond $0{\cdot}1''$. This will be $0{\cdot}1 + (3 \times 0{\cdot}025) = 0{\cdot}175''$. The circular scale is set at 0 or 25, so that in this position nothing need be allowed for it. The dimension, as set, is therefore $0{\cdot}175''$.

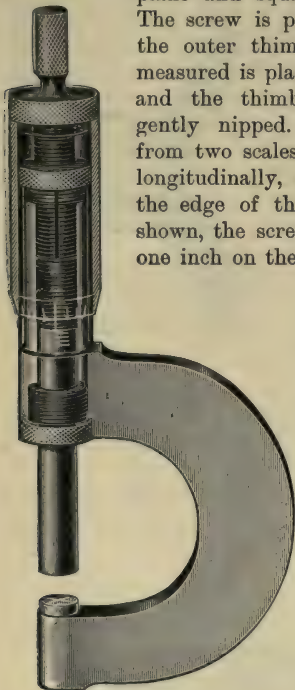


FIG. 6.—Micrometer.

If the circular scale had been beyond the $0{\cdot}175''$ mark on the longitudinal scale, by, say, 14 divisions, then we should have added $0{\cdot}014''$ to the above reading, giving $0{\cdot}175 + 0{\cdot}014 = 0{\cdot}189''$ as the required dimension.

Micrometers should be tested occasionally for zero error by running the screw right home until the points touch, and ascertaining if the 0 or 25 mark on the circular scale comes opposite to the line on the longitudinal scale. If this is not the case, the instrument can be adjusted by the screw point on the opposite jaw, or a *zero error* may be allowed for in subsequent readings.

Verniers.—The *vernier* is a device for subdividing the parts of a scale into divisions that would be too fine to be read by the eye. It consists of a sliding piece fitted to a main scale and having a suitable scale engraved on it. In the case shown in Fig. 7 the vernier scale has 10 divisions of total length equal to 9 divisions on the main scale. Each division on the vernier is therefore $\frac{1}{10}$th shorter than a division on the main scale, so that if set with the arrow opposite a division on the main scale, the next two divisions will be $\frac{1}{10}$th of a division apart, the next pair of divisions $\frac{2}{10}$ths and so on. To read the instrument, note the division on the main scale to the left of the vernier arrow, in the case shown in Fig. 8 this is 0·7; then look along the vernier to find a division on it exactly opposite a division on the main scale and note the vernier division, in the example this is 4; so that the vernier arrow is $\frac{4}{10}$ths of a main scale division beyond the 0·7 mark; the reading is therefore 0·74.

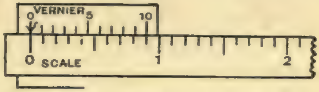


FIG. 7.—Vernier, set at zero.

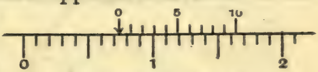


FIG. 8.—Vernier, set at 0·74.

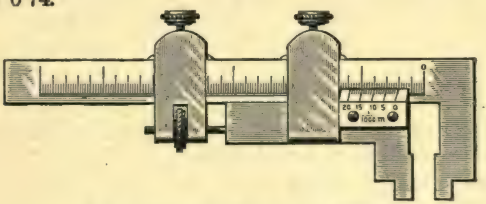


FIG. 9.—Vernier caliper, set at 0·200″.

Calipers for use in the workshop are often fitted with verniers. The one shown (Fig. 9) can read up to about $1\frac{3}{4}''$ by $\frac{1}{1000}$th of an

inch. The main scale has inches divided into tenths, and each tenth is subdivided into five parts, each part being therefore $\frac{1}{50}$ inch. The vernier has 20 divisions of total length equal to 19 divisions on the main scale, so that each vernier division is $\frac{1}{20}$th shorter than a main scale division. The vernier therefore reads to $\frac{1}{20} \times \frac{1}{50} = \frac{1}{1000}$ inch. As set in Fig. 9, the instrument reads 0·200″ on the main scale and 0 on the vernier, the dimension is therefore 0·200″. Had the vernier been set say at 11, the reading would be 0·211″. Readings of this instrument should be corrected for zero error in the same manner as for micrometers.

Other Devices.—**End measuring rods** (Fig. 10) are very useful for calipering holes or distances between parallel faces, when the dimensions are large. The ends of these rods are made spherical, so that they cannot be jammed when in position, no matter how they are turned. The rod is held by the vulcanite handle at its middle. **Flexible steel tapes** may be used for taking the diameter of large cylindrical pieces. By passing these tapes round such a cylinder its circumference can be obtained, from which measurement the diameter of the cylinder may be calculated.

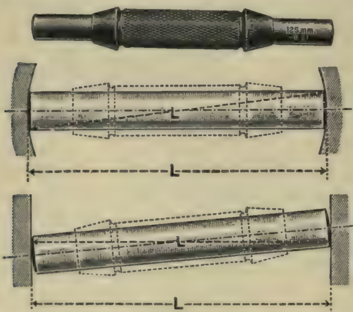


FIG. 10.—End measuring rods, with spherical ends.

MENSURATION.

Determination of Areas.—Some of the ordinary rules of mensuration are given here for future reference.

Square, side s; area $= s^2$.

Rectangle, adjacent sides a and b; area $= a \times b$.

Triangle, base b, perpendicular height h; area $= \frac{1}{2}b \times h$.

Parallelogram, area = one side × perpendicular distance from that side to the opposite one.

Any irregular figure bounded by straight lines; to find its area split it up into triangles, find the area of each triangle separately, and take the sum of these areas for the area of the figure.

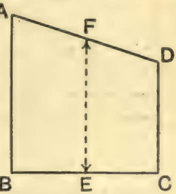


FIG. 11.

Trapezoid, such as $ABCD$ (Fig. 11), area $= BC \times$ average height.

The average height will be EF, drawn from the centre of BC, and will be equal to $\frac{1}{2}(AB+CD)$.

Area of trapezoid $= \frac{BC}{2}(AB+CD)$.

Suppose we have a figure consisting of a number of trapezoids (Fig. 12), all of the same breadth a, of which the area is required. We may proceed to find the area of each separately and to sum for the total area.

$$\text{Thus, area} = \left(\frac{h_1+h_2}{2}\right)a + \left(\frac{h_2+h_3}{2}\right)a + \left(\frac{h_3+h_4}{2}\right)a + \left(\frac{h_4+h_5}{2}\right)a,$$

$$= a\left\{\frac{1}{2}h_1+\frac{1}{2}h_2+\frac{1}{2}h_2+\frac{1}{2}h_3+\frac{1}{2}h_3+\frac{1}{2}h_4+\frac{1}{2}h_4+\frac{1}{2}h_5\right\},$$

$$= a\left\{\frac{h_1+h_5}{2}+h_2+h_3+h_4\right\}.$$

This gives us the **trapezoidal rule** for such areas, viz.—take half the sum of the first and last ordinate, add to this the sum of all the intermediate ordinates, and multiply the result by the common distance between the ordinates.

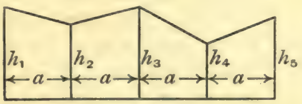


FIG. 12.

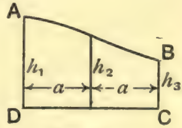


FIG. 13.

Simpson's Rule for finding the area of a plane figure bounded by a curve and end ordinates perpendicular to the base is founded on the assumption that the curve is parabolic. The rule only is given here. For a curve such as $ABCD$ (Fig. 13) divide it by an ordinate h_2 so as to bisect the base DC.

Let a = the distance between the ordinates and h_1, h_2, h_3 = the heights of the ordinates.

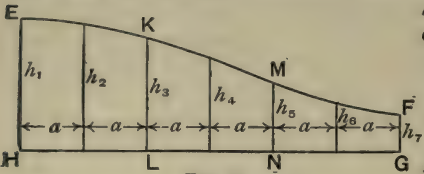


Fig. 14.

Then,

$$\text{area} = \frac{a}{3}(h_1 + 4h_2 + h_3).$$

It is sometimes convenient to take more ordinates as is shown for the area $EFGH$ (Fig. 14). The number of ordinates must always be odd and they must be equidistant.

In this case, $\quad \text{area } EKLH = \frac{a}{3}(h_1 + 4h_2 + h_3),$

$$\text{area } KMNL = \frac{a}{3}(h_3 + 4h_4 + h_5),$$

$$\text{area } MFGN = \frac{a}{3}(h_5 + 4h_6 + h_7),$$

and total area $= \frac{a}{3}(h_1 + 4h_2 + 2h_3 + 4h_4 + 2h_5 + 4h_6 + h_7)$.

This rule may be stated thus:—Add the first and the last ordinates; to this sum add four times the sum of the even intermediate ordinates and twice the sum of the odd intermediate ordinates; multiply this total sum by one third the common distance between the ordinates.

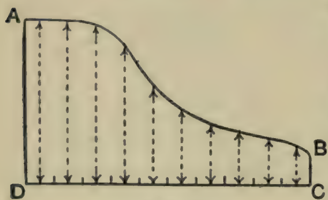


Fig. 15.

These and other convenient rules for finding the areas of figures bounded by curves are more used by naval architects than by engineers. The ordinary process used by engineers for finding the area of a figure such as $ABCD$ (Fig. 15) is to divide DC into 10 equal parts and to measure the height of the diagram at the centre of each part. Sum these heights and divide by 10. The result gives the average height of the

diagram and if this be multiplied by DC the final result will give the area.

Circle, diameter, d or radius, r ; circumference $=\pi d=2\pi r$.

π denotes the ratio of the circumference of a circle to its diameter and is represented by the number 3·1416. For many engineering calculations the value $\frac{22}{7}$ is correct enough.

$$\text{Area of circle}=\frac{\pi d^2}{4}=\pi r^2=0{\cdot}7854\,.\,d^2.$$

Determination of Volumes.—*Cube*, edge, s ; volume $=s^3$.

Prism, having its ends perpendicular to its axis; volume = area of one end × length of prism.

Sphere, radius, r ; volume $=\frac{4}{3}\pi r^3$.

(Area of curved surface $=4\pi r^2$.)

Pyramid, volume = area of base × $\frac{1}{3}$ perpendicular height.

Cone, volume = area of base × $\frac{1}{3}$ perpendicular height.

(Area of curved surface = circumference of base × $\frac{1}{2}$ slant height.)

Right-angled triangle ABC (Fig. 16); $AC^2=AB^2+BC^2$.

Measurement of Angles.—Angles may be measured in *degrees*, or in *radians*.

A **degree** is the angle at the centre of a circle subtended by an arc of $\frac{1}{360}$th of the circumference.

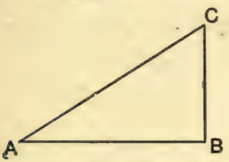


FIG. 16.

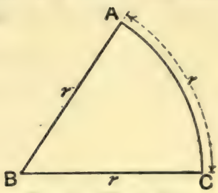


FIG. 17.—The radian.

A **radian** is the angle at the centre of a circle subtended by an arc equal to the radius of the circle; in Fig. 17, ABC is one radian.

In a complete circle there are 360 degrees and 2π radians, therefore

$$2\pi \text{ radians}=360 \text{ degrees},$$

or

$$\pi \text{ radians}=180 \text{ degrees}.$$

An angle expressed in radians can be transformed to degrees by multiplying by $\frac{180}{\pi}$; or if expressed in degrees, can be transformed to radians by multiplying by $\frac{\pi}{180}$.

An angle may be expressed in radians by dividing the length of the circular arc subtending it by the radius of the arc, both being in the same units.

Trigonometrical Ratios.

The following definitions should be understood. Given an angle POM (Fig. 18), take any distance OP and draw PM perpendicular to OM.

Then the following ratios of the sides will be independent of the length OP and will depend only on the magnitude of the given angle.

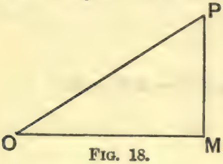


Fig. 18.

$\frac{PM}{OP}$ is called the **sine** of the angle POM, written $\sin POM$.

$\frac{OM}{OP}$ is called the **cosine** of the angle POM, written $\cos POM$.

$\frac{PM}{OM}$ is called the **tangent** of the angle POM, written $\tan POM$.

Values of the sine, cosine, and tangent of angles up to 90° are given in the mathematical tables at the end of the book.

A Table of useful constants will be found on p. 363.

INSTRUCTIONS FOR CARRYING OUT LABORATORY WORK.

General Instructions.—Two Laboratory Note-books are required; in one rough notes of the experiments should be made, and in the other a fair copy of them in ink should be entered.

Before commencing any experiment, make sure that you understand what its object is, and also the construction of the apparatus and instruments employed.

Reasonable care should be exercised in order to avoid damage to apparatus, and to secure fairly accurate results.

In writing up the results, enter the notes in the following order:

(1) The title of the experiment and the date on which it was performed.

(2) Sketches and descriptions of any special apparatus or instruments used.

(3) The object of the experiment.

(4) Dimensions, weights, etc., required for working out the results; from these values calculate any constants required.

(5) Log of the experiment, entered in tabular form where possible, together with any remarks necessary.

(6) Work out the results of the experiment and tabulate them where possible.

(7) Plot any curves required.

(8) Work out any general equations required.

Notes should not be left in the rough form for several days; it is much better to work out the results and enter them directly after the experiments have been performed.

EXERCISES ON CHAP. I.

1. Convert 9 ft. $6\frac{1}{2}$ in. to metres.

2. Convert 2·94 metres to feet and inches.

3. Convert 3 miles 15 chains to kilometres.

4. Convert 53·7 millimetres to inches.

5. A rectangle has sides $4\frac{3}{8}''$ and $2\frac{7}{8}''$. Calculate its area.

6. A triangle, base 8 cms.; perpendicular height, 13·25 cms. Calculate its area.

7. Draw carefully to scale a triangle having sides respectively $4\frac{1}{2}''$, $3\frac{1}{4}''$, and $5\frac{3}{8}''$. Measure its perpendicular height from your drawing and calculate the area of the triangle from this and the length of the base.

8. What is (*a*) the circumference and (*b*) the area of a circle whose diameter is 14 cms.? Take $\pi = \frac{22}{7}$.

9. Calculate the volume of a ball 9″ diam.

10. Draw full size a 5-sided figure *ABCDE* from the following particulars. Take measurements from your drawing and calculate its area.

Sides, $AB = 4''$, $BC = 3''$, $CD = 2''$, $DE = 1\frac{1}{2}''$, $EA = 1''$.
Diagonals, $BD = 3''$, $AD = 1\frac{1}{2}''$.

11. A figure stands on a base *AB* 5″ long. Its heights at intervals of 1″, starting from *A*, are 2″, 4″, $2\frac{1}{2}''$, $3\frac{1}{2}''$, 1″, 5″. Straight lines joining the tops of these ordinates bound the top of the figure. Calculate, using the trapezoidal rule, the area of the figure.

12. Draw at random any figure bounded by a curve at the top, and find its area by applying (*a*) Simpson's rule, (*b*) the ordinary engineering rule.

13. Describe any micrometer screw gauge with which you are acquainted, suitable for measuring to the $\frac{1}{1000}$th of an inch. Sketch and describe carefully the method of graduation and the position of the gauge when set to measure ·374 inch.

14. Describe with sketches the construction and use of external and internal workshop gauges, by means of which the size of a spindle (say 2 inches diameter), and that of a hole into which it fits, may be ensured within specified limits of accuracy. State any advantages due to this system of working.

CHAPTER II.

MATTER, FORCE, WEIGHT.

Definition of terms.—Applied mechanics treats of those laws of force and the effects of force upon matter which apply to works of human art. As science stands at present, it is impossible to state exactly what **force** and **matter** really are, and we are compelled to explain them by reference to some of their properties. Matter is anything which our senses tell us exists. Matter exists in many different forms, and can often be changed from one form to another, but man cannot create it, nor can he annihilate it. Matter always occupies space, and a given piece of matter, occupying a definite space, is called a **body**.

Force may exert push or pull on a body, or may set it in motion, or bring it to rest. The most familiar conception we have of force is obtained from the manner in which our muscles must be exerted when we support a body.

All bodies are measured, as regards the quantity of matter, or **mass**, they contain, by comparison with a standard body. The standard for this country is the quantity of matter contained in a lump of platinum preserved in the Exchequer Office. This quantity of matter is called one **pound**. In countries using the metric system the standard mass is the **gram**, and is the quantity of matter contained in a cubic centimetre of water at the temperature of 4° C.

When two given bodies, of different materials but having the same volume, are found to contain differing quantities of matter, that which contains the greater quantity is said to be more *dense* than the other. The **density** of a material is stated by the quantity of matter, or mass, of a cubic unit of it. Thus, the density of water is about 62·3 pounds per cubic foot; wrought iron has a density of about 480 pounds per cubic foot.

Specific density is the density of a substance when compared with that of a standard, water being taken for this purpose. Thus, the specific density of water being 1, the specific density of wrought iron is 7·7.

The most familiar force we have is **weight**, which all bodies possess. Weight is due to gravitation, which is manifest in the attraction which all bodies have for one another. Gravitational effort is proportional to the product of the masses of the two bodies and inversely proportional to the square of the distance between them. It is very small for bodies of ordinary size, but is perfectly evident when one, or both bodies, possesses a large quantity of matter. Thus, for bodies near the surface of the earth, the gravitational effort is seen by what we call the weight of the body. Weight means the tendency towards the earth's centre possessed by all bodies.

The weight of a body may vary.—Weight, as we have seen above, is proportional to the product of the masses of the earth and of the body. Assuming these to be constant, a given body will always have the same weight at the same place on the earth. Weight, however, is inversely proportional to the square of the distance from the earth's centre to the centre of the body, and therefore any change in this distance will produce a change in the body's weight. Thus, the weight of a given body is slightly less at the top of a mountain than at sea level.

The earth is not perfectly spherical, but is flattened towards the poles. Consequently a body at sea level near the poles will be nearer to the earth's centre than one at sea level near the equator. Therefore a body near the poles has a greater weight than it would possess near the equator.

FIG. 19.—Ordinary balance.

The effect caused by the whirling of the earth on its axis also makes the apparent weight of a body near the poles slightly greater than it would possess near the equator.

Measurement of mass.—Quantities of matter can be measured by comparing their weights, using an ordinary **balance** for this purpose. The balance beam will become horizontal when equal vertical forces acting downwards are

applied to its ends A and B (Fig. 19). These forces are produced by the weights of the bodies placed in the pans, and when the weights, as shown by the beam, are equal, we have equal masses in the pans. Using a standard pound mass in the pan C, we can obtain another pound mass by this means in pan D, and this can be done at any place without variation in the mass measured, as equal masses have equal weights when both are at the same place.

Spring balances (Fig. 20), which measure force applied to them by the extensions of a spring, show the actual weight of bodies placed in their pans. These appliances, therefore, will indicate different readings with the same body placed in the pan at different places on the earth's surface. Thus, it can be shown that a body, the weight of which is 32,088 lbs. at the equator, will have a weight of 32,252 lbs. at the poles. In this country the unit of force in common use is the weight of the standard pound mass, and is called the gravitational unit of force. There is another unit of force, based on the effect produced by it on the motion of a body of unit mass; this unit does not vary, and will be explained later.

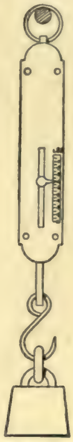


Fig. 20.—Spring balance.

It will be observed from the above numbers that the alteration in the weight of a body by transference from the equator to the poles is too small to affect practical calculations, being about 0·5 per cent. The variation in the value of the gravitational unit of force is, therefore, generally neglected in engineering calculations, although this is no reason why the student should be ignorant of the fact that such alteration exists.

Specific gravity.—Specific gravity is the weight of a given volume of a substance when compared with the weight of an equal volume of water. It is usual in engineering work to measure specific gravities at a temperature of 60° F. The specific gravity of water being 1, the specific gravity of wrought iron would be 7·7, and of lead 11·4. It will be noticed that the number giving the specific gravity of a body will be the same as that giving its specific density. Specific gravity, however, refers to weight, and specific density to quantity of matter.

Some important relations.—The quantity of matter in, or mass of, a given body may be calculated thus:

Let V = its volume in cubic feet,

δ = its specific density.

Then $V \times 62 \cdot 3$ would be its mass if it were water,

and $V \times 62 \cdot 3 \times \delta$ will be its actual mass, in pounds.

The weight of a body may be calculated thus:

Let V = its volume in cubic feet,

ρ = its specific gravity.

Then $V \times 62 \cdot 3$ = its weight if it were water,

and $V \times 62 \cdot 3 \times \rho$ = its actual weight, in lbs.

The specific gravity of a body can be found roughly by first weighing it and then measuring it and calculating its volume from the dimensions.

Suppose V = its volume in cubic feet.

W = its weight in lbs.

Then $W = V \times 62 \cdot 3 \times \rho$,

or
$$\rho = \frac{W}{62 \cdot 3 \,.\, V}.$$

EXAMPLE.—A piece of flat bar iron, 12″ long, section $2'' \times \frac{1}{2}''$ is found to weigh 3·38 lbs. Find its specific gravity.

Here
$$V = \frac{12 \times 2 \times \frac{1}{2}}{1728} \text{ cubic feet.}$$
$$\therefore \rho = \frac{3 \cdot 38 \times 1728}{12 \times 2 \times \frac{1}{2} \times 62 \cdot 3}$$
$$= \frac{3 \cdot 38 \times 144}{62 \cdot 3} = \underline{7 \cdot 8.}$$

Practical Applications.—An important part of the routine work of the engineer is the calculation from its drawings of the weights of various parts of a structure or machine. This he does by first calculating the volume of the part either in cubic inches or cubic feet and then multiplying this volume by the weight of the material per cubic inch or per cubic foot. Or, he may proceed, after having obtained the volume in cubic feet, to multiply this by 62·5,* which gives the weight of the part if made of water, 62·5 lbs. being the weight of 1 cub. ft. of water. If this result be now multiplied by the specific gravity of the material, the

* 62·3 more nearly, but 62·5 is near enough for almost all engineering purposes.

result will be the weight of the part. This procedure has to be followed out, especially in finding the weights of castings.

In finding the **weights of plates**, it is advantageous, if much work has to be done, to tabulate for reference the weights of plates one inch thick, of the substances used per square foot superficial area. If the actual area of the plate in square feet be now calculated, this, multiplied by the tabular number and the thickness of the plate in inches, gives the total weight.

For **bar iron** and **rolled sections** of different materials, the weight of each shape and size of section per foot running length is tabulated; the actual length, in feet, of stuff used multiplied by the tabular number will give its weight.

In estimating the **weights of castings**, it is customary to divide up the drawing of the casting into numerous parts, so chosen as to simplify the necessary mensuration work of finding the volume. Fillets, small bosses, etc., are omitted in this cutting up and allowed for afterwards. In structural work where the parts are riveted together or secured by bolts or pins, the heads of rivets, bolts and pins and nuts are allowed for separately.

The following table gives the weights and specific gravities of some common substances:

WEIGHTS AND SPECIFIC GRAVITIES.

Material.	Weight of		Weight of a sheet 1″ thick, 1 sq. foot area.	Specific Gravity.
	One cub. foot.	One cub. inch.		
	lbs.	lb.	lbs.	
Wrought iron,	480	0·28	40	7·7
Steel, - -	490	0·28	41	7·8
Cast iron, -	450	0·26	37½	7·2
Copper, - -	550	0·32	46	8·8
Brass, - -	525	0·30	44	8·6
Gun metal, -	540	0·31	45	8·7
Aluminium, -	165	0·095	14	2·6
Zinc, - -	450	0·26	37½	7·2
Tin, - -	465	0·27	39	7·4
Lead, - -	710	0·41	59	11·4
Fresh water, -	62·5	0·036	—	1·0
Sea water, -	64	0·037	—	1·024

A few examples are appended to show some of the methods adopted.

EXAMPLE 1. A cast-iron pipe 3″ bore, 6 ft. long, metal of body ½″ thick, circular flange at each end 7½″ diam. × ¾″ thick (Fig. 21). Calculate its weight.

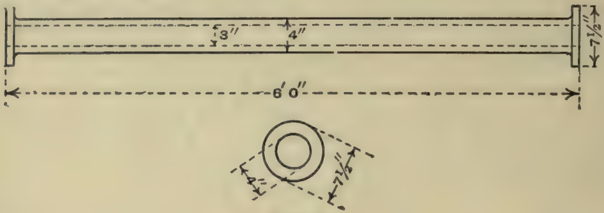


FIG. 21.

Remove the flanges and calculate their volume separately from the volume of the body of the pipe.

$$\begin{aligned}\text{Volume of body of pipe} &= \text{cross sectional area} \times \text{length}\\ &= \pi(2^2 - 1\tfrac{1}{2}^2) \times 72\\ &= 396 \text{ cubic inches.}\\ \text{Volume of each flange} &= \pi(3\tfrac{3}{4}^2 - 2^2) \times \tfrac{3}{4}\\ &= 23{\cdot}7 \text{ cubic inches.}\\ \text{Total volume of metal} &= 396 + 23{\cdot}7 + 23{\cdot}7\\ &= 443{\cdot}4 \text{ cubic inches.}\end{aligned}$$

Now, cast iron weighs 0·26 lb. per cubic inch;

$$\begin{aligned}\therefore \text{ weight of pipe} &= 443{\cdot}4 \times 0{\cdot}26\\ &= \underline{115{\cdot}3} \text{ lbs.}\end{aligned}$$

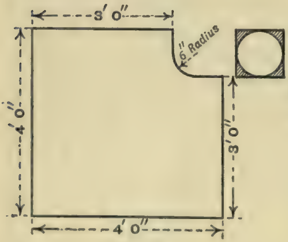


FIG. 22.

EXAMPLE 2. A wrought-iron plate, ½″ thick, originally square, has a piece cut out of the corner as shown (Fig. 22). Calculate its weight.

(i) Area of original square
$= 4 \times 4 = 16$ square feet.

(ii) Area of circle 1 ft. diam.
$= 0{\cdot}7854$ square foot.

(iii) Area of square 1 ft. side
$= 1{\cdot}0$ square foot.

Difference between (iii) and (ii) $= 0{\cdot}2146$ square foot.

This difference is made up of the four shaded pieces, and three of these are cut out together with the circle 1 ft. diam.

$$\text{Area of piece cut out} = 0{\cdot}7854 + (\tfrac{3}{4} \times 0{\cdot}2146)$$
$$= 0{\cdot}9463 \text{ square foot;}$$
$$\therefore \text{ Area of actual plate} = 16 - 0{\cdot}9463$$
$$= 15{\cdot}054 \text{ square feet.}$$

Now, wrought iron weighs 40 lbs. per superficial foot if the plate is 1″ thick;

$$\therefore \text{ weight of plate} = 15{\cdot}054 \times 40 \times \tfrac{1}{2}$$
$$= \underline{301} \text{ lbs.}$$

EXAMPLE 3. A copper float ball 14″ diam. is made of metal $\frac{1}{16}$″ thick. Calculate its weight.

In this case, as the metal is thin compared with the diameter of the ball, we may find the volume of metal near enough for practical purposes by multiplying the spherical area by the thickness of metal. To be quite exact we should have to calculate the volume of metal by taking the volume of a sphere $13\frac{7}{8}$″ diam. from the volume of a sphere 14″ diam.

$$\text{Spherical area} = 4\pi r^2$$
$$= (4 \times \tfrac{22}{7} \times 7 \times 7) \text{ square inches.}$$

Approximate volume of metal $= (4 \times 22 \times 7 \times \frac{1}{16})$ cubic inches, and as copper weighs 0·32 lb. per cubic inch,

$$\text{weight of ball} = 4 \times 22 \times 7 \times \tfrac{1}{16} \times 0{\cdot}32$$
$$= \underline{12{\cdot}3} \text{ lbs.}$$

PRACTICAL WORK.

EXPT. 1.—Using an ordinary caliper and steel rule, find the dimensions of the pieces of bar metal provided.

EXPT. 2.—Find the breadths and thicknesses, or the diameters, of the same pieces of material as in Expt. 1, using a micrometer caliper. First find the zero error, if any, of the instrument and correct your readings accordingly.

EXPT. 3.—Repeat Expt. 2, using a vernier caliper.

EXPT. 4.—Weigh the same pieces of materials; calculate their volumes from the dimensions obtained in Expts. 2 and 3; then calculate the specific gravities of the materials.

EXERCISES ON CHAP. II.

1. Find the weight of a piece of flat bar iron 24″ long, section $2'' \times \frac{1}{2}''$.

2. Find the weight of a wrought-iron bar, 13·16″ long, section $1{\cdot}5'' \times 0{\cdot}4''$.

3. A piece of angle iron, section as in Fig. 23, is 30 ft. long. Calculate its weight, neglecting rounded corners.

4. A circular brass plate, 2 ft. diam., is $\frac{1}{8}''$ thick. Calculate its weight.

5. A hollow cylinder of wrought iron is 4″ inside diameter, $4\frac{3}{8}''$ outside diameter and 10 feet long. Calculate its weight.

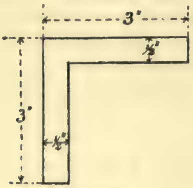


FIG. 23.

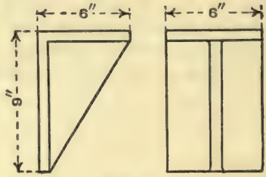


FIG. 24.

6. A solid pyramid of lead, square base 4″ edge, 8″ high. Find its weight.

7. A copper cone has the following dimensions, 10″ diam. of base, 12″ high, metal 0·05″ thick, no bottom. Find its weight.

8. A hollow conical vessel, 6″ inside diam. at top, 9″ deep inside, is full of water. Calculate the weight of water.

9. A cast-iron bracket, metal all over $\frac{1}{2}''$ thick, has dimensions as shown in Fig. 24. Find its weight.

10. Calculate what weight of sheet lead, 0·1″ thick, will be required for lining a timber tank, the internal dimensions of which are 6 ft. long, 4 ft. broad, 3 ft. deep.

11. A solid ball of cast iron is to have a weight of 90 lbs. Calculate its diameter.

CHAPTER III.

FORCES ACTING AT A POINT.

Representation of a force.—To describe completely a force acting on a body we require to state the following particulars, (*a*) its magnitude, (*b*) its point of application, (*c*) its direction, (*d*) its sense, *i.e.* to state whether the force is pushing or pulling at the point of application.

A straight line may be employed to represent a given force, for it may be drawn of any length and so represent to a given scale the magnitude of the force, the end of the line shows the point of application, the direction of the line gives the direction, and an arrow point on the line will indicate the sense of the force.

Thus, a pull of 5 lbs. weight, acting at a point O in a body (Fig. 25) at 45° to the horizontal, would be completely represented by the line OA and arrow point as shown.

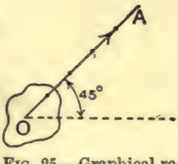


FIG. 25.—Graphical representation of a force.

We often speak, as above, of a force "acting at a point." Of course this must not be understood literally, for no material is so very hard that it would not be penetrated by even a very small force applied to it at a mathematical point. What is meant by the statement, is that the force may be imagined to be concentrated at the point in question without thereby affecting the condition of the body as a whole.

Forces acting in the same straight line.—A body is said to be in equilibrium if the forces acting on it balance one

another. Thus, if two equal opposite pulls P, P (Fig. 26) be applied at a point O in a body both in the same straight line, they will evidently balance one another and the body will be in equilibrium.

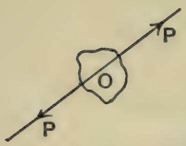


FIG. 26.

FIG. 27.—Tie bar under pull.

A tie bar subjected to two equal opposite pulls of 2000 lbs. weight each (Fig. 27) acting in the direction of its length will be in equilibrium. If one only of these pulls were reduced or increased even by very little the bar would move. This bar could not possibly be imagined pulled with a force of 2000 lbs. weight at one end only and yet to remain at rest, any more than a pull of 5 lbs. weight could be applied by the hand to one end of a piece of string unless the other end were pulled with a force of 5 lbs. weight in the opposite direction.

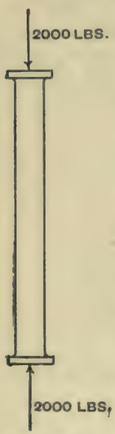


FIG. 28.—Column under push.

In the same way, if a column or strut (Fig. 28) be pushed at one end and remain in equilibrium, there must be an equal opposite push acting in the same straight line at the other end.

It is impossible for a single force to act alone. To every force there must be an equal opposite force, or what is exactly equivalent to an equal opposite force. This equal opposite force is often called a **reaction**.

If several forces in the same straight line act at a point, the point will be in equilibrium if the sum of the forces of one sense is equal to the sum of those of opposite

sense. If these sums are not equal, then a force is required to produce equilibrium; its magnitude will be equal to the difference of these sums, and the force must have the same sense as the smaller sum. In the given case (Fig. 29) these sums are

$$2+3+5=10 \text{ lbs. weight, sense from } A \text{ to } B;$$
$$8+1=9 \text{ lbs. weight, sense from } B \text{ to } A.$$

And a force of $(10-9)=1$ lb. weight of a sense B to A will produce equilibrium.

Two intersecting forces.—If two forces are given acting at a point, their lines of direction intersecting, a single force may

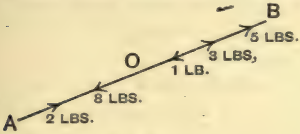


FIG. 29.

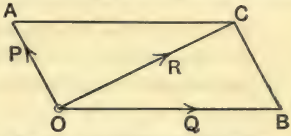


FIG. 30—Parallelogram of forces.

be found which would have the same effect, if applied alone, as the two forces together have. This single force is called the **Resultant** of the given forces, and may be found by the following construction.

Let P and Q be two pulls applied to a nail at O (Fig. 30); their joint tendency will be to carry the nail upwards to the right. Set off OA, to some suitable scale, equal to P, and OB, to the same scale, equal to Q. Complete the parallelogram $OACB$ and draw its diagonal OC. Measure off OC to the same scale of force and this will give the magnitude of R. If a pull R be now applied to the nail along the line OC, it will have the same effect as P and Q together have. This method is called the **Parallelogram of Forces**; P and Q are called **Components** of R.

Let us now remove the forces P and Q, and instead apply R alone to the nail.

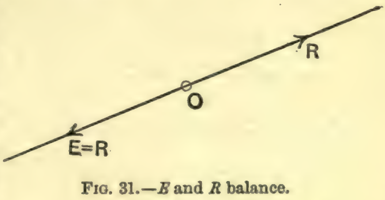


FIG. 31.—E and R balance.

We may balance R by applying another pull E, equal and opposite to R and in the same straight line (Fig. 31), and

if we do so, there will now be no tendency to disturb the nail. But since R is exactly equivalent to P and Q together, we may replace R again by P and Q (Fig. 32), thereby giving three pulls on the nail which will balance one another without any tendency to disturb the position of the nail in the board. E is generally called the **Equilibrant**, meaning the force required to keep the other forces in equilibrium.

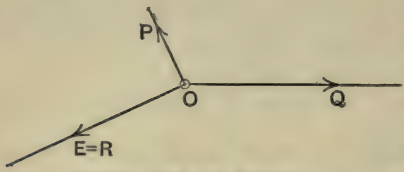


FIG. 32.—P, Q, and E balance.

Experimental verification.—The most satisfactory way for the beginner to verify the truth of the above principle is for him to make an experiment illustrating it.

EXPT. 5.—Procure three wooden pulleys about 2″ or 3″ diameter, having their edges grooved to receive string, and with holes through their centres so that they will run freely on bradawls. Pin a sheet of paper to a vertical board and mount two pulleys at A and B by means of bradawls (Fig. 33). Tie two strings to a small split key ring, pass a bradawl through the ring into the board at O, and lead the strings over the pulleys at A and B. Fasten any bodies of known weights W_1, W_2, to the ends of the strings.

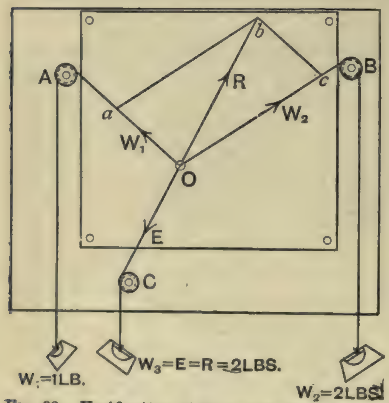


FIG. 33.—Verification of the parallelogram of forces by experiment.

We have now two forces W_1 and W_2 acting on the bradawl at O along the strings OA and OB. Mark the directions

of these strings carefully on the paper and remove them to construct the parallelogram of forces *Oabc*. R, the resultant of W_1 and W_2 acting at O, will thus be found. Produce bO and replace the strings. By means of another pulley and bradawl at C, arrange a third string tied to the ring to lie along bO produced. Tie a body of weight W_3 to the end of this string, W_3 being equal to R in magnitude. This will give a third force $E = W_3 = R$ acting at O. If what has been done is correct, the three forces W_1, W_2, and E should now balance one another, and if they do, we should be able to remove the bradawl at O without the ring changing its position. Try if this is so.

You will probably notice that after the bradawl is removed from O the ring can be made to take up positions some little distance from O. This is due to the stiffness of the strings and the friction of the pulleys on the bradawls, and these causes prevent the perfect success of the experiment when regarded as a means of testing the truth of the parallelogram of forces.

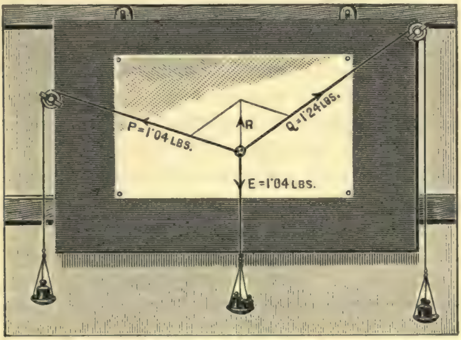


FIG. 34.—Students' apparatus for experiments on forces.

Fig. 34 is reproduced from a photograph of an apparatus arranged for students' use. The pulleys used are of aluminium, with pivot bearings, and may be clamped to any part of the edge of the board. The apparatus may be used also for testing several of the following principles concerning forces. The student should not forget in using it, that scale pans possess weight, and that such weights should be added to those put into

them to get the total pull in the cords. Several experiments should be made by the student, using different weights and pulley positions each time.

Notice that before attempting to apply the parallelogram of forces to find the resultant of two forces, both given forces must be made to act either towards or from the point of application. Thus, given P' pushing and Q pulling at O (Fig. 35),

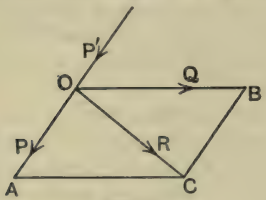


FIG. 35.

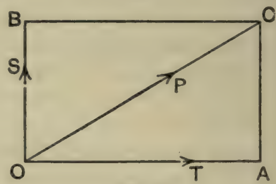


FIG. 36.

the tendency will be to carry O downwards to the right. Substitute $P=P'$, pulling at O, for P' and take OA to scale to represent P, also OB to represent Q. The parallelogram $OACB$ may now be drawn, giving R, the resultant of P' and Q.

Substitution of components for resultant.—Since the resultant produces exactly the same effect as its components, we may use either resultant or components in working out a question. It is often convenient to substitute for a given force its components along two given lines, which are usually taken perpendicular to one another. Thus, if we are given P acting at O, and it would be more convenient instead of P to have forces in OA and OB (Fig. 36), then, by setting off $OC=P$ and completing the parallelogram $OBCA$, two forces $S=OB$ and $T=OA$ are found, which if substituted for P, would have the same effect on O.

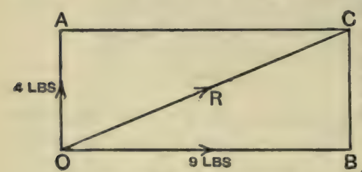


FIG. 37.

EXAMPLE 1. Two forces of 4 lbs. and 9 lbs. weight pull a nail in directions at right angles to one another. Find their resultant.

Draw OA and OB to scale (Fig. 37) to represent the given forces. Complete the parallelogram; its diagonal OC will give R.

R may also be found by calculation. Thus

$$OC^2 = OB^2 + BC^2$$
$$= OB^2 + OA^2 = 9^2 + 4^2;$$
$$\therefore\ OC = \sqrt{81+16} = \sqrt{97},$$

or

$$R = \underline{9 \cdot 8} \text{ lbs. weight.}$$

In solving questions on forces by construction, take care to use a large scale. By doing so you will secure much more accurate results. Thus, in the above case a scale of $\frac{1}{2}''$ to a lb. weight would be suitable.

EXAMPLE 2. A horse exerts a pull of 200 lbs. weight on a tram car at 30° to the direction of the rails as seen in plan. Find the force urging the tram forward, and that tending to pull it off the rails.

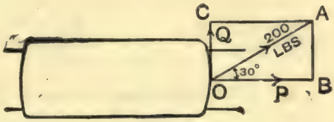


FIG. 38.

Set off OA (Fig. 38), to scale to represent 200 lbs. weight acting at 30° to OB, the direction of the rails. Draw OC perpendicular to OB. Complete the parallelogram $OBAC$; then OB, to scale, gives the force P, tending to urge the tram along the rails, and OC gives Q, the force tending to pull it off.

By calculation, since in the triangle OAB, the angle AOB is 30° and ABO is 90°, the sides have the following proportion:

$$BA : AO : OB = 1 : 2 : \sqrt{3},$$

or

$$OC : AO : OB = 1 : 2 : \sqrt{3};$$
$$\therefore\ Q : 200 : P = 1 : 2 : \sqrt{3};$$
$$\therefore\ Q = \tfrac{1}{2} \times 200 = \underline{100} \text{ lbs. weight}$$

and

$$P = 100 \times \sqrt{3} = \underline{173 \cdot 2} \text{ lbs. weight.}$$

EXAMPLE 3. A load of 14 lbs. weight is hung by a cord 10 ft. long from an overhead beam, the arrangement being that of a pendulum. Find what horizontal force applied to the load will keep it 2 ft. from the vertical through the point of support.

Draw the figure to scale, as shown at OAB (Fig. 39). Let P be the required force; then the three forces acting on the load, P, W, and the pull of the cord, T, keep it balanced, so that T must be equal and opposite to the resultant of P and W. Set off AC to

a suitable scale of force to represent 14 lbs. weight, and complete the parallelogram $ACDE$. T will be represented by AD, and P by AE, which being measured gives $P = 2{\cdot}86$ lbs. weight.

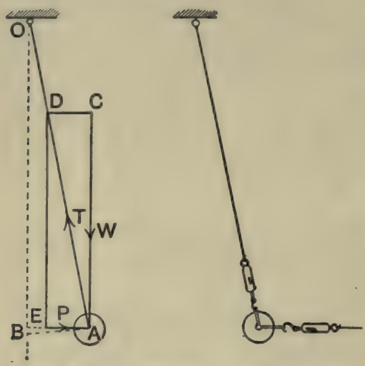


Fig. 39.—Forces acting on a pendulum.

Expt. 6.—Test your construction by actually arranging a cord and weight. Put a spring balance in the cord and apply P by means of another spring balance (Fig. 39). P and T can now be measured directly. If readings of P are taken when W is at different distances from the vertical, a curve may be plotted on squared paper, which will show the values of P for all positions of W. The results for intervals of $\frac{1}{2}$ ft. are given in the table, the curve shown (Fig. 40) being plotted by using the values of AB for abscissae or horizontal distances, and the corresponding values of P for ordinates or vertical distances.

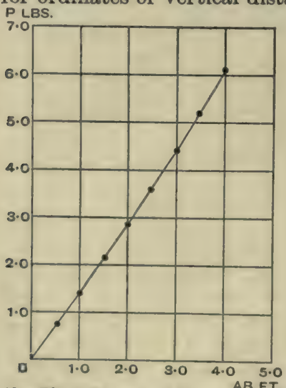


Fig. 40.—Plotted curve of force required to keep the pendulum out of the vertical.

AB	P
ft.	lbs.
0·5	0·7
1·0	1·41
1·5	2·12
2·0	2·86
2·5	3·6
3·0	4·4
3·5	5·2
4·0	6·1

Triangle of forces.—Let us now consider what conditions must be fulfilled in order that three forces, all in the same plane, acting at the same point, may balance one another.

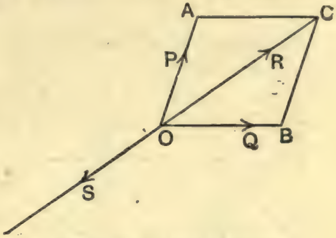


FIG. 41.—P, Q and S balance.

It has been seen already that if three forces, such as P, Q, and S, act at a point O (Fig. 41), one of them must be equal and opposite to the resultant of the other two. Find, by the parallelogram of forces, R, the resultant of P and Q, then S must be equal and opposite to R.

This proportion will evidently be true :

$$Q : P : R = OB : OA : OC.$$

Now $$R = S \text{ and } OA = BC;$$

$$\therefore\ Q : P : S = OB : BC : CO,$$

that is, the three given forces are proportional to the sides of the triangle OBC.

The equilibrium of P, Q, and S, may therefore be tested by seeing whether a triangle can be drawn with sides proportional to these forces. Thus, in Fig. 42, Ob is parallel and proportional to Q, bc to P, and cO to S. If the lines so drawn give a closed triangle, then the given forces will be in equilibrium. This triangle Obc is called the **triangle of forces** for the given forces P, Q, and S.

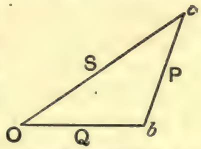


FIG. 42.—Triangle of forces for P, Q and S.

Notice in drawing the triangle of forces that the sides *must be drawn in the proper order* to represent the senses of the forces. Thus, Ob is drawn to the right to indicate that Q acts to the right, bc upwards as P acts upwards, and cO down to the left to indicate the sense of S. So long as attention is paid to this the triangle of forces may be begun with a line parallel and proportional to any one of the given forces. The student should test this fact for himself by actual construction.

EXPT. 7.—Use apparatus as in Figs. 33, 34. Find the equilibrant and the resultant for two given forces by applying the triangle of forces and confirm the results by trial.

Resultant of several forces.—By means of the parallelogram of forces, the resultant may be found of any number of forces acting at a point, all being in one plane. Thus given P, Q, S, T, acting at O (Fig. 43), in the plane of the paper; to find their resultant R, first find the resultant of any pair, such as P and S, by the parallelogram. Call this R_1. Then, take the other pair, Q and T, and find R_2, their resultant. Finally, find the resultant R, of R_1 and R_2. This will clearly be the resultant of the given forces.

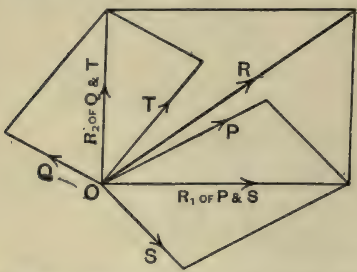


FIG. 43.—Resultant of several forces by the parallelogram of forces.

If we apply E, equal and opposite to R, R would be balanced at O, and therefore, of course, the given forces would also be balanced by E. So that by this method we may also find the equilibrant of a given number of forces.

EXPT. 8.—Try one or two examples of this method on your experimental board. Use five or six cords attached to a ring and led over pulleys, their ends being provided with scale pans. Have different weights in the pans and fix the ring by means of a bradawl at a given position on the board. Find the resultant of these forces, and by means of another cord and pulley apply a force to the ring equal and opposite to R. Now remove the bradawl and see if the ring remains in equilibrium in its original position.

Experimental models of simple structures.—A model **derrick crane** is shown in Fig. 44 having arrangements provided for experimentally finding the forces in its parts. It consists of an upright post firmly secured to a base board, an inclined jib with a compression spring balance fitted to it, and a cord

to serve as a tie and having an ordinary spring balance to indicate the pull. The jib can be fitted with a pulley at the end for the cord supporting the weight to run over, or it may be used without the pulley, the supporting cord being then simply secured to its upper end. The method of using the model is as follows.

EXPT. 9.—Place a known weight in the scale pan and then measure the height of the post from the junction of the jib to the junction of the tie, the length of the jib and the length of the tie. From these dimensions make an outline diagram of the crane and show the vertical line of the weight. This is shown at ABC (Fig. 45). If W is simply hung from A, then by the parallelogram of forces $ADEF$, $AD = W$ being first set off, the pull T of the tie and the thrust Q of the jib will be found. Or, the triangle of forces, as at abc (Fig. 46), may be used. If the pulley is fitted to the jib end, and the supporting cord passed

FIG. 44.—Experimental model of a derrick crane.

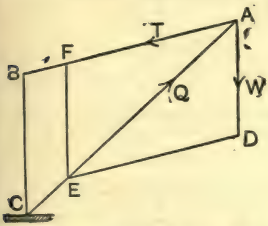


FIG. 45.—Solution of the derrick crane by the parallelogram of forces.

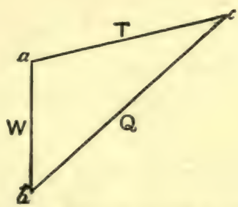


FIG. 46.—Solution of the derrick crane by the triangle of forces.

over it and led to a point D on the post (Fig. 47), this will give an additional force W' which will be nearly equal to W, the

stiffness of the cord and the friction of the pulley making it slightly different. Taking $W' = W$, find their resultant R by means of the parallelogram of forces $Aacb$, in which $Aa = W$ and $Ab = W'$. There are now only three forces acting at A, viz. R, Q and T, and both the latter forces are determined from the parallelogram of forces $Acde$; $Q = dA$ and $T = Ae$. In either case, after the force diagram is drawn and Q and T determined thereby, the spring balances should be read and compared with the scaled results. Generally they will read more than those found by construction, because the weights of the parts have been neglected and these will cause forces in the tie and jib when W is removed from the pan. Take W away and read the balances again; these readings, subtracted from the previous readings, should give results agreeing very closely with the scaled ones.

EXPT. 10.—A **wall crane** (Fig. 48) can easily be arranged, using the jib from the derrick crane. AB is the jib, arranged

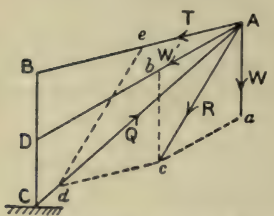


FIG. 47.—Derrick crane having a pulley at A.

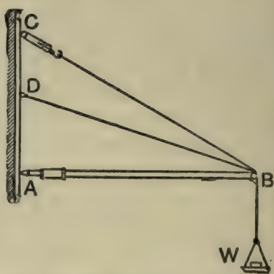


FIG. 48.—Experimental model of a wall crane.

horizontally, BC an inclined tie, fitted with a spring balance. The cord suspending W may either be secured to B or passed over a pulley there and led to any point D between A and C. The method of using this model and the construction for the results are precisely similar to those for the derrick crane.

Inclined Plane.—A model of a small carriage resting on an inclined plane is shown in Fig. 49. The forces required to

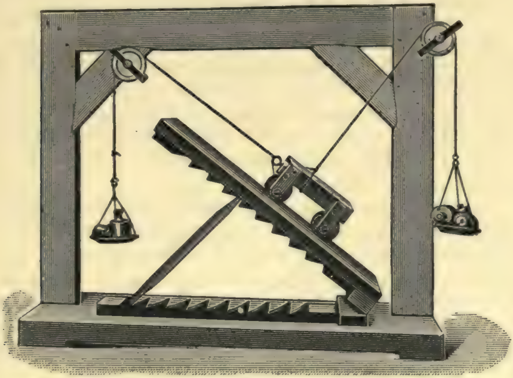


FIG. 49.—Model of an inclined plane.

support it, if the plane were removed, are shown experimentally by the pulls of the two cords, one arranged parallel to the plane and the other at 90° to it. These, with the weight of the carriage, give three forces acting on it and keeping it in equilibrium.

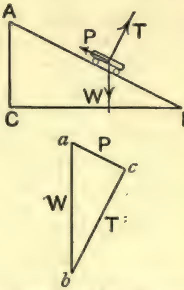


FIG. 50.—Equilibrium of a carriage on an inclined plane.

EXPT. 11.—Arrange the apparatus as shown. Measure the height and length of the plane and make an outline diagram as shown at ABC (Fig. 50). Weigh the carriage to find W and find P and T by construction, using the triangle of forces abc. Check these results by comparison with the weights in the scale pans. Experiments should be made and force diagrams drawn when P is acting parallel to the base, and also when P is applied at any angle to the plane.

EXERCISES ON CHAP. III.

1. Represent graphically a pull of 10 lbs. weight acting at a point, its direction being N.E. Scale $\frac{1}{4}''$ to a lb. weight.

2. Represent graphically two pulls acting at a point, one of 5 lbs. weight, direction S.W.; one of 8 lbs. weight, direction E. Find their resultant. Scale $\frac{1}{2}''$ to a lb. weight.

3. Represent graphically a push of 7 lbs. weight, acting at a point, direction N.; also a pull of 4 lbs. weight acting at the same point, direction S.E. Find their resultant. Scale $\frac{1}{2}''$ to a lb. weight.

4. A pull of 25 lbs. weight and a push of 54 lbs. weight act at a point along the same straight line in opposition to one another. Represent them graphically, and find their resultant.

5. Draw a horizontal line, and mark a point O in it near its centre. Pulls of 2 lbs., 5 lbs., and 9 lbs. weight act at O in the right-hand portion of the line, and pulls of 4 lbs. and 6 lbs. weight together with pushes of 3 lbs. and 12 lbs. weight in the left-hand portion. Find the equilibrant.

6. Two pulls of 6 lbs. and 10 lbs. weight act on a point (a) at 90°, (b) at 120°, (c) at 60°. Find their resultant in each case by construction.

7. A push of 20 lbs. and a pull of 30 lbs. weight act at the same point, their lines making 40° with each other. Find their resultant.

8. The resultant of two forces whose lines are perpendicular to one another is 15 lbs. weight. One is a force of 4 lbs. weight. Find the other force.

9. A force of 100 lbs. weight, acting in a horizontal line has to be balanced by two forces, one of 50 lbs. and the other of 120 lbs. weight. Show their lines of action.

10. Three cords are attached to a ring; one cord carries a weight of 10 lbs. and hangs vertical. The other cords are attached to an overhead beam and are inclined one at 45° and one at 60° to the vertical. Find the pull in each.

11. An overhead pulley has a chain passing over it from a winch, and a load of 5 cwts. is being hoisted. The chain carrying the load hangs vertical and the chain leading to the winch makes 30° with the vertical. Suppose the force in each part of the chain to be 5 cwts., and find the resultant force on the pulley.

12. Prove by diagrams and by experiment that the force required to balance two given equal and opposing forces becomes smaller as the given forces approach, being finally nearly in the same straight line.

13. A barge is pulled along the centre of a canal 60 ft. wide by a horse on the tow-path whose centre is 4 ft. from the bank. The horse pulls the rope, which is 80 ft. long, with a force of 120 lbs. weight. Find, by construction, the force urging the barge along the canal and the force urging it towards the bank.

14. A horse draws a load up an incline of 1 in 20. The traces are inclined at 30° to the horizontal and the pull of the horse on them is 180 lbs. weight. Find, by construction, the backward pull on the horse taken parallel to the incline and the downward pull on the horse taken at 90° to the incline.

15. A man pulls a nail by means of a string in a direction at 30° to the board. If he exerts a force of 20 lbs. weight, calculate the force tending to draw the nail and that tending to bend it.

16. AB and AC are two scaffold poles in the same vertical plane, lashed together at their tops. AB is 20 feet and AC 15 feet long. The distance BC between their feet is 15 feet. Find by construction the push in each pole when a load of 1 ton is hung from the top.

17. The jib of a model derrick crane is 47″ long, the tie 38″, and the post 31″. Find, by construction, the push in the jib and the pull in the tie when a load of 4·7 lbs. weight is simply hung from the end of the jib.

18. A crane jib measures 19 ft., the tie $17\frac{1}{2}$ ft., and the post 9 ft. A load of 50 cwts. is attached to a chain which passes over a single pulley at the top of the jib, then along the tie. Find the push in the jib and the pull in the tie by construction.

19. Answer Question 18 supposing the chain, after leaving the pulley at the top of the jib, to pass along the jib.

20. A is a hinge fixed to a vertical wall 6 ft. vertically over another, B. A triangular frame ABC, $AC=8$ ft., $BC=10$ ft., is attached to A and B, the arrangement forming a wall crane. A load of $\frac{1}{2}$ ton is attached to a chain which passes over a pulley at C, then along CA to a winding arrangement on the other side of the wall. Find, by construction, the forces in AC and BC, indicating whether they are push or pull.

21. In Question 20, turn the frame upside down and answer the same.

22. A carriage mounted on frictionless wheels rests on a plane inclined at 25° to the horizontal. If the carriage weighs 10 lbs., find, by construction, the force required to keep it in equilibrium, (a) when the cord is horizontal, (b) when parallel to the plane, (c) when at an angle of 30° to the plane.

CHAPTER IV.

MOMENTS. PARALLEL FORCES. CENTRE OF GRAVITY.

Moments.—The **moment of a force** means the tendency of the force to turn the body on which it acts about a given axis. The moment of a force is measured by the product of the magnitude of the force and the length of a line drawn from the axis perpendicular to the line of action of the force.

Suppose a rod AB (Fig. 51) to be suspended by means of a bradawl pushed through a hole at A into the experimental board, so that it hangs vertically. Attach a cord to the rod at C, and lead it over a pulley at D, so that the portion CD is horizontal. If a weight W_1 be attached to the end of the cord, this will give a horizontal pull, $P = W_1$, to the rod at C. The effect will be to turn the rod in the same direction as the hands of a clock. By doubling or trebling the load W_1, the tendency to rotate the rod is doubled or trebled. By increasing the distance AC, the turning tendency is increased in the same proportion, so that the turning tendency, or moment of the force P, is proportional jointly to the magnitude of P, and the perpendicular distance AC from the axis at A to the line of action of P.

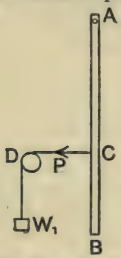


FIG. 51.—Moment of a force.

Notice it is not merely the distance from the axis to the point of application of P that is taken. For if this were so it is easily seen that the calculated moment of P about A would remain the same no matter in what direction P is applied, provided it

always acts at C, whereas, actually, inclining the line of P's action diminishes the turning tendency, until finally if W_1 be hung direct from C so that P is vertical, there will be no tendency whatever to turn the rod. Hence in calculating moments the perpendicular to the line of action of the force must always be taken.

Now suppose a weight $W_1=5$ lbs. hung to the cord, and that AC is 10″ (Fig. 52). The moment of P will be measured by taking the product of 5 and 10; thus the moment of P about $A=5\times 10$ lb.-inch units. In giving the numerical value of a moment, the units of force and distance employed must always

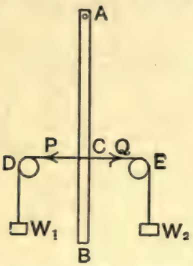


FIG. 52.—Two equal forces, giving equal opposing moments.

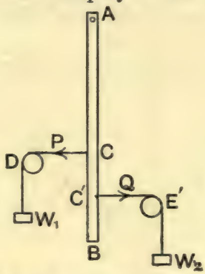


FIG. 53.—Two unequal forces, giving equal opposing moments.

be stated. Thus, ton-foot, cwt.-inch, gram-centimetre, are units of moment. The moment of P will be clockwise. The rod AB may be balanced against rotation if another weight $W_2=W_1$ be applied as shown (Fig. 52), so as to produce a force Q equal to P pulling horizontally at C in the opposite sense to P. Thus, moment of $Q=5\times 10=50$ lb.-inch units, and will tend to turn AB in the opposite direction to the hands of a clock, that is, anticlockwise.

It will be found also, by trial, that the rod will be balanced if the weight W_2 be altered, say diminished, provided at the same time the distance from A at which Q is applied be altered, in the present case increased, viz. AC to AC' (Fig. 53). It will be found in all these cases that $Q\times AC'$ must always amount to 50 lb.-inch units. That is, for AB to be in equilibrium:

Clockwise moment of $P=$ anticlockwise moment of Q.

If the direction of P be altered by raising the left-hand pulley to D' (Fig. 54), the moment of P is now $P \times AM$, clockwise. The rod will be balanced by W_2, provided that matters are so arranged that the product $Q \times AN$, which measures the anti-clockwise moment of Q, is equal to $P \times AM$.

The result may be stated thus : **Two forces which act on a body free to rotate, having equal moments of opposite sign about the axis of rotation, will balance the body.**

EXPT. 12.—Arrange apparatus as shown in Figs. 52, 53. Apply any given load W_1 and calculate the value of W_2 which will produce equilibrium. Apply W_2 and test if the rod will remain vertical. Repeat the Expt. using different forces and distances.

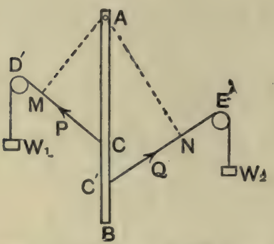


FIG. 54.—Two inclined forces, having equal opposing moments.

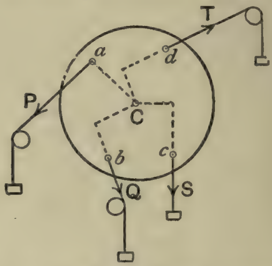


FIG. 55.—Disc in equilibrium under the action of several forces.

Principle of Moments.—EXPT. 13.—By means of a screw at C, mount a circular wooden disc on the vertical experimental board (Fig. 55). Apply any number of forces, such as P, Q, S, T by means of cords attached to the disc at a, b, c, d, led over pulleys and having weights at their ends. Let the disc find its position of equilibrium. Calculate the moment of each force about C by multiplying the magnitude of the force by the length of the perpendicular from C to its line of action, producing this last if necessary. Arrange these moments in two columns, one for clockwise, one for anticlockwise moments. Take the sum of each column, and we should expect to find that these are equal, for, **if the disc is in equilibrium, the total clockwise turning tendency must be equal to the total anticlockwise turning tendency, in order that rotation may not take place.** This principle may be used to solve a great many problems.

EXAMPLE 1. A beam 12 feet long, supported at its ends, carries a load of 2 tons, 4 ft. from one end. Find the reactions of its supports. Neglect meanwhile the weight of the beam itself.

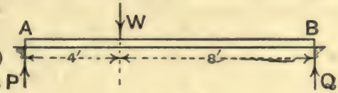


FIG. 56.

Let AB be the beam (Fig. 56) and P and Q the reactions of its supports. Imagine the beam to be free to rotate about B, and take moments about B of the forces acting on it. Find P thus:

Clockwise moment of P about $B = P \times 12$ ton-foot units.

Anticlockwise moment of W about $B = W \times 8 = 2 \times 8 = 16$ ton-foot units.

Q has no moment about B, as its line of action passes through B, and therefore the perpendicular from B to the line of action has no length.

Now the clockwise moment must equal the anticlockwise moment.

$$\therefore \quad P \times 12 = W \times 8,$$
$$12P = 16, \quad P = 1\tfrac{1}{3} = \underline{1{\cdot}33} \text{ tons.}$$

In the same way, take moments about A in order to find Q; then

Anticlockwise moment of $Q = Q \times 12$ ton-foot units.

Clockwise moment of $W = W \times 4 = 2 \times 4 = 8$ ton-foot units.

Anticlockwise moment = clockwise moment.

$$\therefore \quad Q \times 12 = W \times 4,$$
$$12Q = 8, \quad Q = \tfrac{2}{3} = \underline{0{\cdot}66} \text{ ton.}$$

EXAMPLE 2. A beam 20 feet long, supported at its ends, carries loads at intervals as shown (Fig. 57). Find the reactions of the supports, neglecting meanwhile the weight of the beam.

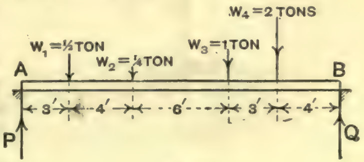


FIG. 57.

Taking moments about B.

Clockwise moment: $P \times 20$ ton-foot units.

Anticlockwise moments:

$$W_1 \times 17 = \tfrac{1}{2} \times 17 = 8{\cdot}5 \text{ ton-foot units.}$$
$$W_2 \times 13 = \tfrac{1}{4} \times 13 = 3{\cdot}25 \quad ,, \quad ,,$$
$$W_3 \times 7 = 1 \times 7 = 7{\cdot}0 \quad ,, \quad ,,$$
$$W_4 \times 4 = 2 \times 4 = 8{\cdot}0 \quad ,, \quad ,,$$

Total anticlockwise moment $= \underline{26{\cdot}75}$ ton-foot units.

Clockwise moments = anticlockwise moments.

$$\therefore \quad P \times 20 = 26{\cdot}75, \quad P = \underline{1{\cdot}3375} \text{ tons.}$$

Taking moments about A,

Anticlockwise moment $= Q \times 20$ ton-foot units.

Clockwise moments:

$$W_1 \times 3 = \tfrac{1}{2} \times 3 = 1\cdot5 \text{ ton-foot units.}$$
$$W_2 \times 7 = \tfrac{1}{4} \times 7 = 1\cdot75 \quad ,, \quad ,,$$
$$W_3 \times 13 = 1 \times 13 = 13\cdot0 \quad ,, \quad ,,$$
$$W_4 \times 16 = 2 \times 16 = \underline{32\cdot0} \quad ,, \quad ,,$$

$$\text{Total clockwise moment} = \underline{\underline{48\cdot25}} \text{ ton-foot units.}$$

$$\text{Anticlockwise moments} = \text{clockwise moments.}$$
$$\therefore \quad Q \times 20 = 48\cdot25,$$
$$Q = \underline{\underline{2\cdot4125}} \text{ tons.}$$

Notice in these questions, that a system of forces exists in each case the lines of which are parallel to one another, and that by the above solutions, the sum of the downward forces

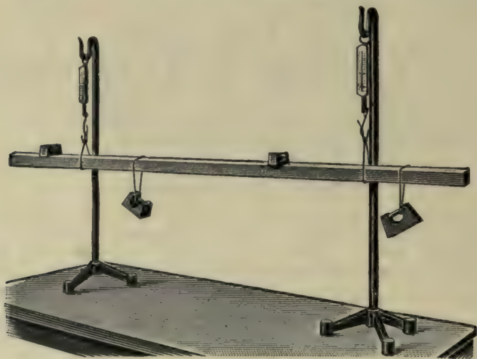


Fig. 58.—Apparatus for determining the reactions of the supports of a beam.

equals the sum of the upward forces. Thus, in Example 1, the downward force is 2 tons and the two upward reactions have a sum of $1\frac{1}{3} + \frac{2}{3} = 2$ tons.

In Example 2, the downward forces give

$$\tfrac{1}{2} + \tfrac{1}{4} + 1 + 2 = 3\tfrac{3}{4} = 3\cdot75 \text{ tons.}$$

The upward reactions give

$$1\cdot3375 + 2\cdot4125 = 3\cdot75 \text{ tons.}$$

Or, **in a given system of parallel forces the sum of the forces of one sense must in every case be equal to the sum of the forces of the**

other sense, otherwise the body will be displaced as a whole in the same sense as the greater sum. Thus, if in the beam examples the sum of the reactions falls below the sum of the loads, the beam will move downwards and *vice versa.* Fig. 58 shows an apparatus for experimentally finding the reactions of a loaded beam.

EXPT. 14.—Arrange apparatus as in Fig. 58. Hang the beam alone from the spring balances and note the pulls; let these be P_1 and Q_1. Place two or three loads on the beam and again observe the pulls; let these be P_2 and Q_2. The spring balance pulls produced by the loads alone will be $(P_2 - P_1)$ and $(Q_2 - Q_1)$. Calculate the reactions which would be produced by the loads alone, neglecting the weight of the beam, and compare these with the observed pulls.

EXPT. 15.—Hang the same rod AB as before vertically in front of the experimental board using this time a long piece of cord for the suspension (Fig. 59). Attach cords at C and D and apply horizontal forces P and Q by means of pulleys and weights. To balance the rod AB, a force acting to the right, of magnitude, by the foregoing, equal to the sum of P and Q, must now be applied. Attach a weight $W_3 = P + Q$ to another cord and by means of a pulley, apply a horizontal force $E = P + Q$ to the rod at F. This will prevent bodily movement of the rod to the left, but, very likely, the rod will not now hang vertically. In this case shift the cord at F, up or down, until such a position is found that the rod hangs vertical under the combined actions of the horizontal forces P, Q and E. E is now the equilibrant of P and Q.

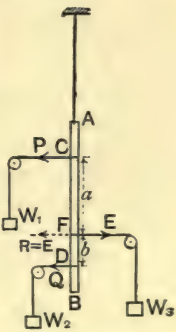


FIG. 59.—Equilibrant of two parallel forces of the same sense.

To find F, imagine the rod to be free to turn about F and take moments of the horizontal forces (which alone tend to rotate the rod) about F. E passes through F and has no moment.

Clockwise moment = anticlockwise moment;

$$\therefore \quad Qb = Pa,$$

or

$$P : Q = b : a.$$

The point F therefore divides the distance CD between the forces P and Q in inverse proportion to the forces.

Since E is the equilibrant of P and Q, if its sense be reversed, it will be the resultant of these forces.

Resultant of parallel forces.—We have, therefore, the following means of finding the resultant of two parallel forces of the same sense: $R=P+Q$; the line of R divides the distance between P and Q in inverse proportion to P and Q.

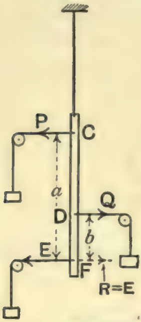


FIG. 60.—Equilibrant of two parallel forces of opposite sense.

If the forces are of opposite sense, the student should verify experimentally the following statements:

(1) R is equal in magnitude to the difference between P and Q; thus, if Q is the greater force, $R=Q-P$.

(2) R acts in the same sense as the greater force.

(3) $P:Q=b:a$. (Fig. 60.)

EXPT. 16.—Do this by actually applying forces P and Q to the suspended rod. Calculate the magnitude of E from (1) and its point of application F from (3). Apply E and see if the rod balances vertically. Notice in this figure, that P, E and Q are arranged in an exactly similar manner to P, Q and E in Fig. 59. So that since P, E and Q balance in both cases, the same results apply to both. Thus:

Given forces of same sense.	Given forces of opposite sense.
(1) $R=E=P+Q$.	(1) $Q=P+E=P+R$; $\therefore\ R=Q-P$.
(2) R acts in same sense as given forces.	(2) R acts in same sense as larger given force.
(3) $P:Q=b:a$.	(3) $P:Q=b:a$.

In (3) reference must be made respectively to Figs. 59 and 60. It is seen that in each case, a is the distance from P to R, and b is the distance from Q to R.

It may now be inferred from the above statements, that **the resultant of any number of parallel forces has a magnitude equal to the algebraic sum of the forces, and its position may be calculated by taking moments about any fixed point in the body.**

Couples.—It has now been seen that the resultant of two parallel forces of opposite sense may be found from

$$R=P-Q \quad \text{.................................} (1)$$
$$P:Q=b:a \quad \text{.................................} (2)$$

and that if R be reversed, giving $E=R$, then P, Q and E will balance.

Notice that if P and Q are nearly equal to one another, that R will become very small [from (1)], and that b and a must be nearly equal to one another and therefore both must be very large. So that as P and Q become more nearly equal to one another, E will become smaller and smaller and will move further away from P and Q. In the particular case of P becoming equal to Q, E will become zero and its distance from P and Q will be infinitely great. In this case P and Q are called a **couple**, and it follows from the above that no single force can balance a couple.

The **moment of a couple** is measured by the product of one of the forces and the perpendicular distance between them, called the *arm* of the couple.

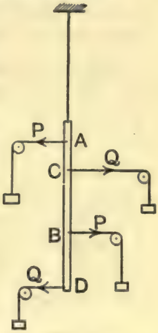


FIG. 61.

EXPT. 17.—Using the suspended rod, Fig. 61, apply two equal horizontal forces P, P, at A and B, thus causing an anticlockwise couple of moment $P \times AB$ to act on the rod. It will now be found impossible to keep the rod vertical by the application of any single force.

To balance, apply Q, Q, at C and D, thereby giving a clockwise couple to the rod. Select the values of Q and the arm CD so that the moments of the clockwise couple and the anticlockwise couple are equal, that is :

$$P \times AB = Q \times CD.$$

The rod now remains balanced vertically.

EXAMPLE. Suppose in the above experiment that P, P, are forces of 6 lbs. each, $AB=10''$ and $CD=15''$. Find the forces Q, Q.

$$P \times AB = Q \times CD,$$
$$6 \times 10 = Q \times 15,$$
$$Q = \underline{4} \text{ lbs.}$$

Fig. 61 shows all the forces horizontal, but this is not essential. **Any two couples of equal moment and opposite turning tendencies will balance the rod.**

EXPT. 18.—Test this statement by inclining the lines of the P couple and also the lines of the Q couple, but at different angles for the two couples, and make their moments equal again by adjusting the weights hung on.

Couples have many interesting and useful properties, but most of them must be reserved until the student is more advanced. One thing in particular should be noticed—a couple applied to a body will not displace it as a whole from its given position, but will only cause it to rotate. Conversely, a body which is beginning to rotate must have a couple acting on it.

Centre of parallel forces.—Let two forces P and Q act on the rod AB (Fig. 62), their directions being perpendicular to AB. The resultant R, found as before, will pass through C, and will divide AB in the proportion

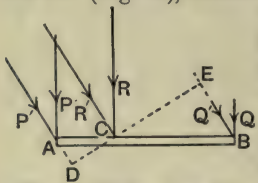


FIG. 62.—Centre of parallel forces.

$$P : Q = BC : AC.$$

Suppose we incline the directions of P and Q, as at P' and Q', without altering their magnitudes. R will now act parallel to P' and Q', its magnitude will be unaltered, and it may be seen, if we draw a line through C, perpendicular to P' and Q', that this line DE is also divided inversely proportional to P and Q, that is,

$$P' : Q' = EC : CD.$$

It therefore follows that R', the resultant of P' and Q', will also pass through C, and it may be shown in the same way, that no matter how P' and Q' are inclined, provided their magnitudes are unaltered and that they are kept parallel to one another, that their resultant always acts through the same point C. This point is called the **centre** of the parallel forces P and Q.

If there are a number of parallel forces it will be easily seen that their resultant also always passes through the same point whatever may be the inclination of the forces.

Centre of gravity.—Suppose now we have a sheet of thin metal. Every particle of the metal is being pulled towards the earth's centre, so that we have a large number of forces acting on the body in lines which are practically parallel to one another (Fig. 63). The resultant of these forces is what we call the weight of the plate, W say. Now no matter how the plate may be turned (which is equivalent to inclining the forces on the particles to their first direction) there will be a point in the plate through which W always acts, this point being the centre of the parallel forces acting on the particles. Let G be this point, then G is called the **centre of weight**, or **centre of gravity** of the plate.

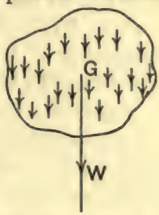


FIG. 63.—Centre of gravity.

In a great many bodies, we may see by inspection where the centre of gravity is. Thus, a *uniform rod* will have its centre of gravity at the middle of its length. A *square*, at the intersection of its diagonals, so also a *rectangle* and a *parallelogram*. A *circle* will have its centre of gravity at its geometrical centre. In the case of a *triangle*, the centre of gravity will be found one-third way up a line from the centre of the base to the opposite corner. *Pyramids* and *cones* have their centres of gravity $\frac{1}{4}$ way up a line from the centre of the base to the apex. In *prisms*, with ends perpendicular to their axes, the centre of gravity will lie at the geometrical centre of the middle cross section.

When we are considering the equilibrium of a given body we may regard its whole weight as concentrated at its centre of gravity.

Centre of gravity by experiment.—Suspend a thin plate of any irregular outline and of any material by a cord attached at A (Fig. 64). The pull in the cord will be P equal to W, the weight of the plate. If the plate be at rest, these forces, being the only two acting on it, must be in the same straight line. It follows therefore that G, the centre of gravity of the plate, must fall vertically under A.

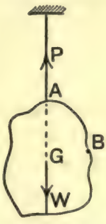


FIG. 64.—A plate hung from A.

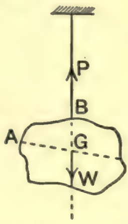


FIG. 65.—The same plate hung from B.

Produce the line of P downwards on the plate as shown, then G is in this line. Now hang the plate from another point in it such as B (Fig. 65). G again must be vertically under B, so if the line of P be again drawn on the plate, the intersection of this line with that first drawn will give G.

EXPT. 19.—Find by experiment the centres of gravity of pieces of cardboard cut into the following shapes: triangle; square with a piece cut off one corner; shape of a letter **H**; shape of a letter **L**; shape of a letter **T**.

Centre of gravity by calculation.—For plates of fairly regular outline, the position of the centre of gravity may be easily calculated by using the principle of moments. Thus, to find the centre of gravity of the plate shown in Fig. 66. Divide it up into rectangles as shown. Then, the centre of gravity of *abcg* is at m, and of *gdef* at n.

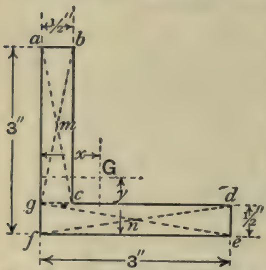


FIG. 66.—Centre of gravity of an angle section.

Also the weights of the rectangles will be proportional to their areas. So that weight of *abcg* is proportional to $2\frac{1}{2} \times \frac{1}{2} = 1\frac{1}{4}$, and weight of *gdef* proportional to $3 \times \frac{1}{2} = 1\frac{1}{2}$ and the weight of the whole plate to $2\frac{3}{4}$.

Let x be the distance of the centre of gravity of the plate from *af*, then taking moments about *af*,

$$\begin{aligned} 2\tfrac{3}{4} \times x &= (1\tfrac{1}{4} \times \tfrac{1}{4}) + (1\tfrac{1}{2} \times 1\tfrac{1}{2}) \\ &= \tfrac{5}{16} + \tfrac{9}{4} \\ &= \tfrac{41}{16}; \\ \therefore\ x &= \tfrac{41}{16} \times \tfrac{4}{11} = \tfrac{41}{44} = 0{\cdot}932''. \end{aligned}$$

Now take moments about *fe*, and let y be the distance of the centre of gravity from *fe*.

$$\begin{aligned} 2\tfrac{3}{4} \times y &= (1\tfrac{1}{2} \times \tfrac{1}{4}) + (1\tfrac{1}{4} \times 1\tfrac{3}{4}) \\ &= \tfrac{3}{8} + \tfrac{35}{16} \\ &= \tfrac{41}{16}; \\ \therefore\ y &= \tfrac{41}{16} \times \tfrac{4}{11} = \tfrac{41}{44} = 0{\cdot}932''. \end{aligned}$$

The centre of gravity is therefore a point $0{\cdot}932''$ from each of the sides *af* and *fe*.

Centre of gravity by a graphical method.—We may proceed in another way. Thus, to find the centre of gravity of the plate shown in Fig 67, divide it into triangles by the line *bd*. Find the centre of gravity of each by construction, *i.e.* bisect *bd* at *e*, join *ea* and *ec* and measure $\frac{1}{3}$rd up each of them from *e*. This construction will give c_1 and c_2, the centres of gravity of *abd* and *bcd*. The centre of gravity of the whole plate must be in the line joining c_1 and c_2. Now divide the plate again by the line *ac*. Find as before c_3 and c_4, the centres of gravity of *abc* and *acd*. Join c_3, c_4. The centre of gravity of the whole plate must be in the line c_3c_4. G will therefore be the point where c_1c_2 and c_3c_4 intersect.

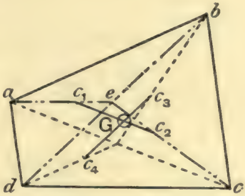


FIG. 67.—Centre of gravity of an irregular plate.

State of equilibrium of a body.—A body is said to be in **stable equilibrium** if, on being slightly disturbed, it tends to return to its original position; **unstable** if it tends to go over further, and **neutral** if it will remain at rest indifferently in any position. We may easily test for a body's equilibrium.

Thus, suppose a **cone** to stand on its base on a horizontal surface (Fig. 68). Its weight being W, then R, the reaction of the

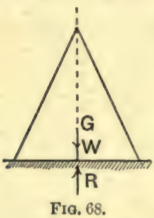


FIG. 68.

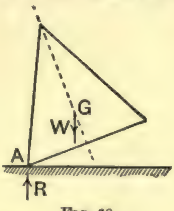


FIG. 69.

Stable equilibrium of a cone.

surface, will be equal to W and in same straight line. Disturb the cone slightly. R shifts along to A (Fig. 69), and R and W now form a couple tending to bring the cone back to its original position. This is therefore a case of stable equilibrium.

Now stand the cone on its apex (Fig. 70). If we disturb it slightly, R and W form a couple tending to upset it (Fig. 71). The position is therefore unstable.

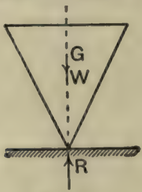


FIG. 70.

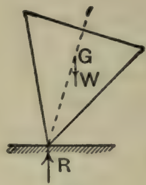


FIG. 71.

Unstable equilibrium of a cone.

A **ball** resting on a horizontal table will be in neutral equilibrium, because R and W (Fig. 72) will always be in the same straight line no matter how the ball is disturbed. It will therefore remain at rest in any position.

FIG. 72.—Neutral equilibrium of a ball.

A **cylinder** lying on its side (Fig. 73) will also be in neutral equilibrium, but if we cut a slice off the top, the equilibrium will be found to be stable. For the centre of gravity G' (Fig. 74) will be brought below G by cutting off the slice, and consequently, if the cylinder be slightly disturbed, as in Fig. 75, R and W will give a couple tending to bring it back again to its original position.

FIG. 73.—Neutral equilibrium of a cylinder.

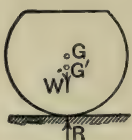


FIG. 74.

FIG. 75.

Portion of a cylinder in stable equilibrium.

It may be seen, from some of the above examples, that another property of the centre of gravity of a body is that, if the body is free to be moved by its weight from its given position, it will always do so in such a way that the centre of gravity is lowered thereby.

EXERCISES ON CHAP. IV.

1. AB is a uniform bar pivoted at C, its centre of length. W is a load of 5 lbs. placed at D, CD being 15″. If we have to restore balance by means of a 3 lb. weight, where must it be placed?

2. A bent lever ACB is pivoted at C; arm AC is horizontal and 9″ long; arm BC is vertical and 39″ long. A load of 300 lbs. weight is hung from A. Find what horizontal force at B will produce equilibrium. Neglect the weight of the lever.

3. The arms of a bent lever ACB are perpendicular to one another, and the lever is pivoted at C. Arm AC is 6″ long, and BC is 27″ long and inclined 30° to the vertical. Find what horizontal force P at B will balance a force $Q=250$ lbs. weight applied at A at 90° to AC. Neglect the weight of the lever.

4. A rod 5 ft. long has a weight of 2 lbs. at one end and 3 lbs. at the other, also a weight of 5 lbs. at its centre. Find the point about which it will balance. Neglect the weight of the rod.

5. Give a dimensioned sketch of a practicable arrangement of levers whereby a weight of ½ ton may be balanced by one of 10 lbs.

6. A beam 12 ft. long, supported at its ends, carries a load of 1¼ tons at a point 4 ft. from one end. Find the reactions of the supports, neglecting the weight of the beam.

7. A beam 20 ft. long, supported at its ends, has a load of 2 tons at the centre of its span, another of 1 ton at 3 ft. from one end, and another of 3 tons at 4 ft. from the other end. Neglect the weight of the beam and find the reactions of the supports.

8. AB is a beam 16 ft. long. It is supported at the end A and at C 4 ft. from the end B. A load of 4 tons is placed 6 ft. from A and another of 2 tons at the end B. Neglect the weight of the beam and find the reactions of the supports.

9. A man whose weight is 160 lbs. can lift, unaided, a load of 3 cwts. Suppose he uses a lever 4 ft. long, the fulcrum being 3″ from one end, find what weight he can raise (a) if his end of the lever is moving down, (b) if his end of the lever is moving up, the fulcrum and weight changing places with each other.

10. A uniform plank 20 ft. long, weight 90 lbs., rests on supports at its ends. A load of 500 lbs. weight rests 8 ft. from one end. Find the reactions of the supports.

11. A uniform beam 12 ft. long supported at its ends, carries a distributed load, including its own weight, of ½ ton per foot run. A concentrated load of 1 ton rests 5 ft. from one end, and another of 3 tons, 4 ft. from the other end. Calculate the reactions of the supports.

12. A uniform beam 16 ft. long weighs 300 lbs. It is supported at one end and at a point 4 ft. from the other end. Calculate the reactions of the supports.

13. A cone 3 ft. diameter of base, 4 ft. high, stands on its base on a horizontal surface. Specific gravity of material $=3$. What horizontal force at the top will turn the cone over?

14. A triangular plate, ABC, of wrought iron 1″ thick, lies on a horizontal surface. $AB=3$ ft., $BC=3\frac{1}{2}$ ft., $CA=4$ ft. Find what vertical lifting force applied at A will raise that corner of the plate.

15. A wall 8 ft. high, 14″ thick, is built of material weighing 130 lbs. per cubic foot. The normal wind pressure on the face of the wall is 50 lbs. per square foot of vertical surface. Consider a piece of the wall one foot long, and calculate the overthrowing moment of the wind on it and also the resisting moment of the weight of the wall. Will the wall stand or fall?

16. Show in a diagram the couples acting on a hinged door, 7 ft. high, 3 ft. wide, weight 90 lbs. There are two hinges, placed one foot from top and bottom of the door.

17. A horizontal beam 10 ft. long weighs $\frac{1}{2}$ ton and is pivoted 4 ft. from one end. Its centre of gravity lies in the longer part, 1 ft. from the pivot. Find where a load of 500 lbs. must be placed to keep the beam balanced.

18. A ladder 24 ft. long weighs 50 lbs. and has its centre of gravity 8 ft. from one end. A bag of tools, weight 100 lbs., is slung at the centre of length of the ladder. A lad and a man carry the whole between them, the lad being at the lighter end of the ladder. Find where the man must be if his share of the load is 90 lbs.

CHAPTER V.

STRESS. STRAIN. ELASTICITY. ULTIMATE STRENGTH.

Stress.—What happens to a piece of material when forces are applied to it in the direction of its length? This is a question which now requires to be studied. Suppose AB (Fig. 76) is a bar the ends of which are subjected to equal and opposite pulls P, P. Imagine the bar to be cut at CD at 90° to its axis, and consider what must be done to preserve the equilibrium of the left-hand portion. In order to balance the force P it will be necessary to apply an equal and opposite force at CD (Fig. 77); let Q be this force. As the cut is only imaginary, the portion to

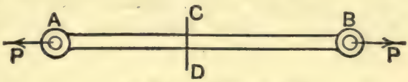


FIG. 76.—Bar under pull.

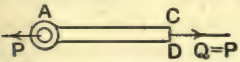


FIG. 77.—Equilibrium of the left-hand portion.

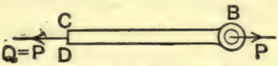


FIG. 78.—Equilibrium of the right-hand portion.

the right of CD in the actual bar must have supplied this force Q, in order to keep the left-hand portion balanced. In the same way, the left-hand portion of the bar exerts a pull Q equal to P, acting towards the left (Fig. 78), on the right-hand portion of the bar.

The force Q in the actual bar will be distributed in some manner over the section of the material, the sum of the pulls on every part of the section being equal to Q (Fig. 79). For sections taken near the ends of the bar, where the forces P, P

are applied, the distribution cannot be definitely stated, but at some little distance from the ends, and right along the bar, the distribution is probably uniform, each square unit of the sections bearing equal forces. This force per unit area of the section is called **stress.** The stress may be easily calculated by dividing the total force on the section by the area of the section. Thus, if the bar is pulled at its ends with forces of 10 tons weight each, and its sectional area is $2\frac{1}{2}$ square inches, the stress will be 10 divided by $2\frac{1}{2}$, or 4 tons per square inch. If the bar is of uniform section, a stress equal to this will be found on any cross section, except those very near the ends where the distribution is unknown.

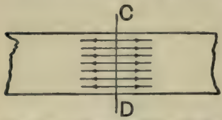


FIG. 79.—Distribution of the stress.

Ties and Struts.—Those portions of a structure which are intended to be under *pull* are called **ties**, and if intended to be under *push*, are called **struts.** Ties are said to be under **tensile stress** when pulled, and struts under **compressive stress** when pushed.

If we consider the case of a tie slightly bent at first and then pulled, we can easily see that the tendency is to straighten it (Fig. 80); in the same way a pulled string becomes straight. Also, if the tie is straight to begin with, there will be no tendency to bend it when pulls are applied. It follows therefore, since no lateral stiffness is required in ties to resist bending, that the shape of the cross section is immaterial. It is very different, however, in struts. If the strut is originally bent, the tendency will be to bend it still more (Fig. 81), consequently the section must be chosen so as to give the necessary lateral stiffness to resist this bending action. If the strut is straight to begin with, and is kept straight when the pushes are applied, then the stress on any section not too near its ends will be uniformly distributed, and will be found as before by dividing the total force on the section by the area of the section.

FIG. 80.—The bar, originally bent.

FIG. 81.—Strut, originally bent.

Both ties and struts should be made straight to begin with, as we have seen that ties tend to become straight if originally bent, and struts will bend more if not at first straight. Hori-

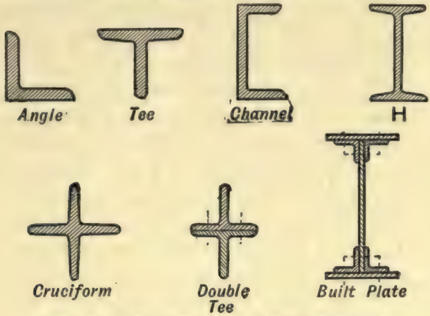


FIG. 82.—Common forms of strut sections.

zontal or inclined ties of great length compared with their cross sectional dimensions will have a bending tendency due to their own weight. Such ties should have a suitable cross section to give stiffness, or should be supported at intervals by suspending rods. Struts are of many different forms depending on their lengths and the magnitude of the loads. A few common sections are shown in Fig. 82.

Columns are vertical pieces designed for the purpose of carrying weights and come under the heading of struts as they are subjected to push forces. Lateral stiffness must be arranged for in columns as in struts. Short blocks used as columns fail by crushing. Very long columns fail by bending and thus breaking. Columns are often made stiffer by putting flanges on the ends, the effect being that the column bends as shown by the dotted lines in Fig. 83, instead of as a whole. But if the

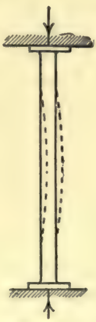


FIG. 83.—Flanged column, load applied centrally.

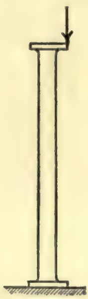
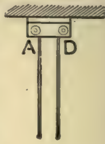


FIG. 84.—Flanged column, load applied at edge of flange.

load is not centrally applied at the ends, the effect may be worse than if there were no flanges. As an extreme case, think of W applied at the edge of the flange (Fig. 84). The bending tendency is much greater than would be the case if the flanges were absent.



Change of length of a loaded bar.—All materials stretch when pull forces are applied and become shorter with push forces. With delicate apparatus, these changes of length can be measured in metals. With material such as rubber, an ordinary scale is sufficient.

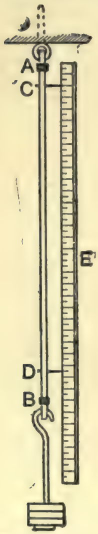


FIG. 85.—Apparatus for measuring the extensions of rubber.

EXPT. 20.—Arrange apparatus as shown in Fig. 85, consisting of a round rod of rubber about $\frac{1}{2}$ inch diameter and 3 or 4 feet long tied to a support at A and having a hook for carrying weights at B. C and D are needles pushed through the rubber; these needles will move on a scale E when loads are applied, and the changes in length of the portion CD can be obtained. With the hook alone, obtain the length CD by deducting the scale reading at C from that at D; call this L. Apply loads increasing by equal increments and note the readings at C and D produced by each. Take off the loads in the same order and again note the readings at C and D. Tabulate the observations:

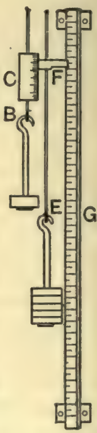


FIG. 86.—Apparatus for measuring the extensions of a pulled wire.

Load, lbs.	Scale readings, inches.				Extensions, inch.	
	Load increasing,		Load diminishing,		Load increasing.	Load diminishing.
	at C.	at D.	at C.	at D.	$(D-C)-L$.	$(D-C)-L$.

Plot graphs showing loads as ordinates and extensions as abscissae, both for loads increasing and loads diminishing.

A simple apparatus for measuring the **extensions of wires** under various loads consists of two wires hung side by side from the same support. One wire AB (Fig. 86) carries a constant load sufficient to keep it taut, and has a scale of inches divided into tenths fixed at C. The other wire DE is the wire under test. The load on it can be varied, and when this is done, a vernier fixed at F will move over the scale C, and will give the changes in length of the portion DF. The arrangement for carrying the scales prevents any drooping of the support at AD from being measured as an extension of the wire under test. The support for the wires should, in order to have a long portion of the wire available for testing, be fixed at the ceiling, or as high up the wall as possible.

Extensions in ordinary test pieces.—Test pieces of ordinary bars or plates used for engineering work must be subjected to

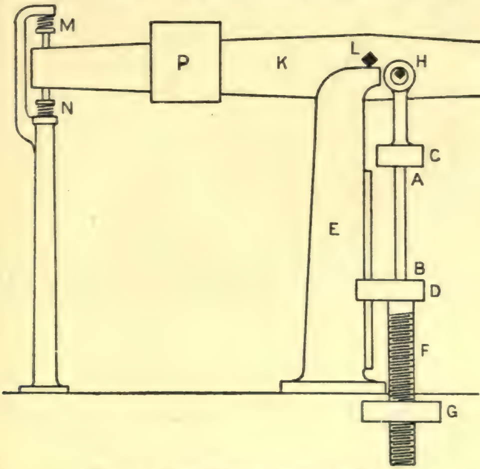


Fig. 87.—Single-lever testing machine.

great loads before a measurable extension is produced. This is done in large testing machines, such as is shown in outline in Fig. 87. A lever K is supported by a knife-edge at L on the top of a column E, and carries a counterpoise P, which may travel along the lever. AB is the test piece, hung from a

knife-edge H, and may be pulled downwards by means of the screw F and nut G. The test piece is thus put under pull by operating the screw, and the pull is balanced by moving the counterpoise along the lever until equilibrium is restored. The pull is measured by observing the position of the counterpoise in relation to a scale attached to the lever. Buffer springs at M and N serve to limit the movement of the lever. The same machine may be arranged to apply push to the test piece, or to bend a beam.

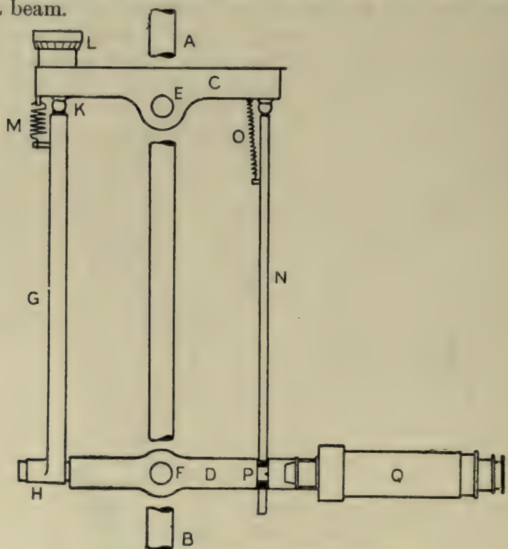


FIG. 88.—Ewing's extensometer.

Changes of length are measured by means of instruments called **Extensometers.** In Sir J. A. Ewing's Extensometer (Fig. 88), the extension of the piece is measured by the movement of a fine wire over the scale of a microscope. AB is the test piece and has pivoted to it two brackets C and D, which may turn on axes E and F. A rod G is pivoted to the lower bracket at H and to the upper bracket by a ball joint at K and a spring M. A rod N has a similar ball joint at its upper end, and carries a fine wire P at its lower end, which is guided to move

in front of a microscope Q. Q is furnished with a micrometer eye-piece for reading the movements of the wire. As the length of G is fixed, extension of the test piece will cause the right-hand ends of the brackets to recede from each other, and the wire P will appear to move over the eye-piece scale. A micrometer screw at L enables the length of G to be changed to a definite extent, and so permits the instrument to be calibrated. It is possible with the instrument to measure a change of length of $\frac{1}{50,000}$th inch on a test piece 8″ long. The following results were obtained by its use, and are given in illustration of an important fact.

Test bar of flat iron 1·501″ wide, 0·492″ thick, 8″ long between the test points. The load was applied in steps of 1000 lbs. until a maximum of 10,000 lbs. was reached. The bar carried this load for 2 or 3 minutes, and then the load was taken off 1000 lbs. at a time.

AN EXPERIMENT WITH EWING'S EXTENSOMETER.

Load, lbs.	Load increasing.		Load diminishing.	
	Scale reading.	Differences per 1000 lbs. load.	Scale reading.	Differences per 1000 lbs. load.
0	3·00		3·00	
		0·18		0·16
1000	3·18		3·16	
		0·18		0·18
2000	3·36		3·34	
		0·18		0·19
3000	3·54		3·53	
		0·19		0·19
4000	3·73		3·72	
		0·21		0·20
5000	3·94		3·92	
		0·19		0·21
6000	4·13		4·13	
		0·19		0·19
7000	4·32		4·32	
		0·20		0·20
8000	4·52		4·52	
		0·20		0·20
9000	4·72		4·72	
		0·19		0·19
10,000	4·91		4·91	

The scale reading was such that one part on the scale corresponds to an extension of $\frac{1}{500}$ inch. Consequently the total stretch for 10,000 lbs. load was $\frac{1 \cdot 91}{500}$ inch.

Plotting on squared paper column 1 for ordinates and column 2 for abscissae, we see that all the points lie practically on a

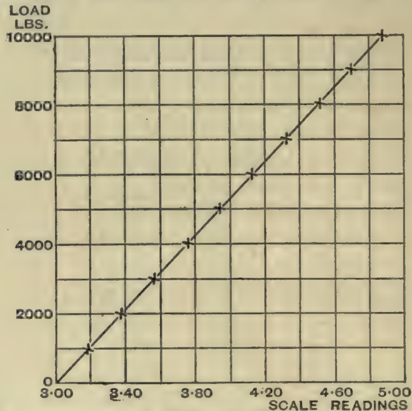


FIG. 89.—Curve showing extensions and loads for a pulled bar.

straight line (Fig. 89). We therefore infer that in this piece **the extensions have been practically proportional to the loads.** The same law is found to hold more or less nearly in all engineering materials and is known as **Hooke's Law.**

Strain.—The term **strain** is used to signify the change of length or other dimensions, or the change of form which occurs in a material when loads are applied. Strain is measured in a pulled or pushed bar by stating the change of length per unit of original length of the bar. To obtain it in any particular case, divide the total extension by the original length of the bar. Thus, in the above experiment,

$$\text{tensile strain} = \frac{1 \cdot 91}{500 \times 8} = 0 \cdot 0004778.$$

Strain is not measured in any units, as it is simply the ratio of two lengths.

Some important definitions.—If we go on loading a test piece, we presently reach a point where Hooke's Law breaks

down, and the extensions cease to be proportional to the load, but increase at a more rapid rate. Up to this point, if the load be removed, the piece will come back to its original length, but if the loading is carried beyond this point the bar will be found to be permanently extended when the load is removed.

Elasticity is that property of matter by virtue of which it tends to return, or spring back, to its original shape and dimensions when the applied forces are removed.

Elastic Limit is the name given to the stress at which Hooke's Law breaks down, and at which the bar takes up a permanent extension. The permanent extension is called **Permanent Set,** and material loaded beyond the Elastic Limit is said to be **overstrained.**

Modulus of elasticity.—Notice that both strain and stress are proportional to the load producing them up to the elastic limit, and that therefore they are proportional to one another for a given material. It follows that if the stress be divided by the corresponding strain, a constant number for the same material will be produced. The fraction $\frac{\text{stress}}{\text{strain}}$ is called **modulus of elasticity**. *Young's Modulus of Elasticity*, is that which refers to a bar pushed or pulled ; it is usually written E. For calculation of E we have the following relations :—

Let W=load applied,
A=sectional area of bar in square inches,
L=length of bar,
e=change of length produced by W,

both lengths being in the same units. Then

$$\text{stress}=\frac{W}{A},$$

$$\text{strain}=\frac{e}{L},$$

and

$$E=\frac{\text{stress}}{\text{strain}}=\frac{W}{A}\div\frac{e}{L}$$

$$=\frac{W.L}{A.e}.$$

The modulus of elasticity will be described as tons per square inch or lbs. per square inch depending on how the stress is

measured. The average values of Young's Modulus for some common materials are given in the following table :

YOUNG'S MODULUS OF ELASTICITY.

Material.	E. (tons per square inch).
Wrought iron, - - -	13,000
Steel, - - - -	13,500
Cast iron, - - - -	6,000
Rolled copper, - - -	6,200
Brass, - - - -	5,700
Gun metal, - - -	5,000
Phosphor bronze, - -	6,000
Aluminium bronze, - -	6,500

Phenomena beyond the elastic limit.—In **ductile** materials, such as iron and steel, if further loading be applied after the elastic limit is passed, a point is reached where the material draws out considerably with no, or very little, increase to the load. This point is called the **yield point.** Further loading produces considerable extension throughout the material which can easily be seen even in a short piece, and it will be also observed, if the diameter of the piece is measured from time to time, that contraction is going on all over the specimen. Presently the load is reached at which the wire is about to break. At the place where fracture is about to occur considerable contraction will be observed, until finally rupture occurs. In materials like cast iron and brass, there is no yield point, and the total extension of specimens of these materials before breaking is much smaller than for iron or steel. In general, the *absence of a yield point and small extension show hard material lacking ductility* ; this is further shown by small local contraction at the fracture. Ductile material, carrying a constant load beyond the elastic limit, goes on extending, or **creeping**, during a long period of time.

It is usual to estimate the **ductility** of a material from the extension on a measured length of the test piece, and also from the local contraction at fracture. Plastic material showing considerable extension before rupture is more suitable for with-

standing shocks when worked into a structure than hard stuff which breaks off short with little extension. The ductility of a material is often tested by bending a bar of it to a given radius, and its ability to resist shocks is tested by repeated blows applied by a weight falling on the middle of the bar, the bar being supported at its ends and turned over after each blow. *In practice the elastic limit of the material should not be approached when the loads are applied*, but the information gained during tensile testing after the elastic limit is reached is valuable for determining the qualities of the material. During testing, the loads should be applied at a fairly uniform rate and without shocks; the material should not be given a rest at any time after loading has once started, as this is liable to alter its qualities.

Ultimate strength.—The **breaking stress**, or **ultimate strength** of a material is measured by dividing the breaking load by the original sectional area of the piece. This is always done for engineering purposes, although it is fictitious, as owing to the contraction at the fracture, the actual area over which the breaking load is distributed is smaller than the original area, and therefore the stress on the section at the fracture is higher than would be shown by the above calculation. It is convenient, however, in practice, to measure the ultimate strength as stated; for we wish to know what load would break a given piece in order, by making the actual load a certain fraction of the breaking load, to prevent that occurring. Thus, if we know that the ultimate strength of mild steel is 30 tons per square inch, then a load of 37·5 tons would be required to break a bar $2\frac{1}{2}''$ by $\frac{1}{2}''$ section. To prevent this, we arrange that the actual load shall be, say, one fifth of this, or 7·5 tons. A table giving the ultimate strengths of some common materials will be found on p. 62.

Factor of safety.—The **factor of safety** is the ratio of the breaking load to the working load. Thus,

$$\text{Factor of safety} = \frac{\text{breaking load}}{\text{working load}}.$$

The magnitude of the factor of safety to be used in any given case depends on the nature of the loading. A low factor of safety may be employed where the load is steady, or is applied

TABLE OF ULTIMATE STRENGTH OF MATERIALS.

Material.	Tensile strength, tons per sq. inch.	Compressive strength, tons per sq. inch.	Shearing strength, tons per sq. inch.
Cast iron,	5 to 15, average 8	25 to 65	6 to 13
Wrought iron—			
Tested in direction of rolling,	20 to 29	16 to 20	22
Tested across direction of rolling,	19 to 24		
Mild steel,	27 to 32	...	21 to 25
Cast steel,	35 to 70	...	...
Copper, cast,	8 to 12	...	...
,, rolled,	15	...	...
,, wire (hard drawn),	28	...	...
Tin,	2	...	...
Zinc, cast,	1 to 3	...	...
,, rolled,	8 to 10	...	...
Lead,	1	...	...
Aluminium, cast,	5	...	...
,, rolled,	6 to 10	...	...
Brass, ordinary,	11	...	...
,, wire,	20 to 25	...	...
Sterro metal,	35	...	...
Delta metal, cast,	22	...	...
,, forged,	34	...	...
,, wiredrawn,	55	...	...
Muntz metal,	22	...	...
Gun metal,	15	...	...
Aluminium bronze,	40	...	...
Phosphor bronze, annealed,	25	...	...
,, unannealed,	up to 70	...	...
Manganese bronze,	28	...	...
Granite,	...	6 to 10	...
Sandstone,	...	2 to 5	...
Portland stone,	...	2	...
Brick, London stock,	...	1	...
,, Staffordshire blue),	...	2 to 6	...
Pine,	5	$2\frac{1}{2}$	$\frac{1}{4}$
Oak,	7	$4\frac{1}{2}$	1
Leather,	2	...	...

and removed very gradually. High factors of safety are employed where the load is applied suddenly, or where the loads are pushes and pulls alternating. The factor of safety allows a margin for shocks and for our ignorance of the possible loads a structure may have to bear.

Factors of safety used in practice.—In practice, live loads, such as the weight of a locomotive and train on a bridge, are usually doubled and added to the dead load which consists of the weight of the bridge itself. The working stress allowed in bridge work of steel having an ultimate tensile strength of 30 tons per square inch ranges from 4 to $7\frac{1}{2}$ tons per square inch, the lower value being used for parts which are alternately pushed and pulled. These stresses correspond respectively to factors of safety of $7\frac{1}{2}$ and 4. Pieces of wrought iron or steel subjected to shocks should have a factor of safety of from 10 to 12. For cast iron the factor of safety ranges from 5 to 15 ; this material is not suitable for withstanding shocks and the latter factor is used for such cases. Timber is liable to sag under loads, consequently factors of safety of from 8 to 20 are used.

EXAMPLE. A certain steel bar belonging to a roof has a steady pull, due to weights of the parts of the roof, of 12 tons. When wind blows on one side of the roof, an additional force of 3 tons pull is produced in the bar. Find the equivalent dead load, and the sectional area of the bar if its breaking strength is 30 tons per square inch, and if the factor of safety is 5.

$$\begin{aligned}
\text{Equivalent dead load} &= \text{dead load} + 2 \text{ live load} \\
&= 12 + (2 \times 3) \\
&= \underline{18} \text{ tons.} \\
\text{Safe stress} &= \tfrac{30}{5} = 6 \text{ tons per sq. inch.} \\
\text{Sectional area of bar} &= \tfrac{18}{6} \\
&= \underline{3} \text{ square inches.}
\end{aligned}$$

Effect of heating and cooling.—The injurious effects of overstraining, or of repeated applications of loads may be got rid of by **annealing**. This consists of heating to redness and then cooling slowly. Bars and plates are usually partly annealed on coming from the makers. This is due to the material leaving the rolling machine hot and then cooling down fairly slowly. Cold rolling and wire drawing produces hard-

ness by setting up overstrain in the material. This can be got rid of by subsequent annealing. Crane chains are occasionally *passed through the fire* so as to anneal them and restore their original qualities. Copper is hardened by mechanical treatment such as wire-drawing or bending; it may be softened and its ductility restored by being heated to redness and plunged into cold water. Steel containing more than 0·2 per cent. of carbon is made very hard by the same treatment.

Tensile tests on wires.—Experiments on pulling wires of various materials until they break are very instructive and are easily performed by students. It has already been shown how the extensions of a wire may be measured for varying loads within the elastic limit. The same apparatus (Fig. 86) may be used, after the elastic limit is passed, if a long scale, divided in inches and tenths, is clamped to a firm support and arranged so that the testing weights may move past it as they descend while the wire is extending. Use the vernier until the elastic limit is reached and the scale after. The slight error produced by any drooping of the support after the material begins to draw out can be safely neglected as it will be only a very small percentage of the total extension.

Expt. 21.—Arrange apparatus as described above (Fig. 86) and carry out tensile tests on wires of copper, brass, iron and steel. In each case, note throughout the test the extensions produced by gradually increasing loads. Plot these on squared paper and work out the results of the tests, using the same method as has been employed in the following record of a test on a copper wire.

Tensile Test on Copper Wire.

Length, 9′ 8½″.

Diameter, 0·036″.

The load was applied in 2-pound increments. Up to the elastic limit the extensions were measured by a vernier on the test wire moving over a scale on another wire hung from the same support. The vernier read to 0·01″. After the elastic limit the extensions were measured by a long boxwood scale, clamped to a fixed support.

Load, lbs.	Scale reading, inches.	Extensions, inches.	Remarks.
1	3·00	0·00	This load was the weight of the hook.
3	3·02	0·02	
5	3·03	0·03	
7	3 05	0·05	
9	3·06	0·06	
11	3·08	0·08	
13	3·09	0·09	Elastic limit reached.
15	3·2	0·20	Scale changed to boxwood rule.
19	3·4	0·4	
21	3·9	0·9	
23	4·8	1·8	
25	6·0	3·0	
27	7·6	4·6	
29	9·6	6·6	
31	12·0	9·0	
33	14·9	11·9	
35	19·0	16·0	
37	26·0	23·0	
39	32·0	29·0	Wire broke.

Columns 1 and 3 plotted give the complete curve as shown in Fig. 90. The curve up to 15 lbs. load has also been plotted in Fig. 91 to an increased scale of extensions.

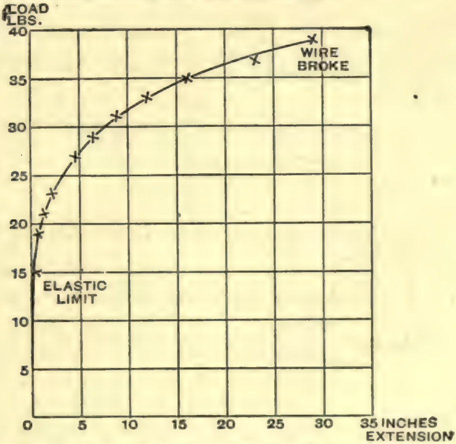


FIG. 90.—Plotted load-extension diagram for a copper wire.

Calculation of results.—Area of wire $=\pi\dfrac{d^2}{4}=0\cdot001018$ sq. inch.

Load at elastic limit $=13$ lbs.

$$\textbf{Elastic limit}=\frac{13}{0\cdot001018}=12{,}770 \text{ lbs. per sq. inch.}$$

$$=\underline{5\cdot7} \text{ tons per sq. inch.}$$

Breaking load $=39$ lbs.

$$=\frac{39}{0\cdot001018}=38{,}310 \text{ lbs. per sq. inch.}$$

$$=\underline{17\cdot1} \text{ tons per sq. inch.}$$

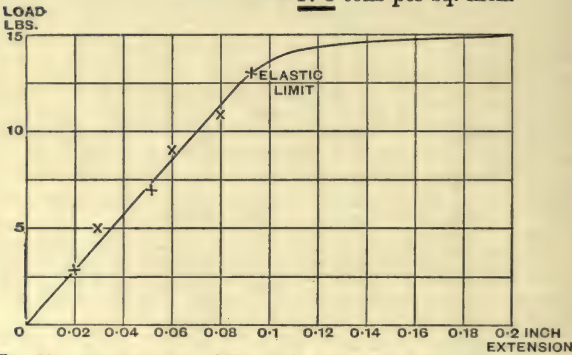


FIG. 91.— Elastic portion of Fig. 113 to an increased scale of extensions.

For **Young's modulus,** 12 lbs. load gives extension 0·09″ on a length of 9′ 8½″.

$$\text{Stress}=\frac{12}{0\cdot001018}=10{,}800 \text{ lbs. per sq. inch.}$$

$$\text{Strain}=\frac{0\cdot09}{116\cdot5}=0\cdot000772.$$

$$E=\frac{\text{stress}}{\text{strain}}=\frac{10800}{0\cdot000772}=13{,}990{,}000 \text{ lbs. per sq. inch}$$

$$=\underline{6250} \text{ tons per sq. inch.}$$

The total extension on 116·5″ was 29″, or

$$\textbf{Extension}=\frac{29}{116\cdot5}\times 100=\underline{24\cdot9} \text{ per cent.}$$

EXERCISES ON CHAP. V.

1. A tie bar 4″ broad, $\frac{5}{8}$″ thick, is under a tension of 7 tons. Calculate the tensile stress.

2. Find the working load for the bar in Question 1 if the tensile stress is not to exceed 5 tons per square inch.

3. A bar of square section, $\frac{1}{4}$″ edge, is 60 ft. long and is found to stretch 0·6″ when a certain pull is applied. Find the strain. Suppose the pull applied to have been 1562 lbs., and find Young's modulus of elasticity.

4. Suppose the tensile stress is not to exceed 4 tons per square inch, find the diameter of a round tie rod which has to resist a pull of 16 cwts.

5. Taking Young's modulus for wrought iron to be 29,000,000 lbs. per square inch, what decrease in length will take place when a column containing 12 square inches in section and 20 ft. high carries a load of 36 tons?

6. What load, in pounds, must be hung to an iron wire 50 ft. long and 0·1″ diameter to make it stretch $\frac{1}{5000}$ inch?

7. An iron tie bar is 50 ft. long, its section being rectangular 4″ × $\frac{3}{4}$″. Its stretch must not exceed $\frac{1}{16}$″; calculate the maximum load it can carry.

8. A copper wire, previously pulled beyond the elastic limit, was tested again, after 15 hours' rest, under tension. Length of wire, 12′ 3″, diameter of wire 0·036″. The following results were obtained:

Load, (lbs.).	Scale reading (inches).	Extension for differences of 2 lbs. (inches).	Remarks.
0	3·03		Load marked zero in column 1 was actually a load of 4½ lbs., hung on to keep wire taut.
2	3·04	0·01	
4	3·06	0·02	
6	3·08	0·02	
8	3·10	0·02	
10	3·12	0·02	
12	3·14	0·02	Elastic limit reached.
14	3·65	0·51	

Plot columns (1) and (2) on squared paper. Calculate the value of Young's modulus for this sample of copper wire.

9. What do you understand by the terms tensile and compressive strength respectively of any material? Define "modulus of elasticity." If a wrought iron bar of 1 square inch sectional area just breaks under a tensile stress of 60,000 lbs., what would be the area of the section of a tie-rod which would just support a load of 20 tons?

10. An iron wire is loaded with gradually increasing tensile loads till it breaks. We want to know its modulus of elasticity, its elastic limit stress and its breaking stress. What measurements and calculations do we make?

CHAPTER VI.

STRENGTH AND STIFFNESS OF BEAMS AND SHAFTS.

Bending of a beam.—**Beams** are parts of a structure, usually supported horizontally, for the purpose of carrying loads applied transversely to their lengths. Suppose we have a beam consisting of a number of planks of equal lengths laid one on the other, and supported at the ends. A load W, applied at the centre of the span, will cause all the planks to bend in a similar fashion, and consequently, as the lengths of all the planks will remain the same, they will overlap at the ends as shown (Fig. 92). Strapping the planks firmly together will prevent this occurring and the beam will now bend as a whole, the ends of the planks remaining in one plane (Fig. 93). Assuming the middle plank to remain the same length as at first, it is clear that the top plank and all those above the middle must have become shorter, and those below the middle one, longer than at first. The planks above the middle must therefore have been subjected to compressive stress in the direction of their length and those below the middle to tensile stress in the same direction. The further we get away from the middle, above or below, the greater will be the change of length of the planks and therefore the greater will be the compressive and tensile stresses producing these

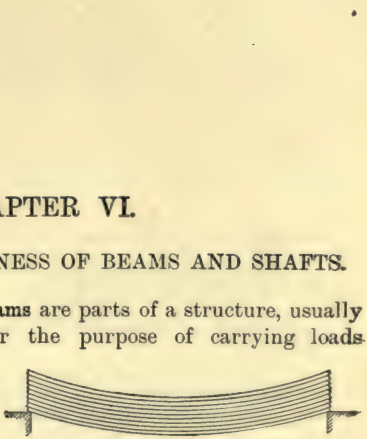


FIG. 92.—Bending of a loose plank beam.

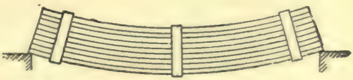


FIG. 93.—Bending of a strapped plank beam.

changes of length. An ordinary solid metal or timber beam may be looked upon as being built up of a large number of fibres cemented together, corresponding to the planks in our model beam. These fibres will be subjected to stresses in the same manner as the planks. Therefore, in a loaded solid beam, such as that in Fig. 94, the upper fibres will be under compressive stress and the lower ones under tensile stress. These stresses are shown at the section *AB*.

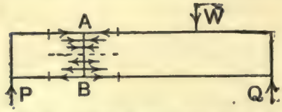


FIG. 94.—Tensile and compressive stresses on the section *AB*.

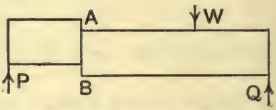


FIG. 95.—Shear at the section *AB*.

Further, the action of P, the pressure of the support, acting on the left-hand side of the section AB, and of W and Q acting on the right-hand side of the section, will be usually to give the material a tendency to slide past at the section as in Fig. 95; this action is called shearing, and the material at the section is under shearing stress as well as the stresses mentioned above.

Models showing the forces in the material.—These stresses may be very well understood by examining models such as those illustrated. Fig. 96 shows a **cantilever**, that is, a beam fixed at one end only and free at the other end. The cantilever has been cut at AB. In order to balance the portion outside of AB it is necessary to put a cord connection at A and a small strut at B. The pull and push forces thereby supplied counteract the bending tendency. In addition, an upward force S has to be supplied to balance the tendency to shear. In the uncut cantilever these forces would be supplied by the resistance of the material at the section. It will be noticed in the cantilever that the tensile stress occurs above, and the

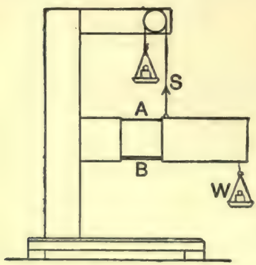


FIG. 96.—Model of a cantilever, cut to show the forces at *AB*.

compressive stress below, the middle. This is just the reverse of what we have seen for a beam supported at both ends. Fig. 97 shows this latter case. This particular model consists of an I section, supported at the ends and cut into two pieces. A compression spring balance at the upper flange of the beam

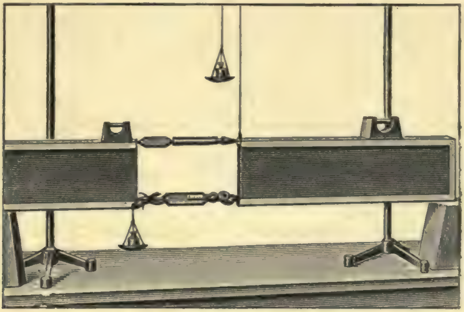


FIG. 97.—Model of a cut beam.

and two ordinary spring balances at the lower flange enable the forces at the section, produced by bending, to be measured. Shearing is balanced by weights applied as indicated, the upward force being supplied by means of a cord passing over a pulley above.

If we again examine the model beam built up of planks, we may notice that when it is bent and the ends of the planks overlap, that they have done so by the planks sliding on one another in the direction of their lengths. The straps obviated this by binding the planks firmly together and so preventing this sliding, or shearing action taking place. So also in a solid beam there are shearing stresses distributed over horizontal longitudinal sections.

The actual distribution and calculation of these stresses is beyond the scope of this book, but sufficient will be said to enable the student to solve many simple practical problems.

Comparative strengths of beams.—In Fig. 98*a* is shown a cantilever of length L inches carrying a load W_1 lbs. weight at its end B. The cantilever is firmly fixed in a wall at A. W_1 has a

tendency to rotate the beam about A, the moment being W_1L lb.-inches. This moment is called the bending moment at A, and the strength of a cantilever or beam is determined from the greatest bending moment it can carry. In Fig. 98*b*, the same cantilever is shown carrying a distributed load W_2 which may be concentrated at its centre of gravity C for the purpose of calculating the moment about A. The bending moment at A in this case will be $W_2 \times \frac{1}{2}L$ lb.-inches. If W_1 is regarded as

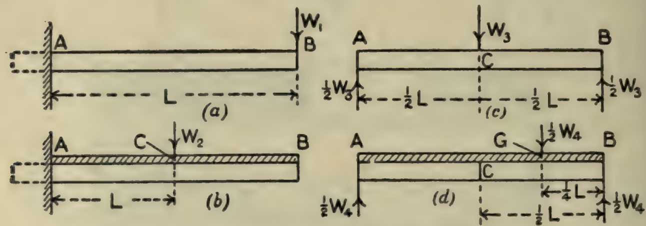


FIG. 98.—Comparative strengths of beams.

being a safe load in Fig. 98*a*, W_2 in Fig. 98*b* may be twice W_1 and will give a bending moment of the same value as W_1 does Fig. 98*a*. In both cases, the cantilever is subjected to the same bending moment and will be equally safe. Hence we may say that a cantilever which can carry a certain load at its end may carry twice this load spread uniformly along it.

In Fig. 98*c* is shown the cantilever of Fig. 98*a* transformed into a beam supported at both ends and carrying a load W_3 at the middle. The reaction of each support will be $\frac{1}{2}W_3$ and we may consider the portion CB in calculating the bending moment. The reaction at B has a moment about C equal to

$$\tfrac{1}{2}W_3 \times \tfrac{1}{2}L = \tfrac{1}{4}W_3L \text{ lb.-inches.}$$

Hence W_3 may be four times W_1 in Fig. 98*a* and will then be a safe load for the beam.

In Fig. 98*d*, the same beam carries W_4 spread uniformly over it. The reactions at A and B will be each $\frac{1}{2}W_4$. Considering CB, and taking moment about C, there will be an anticlockwise moment $\frac{1}{2}W_4 \times \frac{1}{2}L = \frac{1}{4}W_4L$ lb.-inches, produced by the reaction at B. There will also be a clockwise moment owing to half

the total distributed load resting on CB; concentrating this load at its centre of gravity G, the clockwise moment will be $\frac{1}{2}W_4 \times \frac{1}{4}L = \frac{1}{8}W_4L$ lb.-inches. Hence the net moment about C will be $\frac{1}{4}W_4L - \frac{1}{8}W_4L = \frac{1}{8}W_4L$ lb.-inches. The beam may therefore carry a load equal to eight times W_1 in Fig. 98*a*. The comparative strengths of the same piece of material used in these four different ways will be 1 : 2 : 4 : 8.

Beams made of the same material and having rectangular sections, for example, the timber beams used in a house to support floors, may have their strengths compared by the following simple rules. Estimating the strength by the load which may be placed at the middle of the beam, we have:

The central load is proportional to the breadth of the beam and to the square of its depth; the load is also inversely proportional to the span of the beam.

EXAMPLE. Suppose it is found that a beam of cast iron 1″ broad × 1″ deep × 36″ between supports breaks with a load of 6 cwts. at its centre. Calculate the breaking load at the centre of the span for a beam of cast iron 1½″ broad × 3″ deep × 48″ span.

Expressing the above rules for strength in proportional form

$$W_1 : W_2 = \frac{b_1 d_1^2}{l_1} : \frac{b_2 d_2^2}{l_2}.$$

Suffix 1 refers to the given case and suffix 2 to the one to be worked out; this gives

$$6 : W_2 = \frac{1 \times 1^2}{36} : \frac{1\frac{1}{2} \times 3^2}{48},$$

or

$$W_2 \times \tfrac{1}{36} = 6 \times \frac{1\frac{1}{2} \times 9}{48},$$

$$W_2 = \frac{6 \times 3 \times 9 \times 36}{2 \times 48}$$

$$= \underline{60\tfrac{3}{4}} \text{ cwts.}$$

Using a factor of safety of 15, about 4 cwts. would be a safe load for this beam.

The flexibility of a beam may be estimated by the amount of deflection which occurs at the middle when the beam is loaded. For beams of the same material, having rectangular sections and loaded at the middle, the following laws may be noted:

The deflection is proportional to the load and to the cube of the

span; the deflection is inversely proportional to the breadth and to the cube of the depth.

Experiments on beams.—Students should carry out for themselves some experiments on the stiffness and strength of beams. Metal beams are tested best to breaking in a large

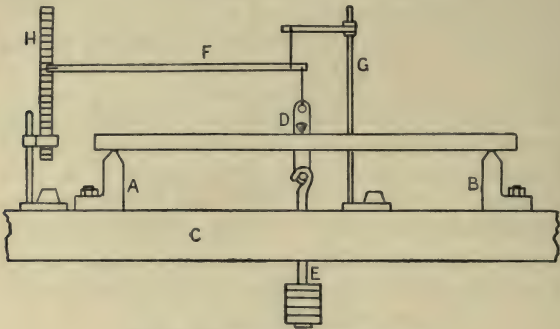


FIG. 99.—Apparatus for testing beams.

testing machine; wooden beams having a section 1″ × 1″ and 36″ span can be broken easily with apparatus similar to that described below, and the same apparatus will do for experiments on the deflection of both metal and timber beams. The apparatus (Fig. 99) consists of two cast iron brackets *A* and *B* which can be clamped anywhere to a lathe bed *C*, or other rigid

FIG. 100.—Timber test beam.

support. These brackets have knife edges at the tops, and the test beams rest on these. A wrought iron stirrup *D*, with a knife edge for resting on the beam, carries a hook *E* for applying a load anywhere to the beam. Deflections may be measured by means of a light lever *F*, pivoted to a fixed support *G*, and attached by a fine wire at its shorter end to the stirrup, the longer end moving over a fixed scale *H* as the beam deflects.

This lever has arms having a ratio of 1 to 10, so that the deflection is multiplied 10 times at the fixed scale. Using a scale of inches divided into tenths, the deflection with this apparatus may be read easily to $\frac{1}{200}$ inch. If the test beam is of timber, small iron plates should be introduced at *a*, *b* and *c* (Fig. 100) in order to prevent indentation of the soft material by the knife-edges, and so to obviate errors in the deflection observations.

Expt. 22.—You are provided with a flat iron bar to be tested for deflection. Arrange the apparatus as shown in Fig. 99; measure the breadth and depth of the bar, using a micrometer; measure the span. Place the stirrup for applying the loads at the middle of the span, hang on the hook, and note the scale reading of the light lever. Apply a series of gradually increasing loads and note the scale reading after the application of each load. Take off the loads in the same order and again note the scale readings after the removal of each load. Tabulate in the following manner:

Test on the Deflection of a Beam.

Load, lbs.	Scale readings, inches.		Deflection in inches.	
	Load increasing.	Load diminishing.	Load increasing.	Load diminishing.
Hook and stirrup only = 0			0	0

Reduce the scale readings to deflections in inches, taking zero deflection to be that produced by the hook and stirrup only, as shown in the table.

Plot graphs showing loads as ordinates and deflections as abscissae; do this both for increasing and diminishing loads. If these graphs are straight lines, it may be inferred that the deflection is proportional to the loads.

Select a point on your graph and note, from the graph scales, the load W and the deflection D corresponding to this point. Use these values in order to calculate Young's modulus for the material of the beam as indicated below.

Let W = load applied, in lbs.
L = length of span, in inches.
D = deflection produced by W, in inches.
b = breadth } of beam of rectangular section,
d = depth } both in inches.

Then, for a test piece used as a beam simply supported at both ends, with the load applied at the middle of the span,

$$E = \frac{1}{4} \cdot \frac{WL^3}{D \,.\, b \,.\, d^3} \text{ lbs. per square inch.}$$

The theory involved in this equation for E is too complicated to be dealt with here.

Repeat the experiment using a timber beam.

Expt. 23.—You are supplied with an iron beam similar in all respects to that used in the above experiment, but of double the breadth. Test it in the same way, and verify the law that the deflection produced by a given load is inversely proportional to the breadth of the beam.

Expt. 24.—An iron beam is supplied similar to that used in Expt. 22, but of double the depth. Test it in the same manner, and verify the law that the deflection produced by a given load is inversely proportional to the cube of the depth.

Expt. 25.—Another similar iron beam is supplied, but of one-half the span. Test it as before, and verify the law that the deflection is proportional to the cube of the span.

Expt. 26.—Prepare several timber beams about 42 inches long and of various breadths and depths. These should be selected as free from knots and shakes as possible, and should be cut from the same timber plank. Test each as a beam supported at the ends and loaded in the middle of the span; increase the load gradually until fracture occurs. A box containing sand or sawdust should be placed on the floor to receive the weights on rupture. From the results, verify the laws of proportional strength given on p. 73. Exact agreement must not be expected; timber is of too variable a nature to enable concordant results to be obtained.

Twisting moment on a shaft.—Shafts are pieces used for the transmission of motion and power from one place to another. They are usually made *round*, and receive a moment tending to rotate them at one place, which moment is transmitted, by stresses in the material of the shaft, to the desired place. Let us consider a shaft AB (Fig. 101) one end of which, A, is fixed in some way, and having an arm BC, mounted at the other end. If a force P is applied at the end of the arm BC, two things will evidently occur: the shaft will tend to *rotate*, or *twist*; also it will tend to *droop*, showing that there is a *bending action*. We may get rid of the tendency to bend by prolonging the arm on the other side of B (as shown dotted in Fig. 101) and applying a force equal and opposite to P at the end E. These forces P, P, now form a couple acting on the shaft, which in consequence is called upon to resist rotation only, and its material will be subjected to pure twist. The moment of the couple is called the **Twisting Moment,** or **Torque.** Thus

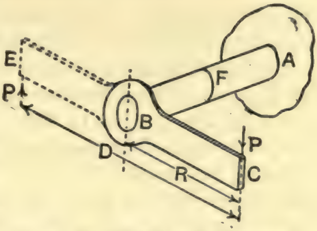


FIG. 101.—Shaft under torque.

$$\text{Torque} = T = P \times D.$$

If we measure the strength of a shaft by the torque which may be safely applied to it, the theory of shafts shows us, that if the shaft is under pure twist, without bending action, **its strength will be independent of its length** and **directly proportional to the cube of its radius.** Thus, a solid shaft 4″ diameter could withstand safely 8 times the torque which could be safely applied to a shaft of the same material, but only 2″ diameter.

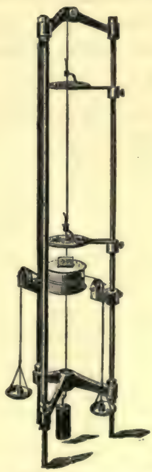


FIG. 102.—Apparatus for experiments on the torsion of wires.

Stiffness of wires under torsion.—The elastic properties of a wire under torsion can be examined by a

self-contained machine such as is shown in Fig. 102. This particular machine was not specially designed, but made out of some materials which happened to be at hand. It consists of an upper bracket for holding the top end of the wire under test, supported by three mild steel rods, which are tied together near their lower ends by another similar bracket. Two pointers can be clamped to any part of the wire, and move over circular scales divided in degrees. The difference of the readings gives the angle of twist of the portion of the wire between the pointers. The torque is applied by two cords coiled round a drum 5″ diameter, clamped to the wire, the cords being led over pulleys and having scale pans at their ends. If equal loads are placed in the pans, they will produce a couple, giving pure twist to the wire. A permanent weight hung to the end of the wire keeps it taut. This machine can be used for verifying the following elastic properties of wire under torsion.

The angle of twist is proportional to the torque applied, directly proportional to the length of the wire, and, for wires of the same material but of different diameters, inversely proportional to the fourth power of their diameters.

Expt. 27.—The method of carrying out an experiment on torsion is indicated sufficiently in the following record of a test.

An Experiment on Twisting.

Steel wire, annealed, 0·065″ diam.

Load in each pan + weight of pan; W lbs.	Torque $= W \times 5$ lb. inches.	Angle of twist on 4·5″ length. Degrees.	Angle of twist on 20·8″ length. Degrees.
0·0	0·0	0·0	0·0
0·20	1·0	1·0	6·0
0·80	4·0	5·0	23·0
1·05	5·25	7·5	29·0
1·55	7·75	9·0	41·0
2·05	10·25	11·0	53·5
2·80	14·0	15·5	73·0
3·00	15·0	16·0	79·0
3·50	17·5	19·0	92·0
3·60	18·0	20·0	94·0
3·85	19·25	21·5	101·0

Columns 2 and 3 when plotted, and also columns 2 and 4, give very nearly a straight line, showing that the angle of twist is proportional to the torque (Fig. 103).

Selecting two values from the plotted curves, when the torque is 14 lb.-inches, the angle of twist on a length of 4·5″ is 15·5 degrees, and the angle for a length of 20·8″ is 73·5 degrees.

$$\text{Ratio of angles of twist} = \frac{73{\cdot}5}{15{\cdot}5} = 4{\cdot}7.$$

$$\text{Ratio of lengths of wire} = \frac{20{\cdot}8}{4{\cdot}5} = 4{\cdot}6.$$

These results are in fair agreement with the law that the angle of twist is proportional to the length of the wire.

Angle of twist after passing the elastic limit.—Test pieces of wrought iron or mild steel subjected to torsion, after the

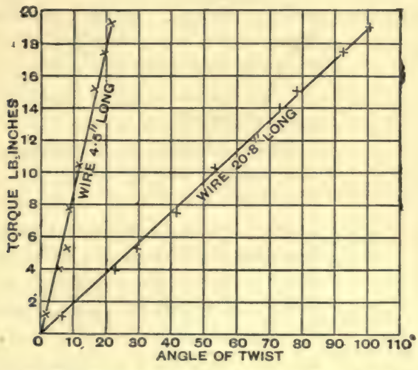


FIG. 103.—Plotted diagram, showing torque and angles of twist for a wire under torsion.

elastic limit of the material for shearing stresses is passed, do not twist through angles proportional to the torque. After this, the angle is much larger proportionally, and a piece of great length compared to its diameter will twist through many complete turns before fracture occurs. For this reason, it is convenient to use rather short pieces of the material for such testing purposes.

Springs.—Springs are pieces of material intended to take a relatively large amount of strain, although any body which can be strained and shows a strong tendency to recover its original shape *freely* may be called a **spring**. Thus, a bar of iron or steel, pulled within its elastic limit, may be called a spring. The forms taken by springs depend on the purpose for which they are intended. For spring balances, used to measure forces, **helical springs** are used (Fig. 104). In these, the forces are applied in the direction of the axis of the spring and the material of the spring is under torsion. Springs are used for measuring forces, for storing energy, and for minimising the effects of shocks. In all cases, the loading is kept within the elastic limit, and consequently, as a rule, the spring is distorted. or changes its length, by an amount proportional to the applied forces.

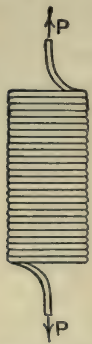


FIG. 104.—Helical spring.

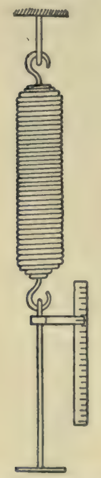


FIG. 105.—Apparatus for measuring extensions of a spring.

Elastic extension of springs. EXPT. 28. —Arrange a helical spring as shown in Fig. 105, and load it with gradually increasing weights; the extensions are given by a pointer attached to the spring and moving over a scale. Plot extensions and loads. It will be found that, if several springs of the same material are available for testing, the following laws are approximately true:

The extensions are proportional to the load, to the cube of the radius of the helix, and to the number of complete turns in the helix; also inversely proportional to the fourth power of the radius of the wire of which the helix is made.

In these experiments, the wire is assumed to be round, and the springs made so that the coils lie close together.

The proportional laws may be represented by an equation in this way.

Let

W = load applied, lbs., (Fig. 106).

R = radius of helix to centre of wire, in inches.

N = number of complete turns.

r = radius of wire, in inches.

X = extension produced by W, inches.

Then

$$X = c\frac{WR^3N}{r^4},$$

where c is a constant depending on the elastic qualities of the material. Its value for round steel wire is approximately

$$c = 0{\cdot}00000033.$$

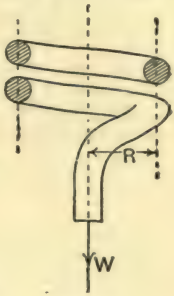


FIG. 106.

EXERCISES ON CHAP. VI.

1. A timber beam, 10 ft. span, supported at its ends, carries a load of 600 lbs. at its middle. Calculate the Bending Moment at its middle.

2. A cast iron cantilever projects 6 ft. from a wall and carries a load of 500 lbs. at its end. Calculate the Bending Moment at the wall.

3. A cantilever projects 8 ft. from a wall and carries a load of 400 lbs. uniformly distributed. Calculate the Bending Moment at the wall.

4. A beam, 12 ft. span, supported at its ends, carries a uniformly distributed load of 2,400 lbs. Calculate the Bending Moment at the middle.

5. It is found that a certain beam may carry safely a load of 2 tons at the middle of the span. Supposing that the same piece of material were used as a cantilever carrying a load (*a*) at its end, (*b*) uniformly distributed, what loads would be safe?

6. A cast iron cantilever 1″ long, 1″ broad, 1″ deep breaks with a load of 30 cwts. at its end. Calculate the safe load for a cantilever 4″ broad, 1¼″ deep, 3″ long, taking a factor of safety = 12.

7. A bar 2 ft. long, 2″ × 2″ square section is supported at its ends and breaks when a load of 3 tons is placed at its middle. Calculate the working load of a bar 5 ft. long, 6″ deep, and 4″ broad, taking a factor of safety = 12. What distributed load would be safe?

8. A bar of mild steel, section $1'' \times 1''$, is supported at points 40″ apart. A load of 10 lbs. being applied at the middle of the span, the deflection is observed to be 0·0053″. Calculate the value of E for the material.

9. A beam of wood, rectangular in section, is fixed at one end and loaded at the other. What is occurring at various places in any imaginary cross section? Sketch anything you have seen or used which illustrates your ideas about bending.

10. How would you carry out tests for verifying the comparative laws of deflection of beams? What results would you expect to find?

11. It is found that a shaft 2″ in diameter may carry a torque of 12,000 lb.-inches. What torque would be safe for a shaft 3″ in diameter and made of the same material?

12. Describe, with sketches, how you would verify the laws that the angle of twist of a wire is proportional to the torque and to the length of the wire.

13. Describe how you would verify the law for a helical spring that the extension is proportional to the load. How would you expect to find the actual extension affected by the dimensions of the spring?

CHAPTER VII.

WORK. MECHANICAL ADVANTAGE. VELOCITY RATIO OF MACHINES. ENERGY. POWER. EFFICIENCY. DIAGRAMS OF WORK. FRICTION.

Definitions of terms.—A force is said to be doing **work** when it acts through a distance, overcoming resistance. The **quantity of work** done is proportional jointly to the magnitude of the force and the distance through which it acts, the distance being always measured along, or parallel to, the line of action of the force. The **unit of work** in general use in this country is the **foot pound, and is that quantity of work which is done when a force of one pound acts through a distance of one foot in its line of action.** The **inch-ton** and **foot-ton** are also sometimes used, these being the work done when a force of one ton acts through a distance of one inch or one foot respectively.

The work done by any force is calculated by taking the product of the magnitude of the force and the distance through which it acts.

EXAMPLE 1. If a weight of 4 tons has to be raised from the bottom of a shaft 100 fathoms deep, find the work done.

$$\begin{aligned}\text{Work done} &= \text{force} \times \text{distance}\\ &= 4 \times (100 \times 6)\\ &= \underline{\underline{2400}}\ \text{foot-tons.}\end{aligned}$$

EXAMPLE 2. A load of 2 cwts. is dragged along a level floor through a distance of 10 feet by means of a rope inclined at 30° to the floor (Fig. 107). The pull P is found to be 80 lbs. Calculate the work done.

First Solution. Notice that P does not act through a distance $AB = 10$ feet, but through $AC = 5\sqrt{3}$ feet (Fig. 107). Hence,

$$\text{work done} = 80 \times 5 \times 1{\cdot}73$$
$$= \underline{692} \text{ foot-lbs.}$$

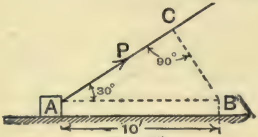


FIG. 107.

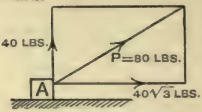


FIG. 108.

Second Solution. Or, we may solve the question in another way: Take horizontal and vertical components of the pull in the rope (Fig. 108) giving $40\sqrt{3}$ lbs. for the horizontal and 40 lbs. for the vertical component.

This vertical component, while the weight is being dragged along, will always be acting in parallel vertical directions, and its point of application will move neither up nor down. Consequently, as it does not act through a distance measured along or parallel to its line of direction, no work is done by it. The horizontal component acts through a distance of 10 feet measured along its line of direction, consequently work done will be

$$40\sqrt{3} \times 10$$
$$= \underline{692} \text{ foot-lbs., as before.}$$

The same quantity of work may be done either by a small force acting through a large distance, or a large force acting through a small distance. Thus, suppose we have to do 1,000 ft.-lbs. of work, we may use a force of 1 lb. acting through 1,000 feet, or 500 lbs. acting through 2 feet, obtaining the desired result in each case.

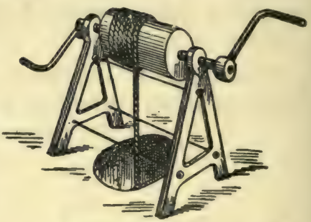


FIG. 109.—Simple winch.

Machines. — **Machines** are arrangements receiving work from some outside source of supply, which work is modified by the machine and delivered in some form more suitable for the purpose required.

As an example of a machine, we may take a **simple winch** (Fig. 109) used for raising loads. This machine takes in work from the pushes and pulls of two

men at the handles, turning the drum. The drum produces a modified pull on the lifting rope by which work is done on the ascending weight.

Usually in machines such as hoisting tackle, machine tools, etc., the force delivering work to the machine is smaller than the resistance which is overcome by the use of the machine. The **Mechanical Advantage** of a machine is the ratio of these two forces. Thus, in the above winch, suppose that each man exerts a constant force of 30 lbs. applied always tangential to the path of his hand (Fig. 110), the load raised being 300 lbs., then the Mechanical Advantage will be $\frac{300}{2 \times 30} = 5$.

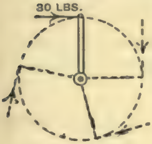


FIG. 110.

The **Velocity Ratio** of a machine is measured by dividing the distance through which the force applied to the machine acts, by the distance through which the resistance is overcome in the same time. Thus, in the above winch, if the men's hands move through a circumferential distance of 6 feet while the weight is being raised a height of one foot, then the velocity ratio is 6.

Energy and its transformation.—If work is imparted to a body so that it stores it up and is capable of giving it out again, the body is said to possess **Energy.**

Energy means capability of doing work. If we raise a one pound weight from the table a height of one foot, we have done one foot-pound of work on the weight, and this work it can give out again if we allow it to descend. The raised weight possesses energy to the amount of one foot-pound. The individual who raised the weight has had to part with one foot-pound of his store of internal energy, and if he goes on for some time raising weights, say for 4 or 5 hours, he will find that there is need to replenish his store of energy by absorbing some food and resting a little. Food possesses energy which, transformed by the organs of our body, enables us to do mechanical work. The energy of food is liberated in our bodies in the form of heat by a process of slow combustion. Coal also possesses heat energy which is utilised by a much more rapid combustion in boiler furnaces or otherwise. Water at an elevation possesses energy which can be transformed into mechanical work while the water is descending,

and the atmosphere has energy also when in motion as wind. The principal source of all these stores of energy is the heat of the sun, which (i), in past ages, caused the vegetation to grow from which we at present derive our coal supply, and (ii) now gives us our food supply by giving life to plants. The heat of the sun also raises water by evaporation and so gives it its store of energy, and it also sets the atmosphere in motion as wind.

Conservation of energy.—The principle of the **Conservation of Energy** asserts that *man is not able to create or destroy energy, he can only transform it from one form into another.* This principle is the result of the observation and experiment of many people, including those who have sought in vain for perpetual motion. To the engineer, it is of extreme importance. Thus, if we impart to any machine a certain quantity of energy, and no energy is lost in the machine or used to coil up a spring belonging to the machine or do any other form of work in the machine, then the machine will deliver an exactly equal quantity to that given to it. It cannot deliver more, for then it would create energy; nor can it deliver less, for then energy would be destroyed in the machine. This principle is sometimes referred to as the **principle of work.**

Actually, it is impossible to construct a machine in which there is no energy lost, whether by the rubbing of surfaces on one another, by churning the atmosphere, or by the development of sound, and other causes. But we can assert about all machines:

Energy supplied = Energy given out + energy lost in overcoming resistances in the machine.

If the machine is *running light*, *i.e.* doing no useful work against resistances, then we must supply energy sufficient to make good that lost by resistances in the machine. If the machine is *doing useful work*, then we must in addition supply energy equivalent to this useful work.

Efficiency of machines.—The **efficiency of any machine** is measured by the ratio of energy actually given out as useful work to the energy supplied. Or,

$$\text{Efficiency} = \frac{\text{useful work done}}{\text{energy supplied}}.$$

EXAMPLE. A certain machine is found to give out 125 foot pounds of useful work when 180 foot pounds of energy are supplied to it. Find the efficiency of the machine.

$$\text{Energy lost in machine} = 180 - 125$$
$$= 55 \text{ foot pounds.}$$
$$\text{Efficiency} = \tfrac{125}{180} = \underline{0{\cdot}69.}$$

The efficiency may be stated as a percentage by multiplying this by 100, giving

$$\text{Efficiency} = 0{\cdot}69 \times 100$$
$$= \underline{69} \text{ per cent.}$$

Useful forms of energy.—The principal forms of energy that have to be dealt with are : **Potential Energy**, such as the energy of a raised weight or a coiled spring ; **Kinetic Energy**, which a body possesses when it is in motion, and can give out as mechanical work while it is coming to rest ; both these forms of energy are stated in foot pounds. One foot pound of potential energy is exactly equivalent to one foot pound of kinetic energy. For example, a raised weight possesses potential energy, and if it is allowed to fall freely, doing no work against any resistance, the potential energy will be converted into an exactly equal quantity of kinetic energy.

Heat is a form of energy which can be converted into mechanical work. The amount of mechanical work equivalent to a given quantity of heat is known with considerable accuracy from the experiments of Dr. Joule and others. Thus, 778 foot pounds of mechanical work transformed into heat would, if all the heat passed into one pound of water, raise its temperature one degree Fahrenheit. The quantity of heat which would raise the temperature of one pound of water through one degree Fahrenheit is the unit of heat used by British engineers, and is called the **British Thermal Unit**.

Electrical energy is measured in *kilowatts performed per hour.* The **kilowatt** is an electrical *power unit*, and consequently gives the rate of energy production, corresponding to the horse-power (p. 89). A kilowatt is equal to 1000 watts, the **watt** being the rate of working when an electric current of one ampere flows from one point of a conductor to another, the potential difference between which is one volt. The product of amperes and volts

is expressed in watts. 746 watts are equivalent to the mechanical horse-power. It will be understood that just as we have to state the time during which a given horse-power has to be maintained in order to produce a certain amount of work, so we must state, as above, the time during which one kilowatt has to be maintained in order to produce a given amount of electrical energy. The Board of Trade unit of electrical energy is that given above as one kilowatt maintained for one hour.

Loss of useful energy.—It should be clearly understood that although energy in one form may be exactly equivalent to a certain quantity of energy in another form, that we never succeed, in any transformation of energy, in obtaining an exactly equal quantity in the new form. There are always losses—sometimes great losses—which are inevitable. As a common example, and one which gives a fair idea of the magnitudes of some of these losses, take the following case of electrical power production. Suppose 100 units of energy to be liberated from some coal in the boiler furnace. About 75 of these will enter the steam and the remaining 25 will be lost by the passage of the smoke and heated gases up the chimney, or by radiation and other causes. Of the 75 units of energy reaching the engine in the steam, about 6 will be converted into mechanical energy and the remaining 69 will be lost. The 6 units of mechanical energy given to the dynamo will produce about 5 units of electrical energy, 1 unit being lost. If these 5 units be reconverted into mechanical energy by an electrical motor, about 4·5 units will be produced. We utilise in this way about 4·5 per cent. of the original energy and lose 95·5 per cent.

Variation of the actual mechanical advantage.—From what has been said it will be seen that the velocity ratio of a machine, which depends solely on the arrangement and nature of its parts, does not change provided the arrangement remains the same. On the other hand, the mechanical advantage, or the ratio of the resistance overcome to the force delivering energy to the machine, depends on the extent of the losses in the machine; these again are variable, depending on the load and on the condition of the machine as regards lubrication and state of the bearing surfaces.

Let P = the force delivering energy to a machine, and W = the

resistance overcome, both in same units of force. Let D=distance through which P acts while W is being overcome through a distance=d, D and d being in the same units of distance. Then, if there were no losses in the machine,

$$\text{Energy supplied} = \text{useful work done},$$
$$P \times D = W \times d,$$
$$\frac{D}{d} = \frac{W}{P}.$$

Now $\frac{D}{d}$ is the velocity ratio of the machine and $\frac{W}{P}$ is the mechanical advantage, so that in this hypothetical case, the velocity ratio and mechanical advantage are equal numerically. Actually, however, W will always be less than its value assumed above, and consequently the actual mechanical advantage will always be less than the velocity ratio for any machine. In Chap. VIII. it will be seen how the actual mechanical advantage of a machine can be obtained. The velocity ratio can be calculated from a knowledge of the mechanism, or by direct measurement, at the places where P and W are applied, of the distances through which they act.

Power.—If we state not only the quantity of work done by a force or forces, but also the time in which it is done, this will give us the rate at which work is being performed. **Power**, or **activity**, is the name given to the rate of performing work. The **unit of power** used generally by engineers in this country is produced when 33,000 foot-pounds of work are done in one minute. This unit was defined by James Watt, who found that the average horse could do about 22,000 foot-pounds of work in a minute. Watt added 50 per cent. to this and took 33,000 foot-pounds per minute as the unit of power to be used in measuring the performance of his steam engines.

The horse-power developed in any given case will be ascertained by first calculating the work done per minute, in foot-pounds, and dividing the result by 33,000.

Thus, suppose we take the previous example (p. 83), in which 4 tons were raised from a depth of 100 fathoms. If the work is performed in 40 seconds, the horse-power required may be calculated thus :

$$\text{Work done in 40 seconds} = (4 \times 2240) \times (100 \times 6) \text{ ft.-lbs.}$$

$$\text{Work done in 60 seconds} = 8960 \times 600 \times \frac{60}{40} \text{ ft.-lbs.}$$

$$= 8{,}064{,}000 \text{ ft.-lbs.}$$

$$\text{Horse-power} = \frac{8{,}064{,}000}{33{,}000}$$

$$= \underline{244 \cdot 4.}$$

In calculating the horse-power required in any given case, the efficiency of the machine employed must be considered, as power will be required for overcoming losses in the machine. Thus, suppose in the above case, that the efficiency of the mechanism of the winding engine employed to raise the 4 tons is 60%. This means that 60% of the energy given to the engine in a given time, say one minute, is turned into useful work. Consequently, the 8,064,000 ft.-lbs. useful work above done per minute is only $\frac{6}{10}$ths of the energy that must be given to the engine per minute.

$$\text{Energy supplied to engine per min.} = \frac{8{,}064{,}000 \times 10}{6} \text{ ft.-lbs.,}$$

$$\text{and, Actual Horse-power required} = \frac{8{,}064{,}000 \times 10}{6 \times 33{,}000}$$

$$= \underline{407.}$$

Graphic representation of work.—Since work is measured by the product of two quantities, force and distance, we may represent it by the area of a diagram. Thus, supposing a uniform force P to act through a distance D, the work done will be $P \times D$. If we set off D to scale in a diagram (Fig. 111), and erect ordinates of constant height equal to P to scale, we obtain a rectangle of area equal to PD, which therefore represents the work done.

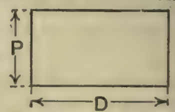


FIG. 111.—Work done by a uniform force.

If the work is done by a varying force, and if the latter is known at various points in the distance acted through, we may set up ordinates to represent its value at these points, giving a diagram as shown in Fig. 112 bounded by a curve at the top. The work done in this case will be equal to the average value of P multiplied by D. Now the average value of P will be represented by the average height of the diagram to

scale; and as the average height multiplied by D gives the area of the figure, it follows that in this case also, the area of the figure represents the work done. The average height of a diagram of work resembling that given in Fig. 112 may be found by Simpson's rule or by the engineer's rule stated on p. 8.

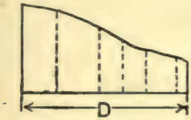


FIG. 112.—Work done by a varying force.

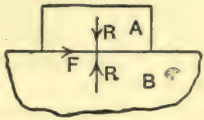


FIG. 113.—Frictional resistance to sliding.

Friction.—When two bodies are pressed together, resistance has to be overcome before they can be made to slide on one another. This resistance is called the **force of friction.** The force which friction gives to a body always acts contrary to the direction of motion of the body and tends to arrest the motion, or, if the body is at rest, to prevent motion taking place. In many practical problems, frictional forces have to be considered with a view to their reduction; in others, friction is useful by preventing slipping taking place.

If two bodies A and B (Fig. 113) are pressed together so that the mutual pressure perpendicular to the surfaces in contact is R, and if F is the force of friction which has to be overcome before sliding will take place, then $\frac{F}{R}$ is called the **coefficient of friction of rest,** or the **static coefficient of friction** of the bodies. If the bodies are sliding steadily on one another, and F' is the steady resistance to sliding, then $\frac{F'}{R}$ is called the **coefficient of friction of motion,** or the **kinetic coefficient of friction** of the bodies.

Conditions influencing friction.—The value of the coefficient of friction for two given bodies depends on the nature of the materials of which they are made, especially *on their hardness and ability to take on a smooth regular surface, and on the state of the rubbing surfaces as regards cleanliness and lubrication.* With

dry, clean surfaces, the force of friction is produced largely by roughnesses on the surface of one body interlocking with roughnesses on the surface of the other. The surfaces which have to be dealt with in engineering work are usually of fair shape and satisfactorily fitted to one another, but even these do not bear on one another all over, but only in places, and when sliding takes place, the projections on one body have to get over, or, if the forces pressing them together are large enough, to cut away, or abrade, the projections on the other. That body which is the more easily replaced is generally made of softer material, in order to confine the wear principally to it.

When clean, dry surfaces, well fitted to one another, are brought together, a film of air may exist between them and thus prevent the bodies from actual contact. This is very noticeable when one surface plate is laid on another. If the cast iron surfaces are perfectly clean, the upper plate seems to float on the lower one. By pressure and working a little, the air film may be eliminated. The plates then adhere strongly together, or *seize*, partly on account of the vacuum between them, but more, since the effect takes place even in a good vacuum, on account of molecular forces of attraction being brought into play. Seizing takes place more readily with bodies of the same than with those of different materials. In practice a film of lubricant is used to keep the rubbing bodies as far as possible, from contact with one another, and the working load is such that there is no danger of the film being squeezed out.

Laws of friction for dry surfaces.—For bodies with rubbing surfaces dry, and perfectly clean or only slightly contaminated by films of foreign matter, the following laws of friction have been deduced from the results of experiments: **The static coefficient of friction is greater than the kinetic coefficient of friction**; in other words, **the resistance offered to sliding when the bodies are at rest is greater than that after steady rubbing has been attained.**

The force of friction is practically proportional to the perpendicular pressure between the surfaces in contact and is independent of the extent of such surfaces and of the speed of rubbing, if moderate.

From this we infer that for two given bodies, **the coefficient of friction is practically constant for moderate pressures and speeds.**

Considerable increase in the speed lowers the value of the

coefficient of friction, and heating of the bodies produces the same effect. It has also been found that the coefficient of friction is a little greater for light pressures on large areas than for heavy pressures on small areas.

Experiments on friction.—Students should verify experimentally as many as possible of the above statements.

Expt. 29.—Set up a board AB (Fig. 114) as nearly horizontal as possible, and arrange a slider C (which can be loaded to any

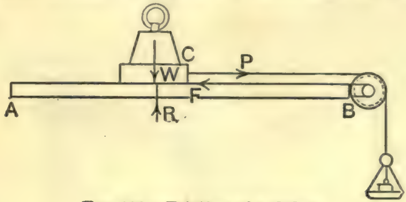


Fig. 114.—Friction of a slider.

desired amount) with a cord, pulley, and scale pan, so that the horizontal force P required to overcome frictional resistances to sliding may be measured. Under these conditions, the perpendicular pressure between the surfaces in contact will be equal to the weight of the slider and loads placed on it, W say, and its actual distribution over the surfaces in contact need not concern us at present. The force of friction F will be equal to P, and this will very nearly equal the weight of the scale pan and loads placed in it, provided the pulley used is finely mounted on pivot bearings and oiled so as to run very freely.

The coefficient of friction will be $\frac{P}{W}$.

First, make a number of experiments on the static coefficient of friction, using different loads on the slider. In each experiment place loads carefully into the scale pan so as to avoid jerks until the slider starts off. From the observations the static coefficient will be found.

Using the same loads on the slider, perform the same processes, only this time help the slider to start by jerking it. Adjust the load in the scale pan until steady uniform motion, as nearly as you can judge, has been obtained. From these observations the kinetic coefficient will be found.

Some results are given in order to show the method of recording.

An Experiment to Determine Coefficients of Friction.

Material of slider—mahogany.

Material of board—teak.

Rubbing surfaces—slightly contaminated with dust and finger marks.

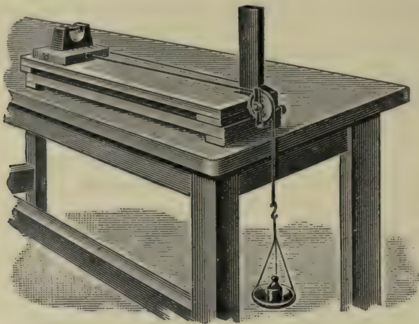


Fig. 115.—Apparatus for determination of the friction of a slider.

Area of sliding surface of slider $=6'' \times 6''=36$ square inches.
Weight of slider $=0{\cdot}701$ lb.
Weight of scale pan and hook for applying $P=0{\cdot}701$ lb.

W lbs.	Grain of slider parallel to direction of motion.				Grain of slider perpendicular to direction of motion.	
	Static Values.		Kinetic Values.		Kinetic Values.	
	P_1 lbs.	$\frac{P_1}{W}$	P_2 lbs.	$\frac{P_2}{W}$	P_3 lbs.	$\frac{P_3}{W}$
2·701	0·74	0·274	0·49	0·181	0·51	0·188
4·701	1·43	0·304	0·901	0·191	0·901	0·191
6·701	2·69	0·401	1·301	0·194	1·301	0·194
8·701	3·04	0·35	1·701	0·195	1·65	0·19
10·701	3·54	0·335	2·201	0·206	2·09	0·195
12·701	4·501	0·355	2·501	0·197	2·401	0·189
14·701	4·701	0·32	2·901	0·197	2·90	0·197
16·701	5·04	0·301	3·501	0·209	3·40	0·203

A graph should be drawn showing the plotted results for the experiments in which the grain of the slider was parallel to the direction of motion, when it will be found that the plotted points, especially those for the kinetic values, fall approximately on the straight lines which may be drawn to lie fairly among the plotted observations. The plotted points would all lie exactly in a straight line had the friction been proportional to the load and the experiment been perfectly performed.

Expt. 30.—For showing any difference in the coefficient of friction produced by changing the *extent of the surfaces in contact*, four sliders cut from the same teak plank may be used. Each slider measures 6″ × 6″. One has the full surface, the others are cut away (Fig. 116) on the under side so as to have rubbing surfaces respectively of 27, 18 and 9 square inches. The rubbing surfaces are thus in the proportion 1 : 0·75 : 0·5 : 0·25.

Fig. 116.—Sliders having different areas of rubbing surfaces.

The average results of some experiments are given, the sliders having their grain parallel to the direction of motion and sliding on a teak board.

Effect of Extent of Surface on the Coefficient of Friction.

Slider.	Proportional Area of Rubbing Surface.	Weight of Slider, lb.	Average Coefficient of Friction.
A	1·0	0·88	0·225
B	0·75	0·83	0·213
C	0·5	0·78	0·206
D	0·25	0·718	0·200

It will be observed from these figures that the coefficient of friction is rather less for the smaller rubbing surfaces, that is, it diminishes as the pressure per square inch rises. The law that *friction is independent of the extent of the surfaces in contact* is, however, shown to be approximately true.

Fluid friction.—The laws of friction for fluids differ considerably from those stated for dry surfaces. Experimental data is generally obtained by investigating the flow of fluids through pipes, under varying conditions of speed, temperature and roughness of surface. For liquids such as water and oils the laws have been shown experimentally to be as follows:

The resistance is proportional to the extent of the surface wetted by the liquid.

The resistance is independent of the material of which the boundary is made and of its surface, provided it is not too rough.

The resistance is independent of the pressure to which the liquid is subjected.

Rise of temperature of the liquid diminishes the resistance.

At slow speeds the resistance is very small.

Below a certain critical speed, the resistance is proportional to the speed; at speeds above this, the resistance is proportional to some power, approximately the square, of the speed.

The critical speed depends on the liquid used and its temperature. Below this speed the motion of the liquid is steady; above it, the liquid breaks up into eddies.

Liquids which can flow, or change their shapes more easily than others, are said to be less **viscous**, or to possess less **viscosity.** All liquids have one property in common—*they are unable to resist shearing forces and yet remain at rest.* Now friction is always manifest as a tangential or shearing force, and it therefore follows that if a liquid is at rest there can be no frictional resistances of any kind present.

Laws of friction for ordinary bearings.—The laws of friction for ordinary bearings are intermediate between those for liquids and for dry surfaces. In bearings running in a bath of oil, the laws have been shown to be approximately the same as those of fluid friction, and in other bearings the resistances experienced depend on the success which is achieved in getting the oil into the bearing and in preserving the oil film. It is not usual, in investigating the losses due to frictional resistances of a number of bearings such as we find in any machine, to attempt to ascertain how much is lost at each bearing, but to find simply how much is lost at all the bearings collectively. *Almost always it is found that the frictional losses, when the*

machine is loaded, are equal to those of the unloaded machine together with a small fraction of the load. If we were to find that the frictional losses in a machine were constant for all loads, then we might deduce that the frictional resistances have been altogether due to fluid friction, this being independent of the load.

Effect of friction in machines.—It is useful, in dealing with simple machines such as hoisting tackle, to deduce an equation connecting the **effect of friction** in the machine with the actual load applied.

Let P = force applied to work the machine;

v = velocity ratio of machine;

W = actual load raised;

P and W being in the same units.

Suppose W to be raised one foot, then P will act through v feet.

Energy supplied to machine $= Pv$;

Useful energy obtained from machine $= W \times 1 = W$.

Imagine the actual frictional resistances of the machine to be removed, and an equivalent addition to the load W to be made, so that P is unaltered. Call this additional load F = effect of friction, and measure it in the same units as P and W.

Energy lost in overcoming frictional resistances $= F \times 1 = F$.

By the principle of the conservation of energy, energy supplied = useful energy obtained + energy lost.

$$\therefore \; Pv = W + F,$$

or,

$$F = Pv - W \qquad (1)$$

If experiments have been carried out on a given machine, a series of values of P and W will have been obtained. From (1), corresponding values of F can be calculated. On plotting the values of F and W so found, it will generally be found that the plotted points lie approximately on a straight line, showing that the connection between F and W can be represented by the equation

$$F = aW + b, \qquad (2)$$

where a and b are constants for the machine.

EXERCISES ON CHAP. VII.

1. A load of 2 tons is raised from the bottom of a shaft 300 ft. deep. How much work is done? Draw a diagram to represent this work.

2. In Question (1) the load is raised by a wire rope weighing 3 lbs. per foot length. Calculate the work done in raising the rope alone, and draw a diagram of work done.

3. A loaded truck, weight 12 tons, is pulled along a level track. If the resistances to motion are 11 lbs. per ton weight, calculate the work done in pulling the truck a distance of one mile.

4. A bridge girder weighs 15 tons, and is to rest on supports 25 ft. above the level of the ground. Calculate the work done in raising the girder into position.

5. A man weighs 140 lbs. Calculate the total work he has to do in carrying his bicycle, weight 30 lbs., upstairs to a room 20 ft. above the street level.

6. A man exerts a constant force of 30 lbs. in turning a handle of 14″ radius; calculate the work done per revolution if (*a*) the force is always exerted in a horizontal direction, (*b*) the force is always exerted tangential to the circle described by the handle.

7. A tank measures 10 ft. long × 6 ft. wide × 3 ft. deep, and is at a height of 200 ft. above the level of the pump used for filling it with water. Calculate the work done in filling it, taking one cubic foot of water to weigh 62·5 lbs.

8. What work is done in raising a bucket, weight 2 lbs., containing 25 lbs. of water from a well the surface level of which is 12 feet below ground level?

9. A shaft, 10 ft. diam., 100 feet deep, is full of water. Calculate the work done in emptying it.

10. The weight of a pile driver is 1250 lbs., and it is raised 6 ft. above the pile head before delivering a blow. Calculate its potential energy when raised.

11. One cubic foot of a gas contains 600 British thermal units. To how much mechanical work is this equivalent?

12. Find the mechanical work equivalent to the heat contained in a pound of petroleum of heating value 20,000 British thermal units.

13. A horse exerts a constant pull of 80 lbs. in dragging a cart along a level road. If he walks at the rate of 3 miles an hour, what horse-power is he developing?

14. A locomotive exerts a steady pull of 2500 lbs. in hauling a train along a level track. If the speed is 4200 feet per minute. calculate the horse-power.

15. Calculate the horse-power required to pump 5000 gallons of water per minute from a well 40 feet deep to the surface of the water if the efficiency of the machinery employed is 60 per cent.

16. 15,000,000 ft.-lbs. of energy are given to an engine per hour, and the horse-power developed is $1\frac{1}{2}$. What is the efficiency of the engine?

17. A man of 150 lbs. climbs a hill regularly 1200′ vertically per hour; at another time he climbs a staircase at $2\frac{1}{2}'$ per second; in each case find the useful horse-power in lifting himself.

18. A centrifugal pump is to lift 6·2 cubic feet of water per second to a height of 7 feet; how much horse-power must be supplied to it if its efficiency is 45 per cent.?

It is direct driven by a continuous current electro-motor which works at 200 volts. How many amperes of current must be supplied to the motor, if its efficiency is 85 per cent.?

19. It is found that a horizontal force of 8 lbs. can keep a load whose weight is 30 lbs. in steady motion along a horizontal surface. What is the coefficient of friction?

20. A block whose weight is 10 lbs. rests on a horizontal table; it is found that a pull of 4 lbs., applied at 30° to the table, just starts it off. What is the static coefficient of friction?

21. Answer Question 20 supposing the force applied had been a push.

22. A train whose speed is $\frac{1}{2}$ mile per minute has frictional resistances amounting to 12 lbs. per ton weight of train. If this weight is 150 tons, calculate the pulling force required and the horse-power of the engine.

23. Define "force," "work," "foot-pound," and "horse-power." A small metal planing machine, the table of which weighs 1 cwt., makes 6 backward and 6 forward strokes each of $4\frac{1}{2}$ feet in a minute, and the coefficient of friction between the sliding surfaces is 0·07. What is the work performed in foot-pounds per minute in moving the table?

24. An express train going at 40 miles per hour weighs 150 tons; the average pull on it is 12 lbs. per ton, what is the horse-power exerted? This power is only 40 per cent. of the total indicated power of the engine; find the indicated power.

25. Describe any experiments you have made or seen for finding the laws of solid friction. What are the laws so found? Are they quite true? How do they differ from the laws of fluid friction?

CHAPTER VIII.

SIMPLE MECHANISM.

Driving by belt.—Motion may be transmitted from one shaft to another in many different ways. If the shafts are parallel to one another and a considerable distance apart, the most convenient way is by the use of a belt, or rope, lapping round pulleys fixed to the two shafts. Both shafts will rotate in the same direction if the belt is open, as in Fig. 117, and in opposite directions if the belt is crossed, as in Fig. 118. A, the shaft supplying motion, is called the **driver**, B is the **driven shaft.** The action of driving is possible by reason of the frictional resistance to slipping of the belt on the pulleys, but there will always be a certain amount of slipping, introducing some loss of motion. Neglecting this, the velocity ratio of the shafts may be found thus.

Fig. 117.—Driving by open belt.

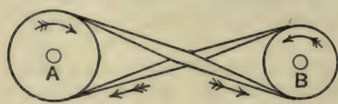


Fig. 118.—Driving by crossed belt.

Let R_A = radius of the pulley on A.
R_B = " " " B.

Then, if there is no slipping, the circumferences of both pulleys will move through the same distance in a given time, for each will have the same speed as the belt. Suppose, then, that A turns once; its circumference will travel a distance $= 2\pi R_A$.

The circumference of B will move through an equal distance, and consequently B will turn through $\frac{2\pi R_A}{\text{circumference of } B}$ revolutions, or

$$\text{Revolutions of } B \text{ for one of } A = \frac{2\pi R_A}{2\pi R_B} = \frac{R_A}{R_B}.$$

If then, A rotates N_A times in a minute, and B rotates N_B times also in a minute,

$$\frac{N_B}{N_A} = \frac{R_A}{R_B},$$

or the revolutions of the shafts are inversely proportional to the radii of the pulleys mounted on them.

The power transmitted in any given case can easily be calculated if we know the ratio of the tensions in the two parts of the driving belt. This ratio may be taken to be about 0·4 for a leather belt on a cast iron pulley.

Let T_1 = pull in the tight part of the belt, lbs.
T_2 = „ slack „ lbs.
V = distance travelled in one minute by a point on the belt, in feet.

Then, considering the driver A (Fig. 119), T_2 is assisting the pulley to turn and T_1 is retarding it, so that the driver delivers a net pull $(T_1 - T_2)$ pounds, by means of the belt, to the driven pulley. The work done in one minute will be W foot-pounds.

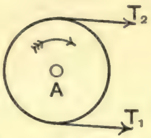


Fig. 119.—Pulls transmitted by the belt.

$W = (T_1 - T_2) \times V$ ft.-lbs. per minute, and

$$\text{Horse-power transmitted} = \frac{(T_1 - T_2)V}{33{,}000}.$$

Let us take roughly $T_2 = 0·4 \,.\, T_1$, then

$$T_1 - T_2 = 0·6 \,.\, T_1$$

and $$\text{Horse-power transmitted} = \frac{0·6\,T_1}{33{,}000} \,.\, V.$$

Example. A belt running at 900 ft. per minute has a pull in its tight part of 400 lbs. Calculate the horse-power transmitted.

$$\text{H.P.} = \frac{0·6 \times 400 \times 900}{33{,}000}$$
$$= \underline{6·5.}$$

If, instead of the speed of the belt, we are given the diameter, D ft., of the pulley, and N, the number of revolutions per minute of the shaft on which it is mounted, then

$$V = \pi D \times N,$$

and
$$\text{Horse-power transmitted} = \frac{(T_1 - T_2)\pi . D . N}{33,000}.$$

Slipping of the belt introduces a loss of energy in overcoming frictional resistances between the belt and pulley. The amount of slipping is variable, and depends on the power transmitted and the tightness of the belt.

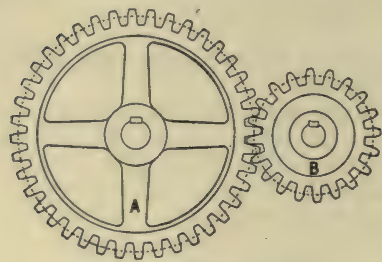


FIG. 120.—Toothed wheels in gear.

Toothed wheels.—Driving by use of toothed wheels (Fig. 120), has an advantage over belt driving in that there is no slip possible. The velocity ratio of the wheels may be found in the following manner :

Let
n_A = number of teeth on A's rim.
n_B = " " B's "
N_A = revolutions per min. of A.
N_B = revs. per min. of B.

If A makes one revolution, it will cause n_A teeth on B to pass the point of contact of the wheels. As there are n_B teeth on B, B will rotate $\frac{n_A}{n_B}$ times ; hence

$$\frac{N_A}{N_B} = \frac{n_B}{n_A},$$

or, **the revolutions per minute of the two wheels are inversely proportional to their numbers of teeth.**

The dotted **pitch circles** (Fig. 120) touch one another and are used in drawing the teeth ; the **pitch** is the distance centre to centre of the teeth along the pitch circle. n_A and n_B will be proportional to the pitch circle radii R_A and R_B, hence

$$n_B : n_A = N_A : N_B = R_B : R_A.$$

Use of idle wheels.—Two toothed wheels in gear with one another must rotate in opposite directions. If both are required to rotate in the same direction, then another wheel, mounted on an intermediate shaft, and gearing with both driver and driven wheels, is required. This is shown in Fig. 121. A and B will now rotate in the same direction, and since the speed of the circumferences of all three pitch circles will be the same, it follows that

$$\frac{N_A}{N_C}=\frac{R_C}{R_A};$$

also

$$\frac{N_C}{N_B}=\frac{R_B}{R_C}.$$

Multiplying the left-hand sides of these equations together and also the right-hand sides, we get

$$\frac{N_A}{N_C}\times\frac{N_C}{N_B}=\frac{R_C}{R_A}\times\frac{R_B}{R_C},$$

or

$$\frac{N_A}{N_B}=\frac{R_B}{R_A}.$$

FIG. 121.—A drives B through the idle wheel C.

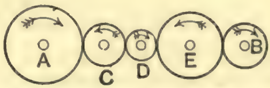


FIG. 122.—C, D, and E are idle wheels.

The relative speeds of rotation of the driver and driven wheels is therefore the same as if they geared direct. The only object of C is to change the direction of rotation, and as it does not alter the velocity ratio it is generally called an **idle wheel.** In the same way we may show that any number of idle wheels, as in Fig. 122, may be interposed without affecting the velocity ratio.

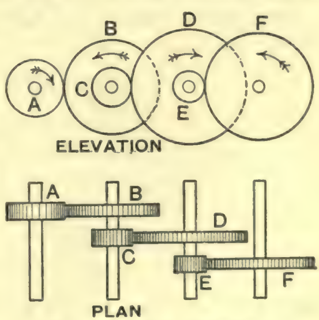


FIG. 123.—Train of wheels.

Trains of wheels.—Where a considerable velocity ratio is required, trains of wheels arranged as in Fig. 123 may be

employed. In this case, the velocity ratios of the various pairs in gear will be respectively,

$$\frac{R_B}{R_A}, \frac{R_D}{R_C}, \frac{R_F}{R_E};$$

or

$$\frac{n_B}{n_A}, \frac{n_D}{n_C}, \frac{n_F}{n_E}.$$

Supposing F to rotate once, the revolutions of E will be $\frac{n_F}{n_E}$, and D will have the same number of revolutions. For one revolution of D, C will have $\frac{n_D}{n_C}$ revolutions, and consequently for one of F, C will have $\frac{n_D}{n_C} \times \frac{n_F}{n_E}$ revolutions, and B will have the same number. For one revolution of B, A will have $\frac{n_B}{n_A}$ revolutions, and therefore for one revolution of F, A will have

$$\frac{n_B}{n_A} \times \frac{n_D}{n_C} \times \frac{n_F}{n_E} \text{ revolutions.}$$

We see, therefore, that **the velocity ratio of the first and last wheels in the train is found by taking the product of the numbers of teeth on all the drivers, F, D and B, and dividing this by the product of the numbers of teeth on all the driven wheels, E, C and A.**

Bevel wheels are evolved from cones by giving teeth to the cones in the same way as for ordinary toothed wheels.

The screw consists of two portions, one A (Fig. 124) cylindrical and free to rotate but not to slide axially, and the other B,

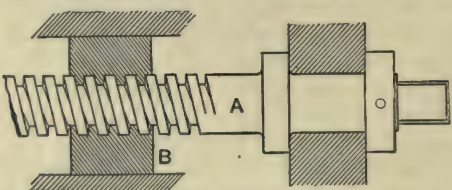


FIG. 124.—Section through a nut, B, showing screw, A.

called the nut, free to slide axially but not to rotate. A helical thread is cut on the outside of A and a corresponding thread on the inside of B, so that A may fit in B. If A is rotated, B will slide axially. Combinations of the screw and nut are very

often used and take many forms. The threads also take many different shapes. As regards the relative motions of A and B. Let B have N threads per inch, then the distance from thread to thread, measured from corresponding places on the threads, will be $\frac{1}{N}$. This distance is called the **pitch of the thread**, and may be written p inches. For one revolution of A, B will slide a distance $= p$ inches.

EXAMPLE. The ordinary bolt and nut is an example of a screw. Supposing a $\frac{1}{2}''$ bolt, 12 threads per inch, to be screwed down by a spanner, the turning force being applied $7''$ from the axis of the bolt, and equal to 20 lbs., what pull will be produced on the bolt, neglecting friction?

Let P = force on spanner, lbs.,
R = radius of P, inches,
Q = pull on bolt, lbs.,
p = pitch of screw, inches.

Then, if there is no friction,

Work done by P in one revolution = Work done in overcoming Q through a distance equal to the pitch.

$$P \times 2\pi R = Q \times p,$$

$$Q = \frac{20 \times 2 \times 22 \times 7 \times 12}{7}$$

$$= \underline{10{,}560} \text{ lbs.}$$

If 75 per cent. is lost in overcoming frictional resistances, then, pull on the bolt will be

$$\frac{25}{100} \times 10{,}560 = \underline{2640} \text{ lbs.}$$

Worm-wheel gearing.—A useful application of the screw is to be found in the **worm and worm wheel.** A short screw of comparatively large diameter is mounted on the driving shaft, and this gears with specially shaped teeth on the rim of a wheel mounted on a shaft, the axis of which is perpendicular to the driving shaft. On the driving shaft rotating, the worm mounted on it will drive the worm wheel, one tooth being advanced for every revolution of the worm. The relative velocities of rotation will therefore be simply equal to the number of teeth on the worm wheel. The driving shaft must be prevented from axial

movement under the thrust produced by driving the worm wheel. In Fig. 125 the driving shaft is fitted with ball thrust bearings so as to minimise as far as possible frictional losses

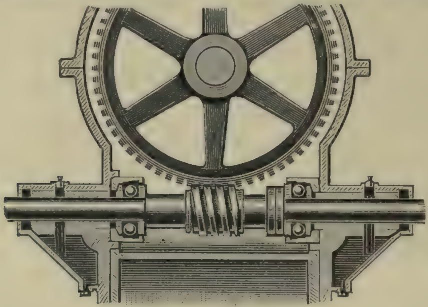


FIG. 125.—Worm and worm wheel.

due to collar friction. The worm and worm wheel is often employed for reducing from high to low speeds of rotation, and is specially adapted for this on account of the large velocity ratio easily obtainable.

As an illustration of a combination of some of the mechanisms described above, the following example is included:

EXAMPLE. A train of wheels, arranged as shown in Fig. 123, is driven by means of a worm which gears with a worm wheel having 40 teeth, the worm wheel being fixed to the same shaft as the wheel A. The numbers of teeth are as follows: A, 12; B, 36; C, 15; D, 50; E, 18; F, 60. The shaft carrying the wheel F has a screw cut on it, 3 threads per inch, and is fitted with a nut free to move axially but not to rotate (Fig. 124). Find how many revolutions of the worm are required in order to cause the nut to move through a distance of one foot.

Pitch of the thread $= p = \frac{1}{3}$ inch.

Revs. of F required to move the nut 12 inches $= \frac{12}{p} = 36$.

$$\text{Revs. of } A \text{ for one rev. of } F = \frac{n_B}{n_A} \times \frac{n_D}{n_C} \times \frac{n_F}{n_E}$$
$$= \frac{36 \times 50 \times 60}{12 \times 15 \times 18}$$
$$= 33\tfrac{1}{3}.$$

$\therefore$ Revs. of A to move the nut 12 inches $= 36 \times 33\frac{1}{3}$

$= 1200.$

Revs. of worm for one rev. of worm wheel $= 40.$

$\therefore$ Revs. of worm to move the nut 12 inches $= 1200 \times 40$

$= 48{,}000.$

Experiments on Machines.—We now proceed to describe some simple machines suitable for experimental work. The scope of the student's work will depend on the machines available. It is recommended that the pulley blocks shown in Figs. 127 and 131, together with a small crab (Fig. 135), should at least be accessible for students' use.

Simple pulley block.—The simplest means we have for raising loads consists of a pulley suspended from an overhead beam, with a rope passing over it (Fig. 126). A load secured to one end of the rope may be raised by pulling on the other end. If there were no losses by friction, stiffness of rope, etc., the greatest load a man could support would be equal to his own weight, and to do this he would have to lift himself off the floor by pulling on the rope. Frictional losses always prevent so great a load from being raised. By attaching two scale pans to the rope ends, A and B, and placing loads in them, the force P required to steadily raise a load W may be found. If equal loads are placed in the pans, no movement will result, but if one load is increased until, by slightly pulling the rope so as to start motion it is found to be sufficient to maintain steadily the movement so produced, the weights in the pans will give the value of P required for this load W. In this arrangement the velocity ratio is clearly 1; the mechanical advantage for any load W is equal to $\frac{W}{P}$, which will always be less than 1, as W is always less than P. The imaginary load F, which, if placed in

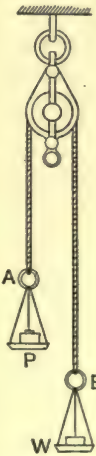


FIG. 126.—Simple pulley block.

the same pan as W, would be equivalent to the frictional resistances of the machine (p. 97), will be equal to $P-W$, for if there were no losses, a force P would raise a load P. The efficiency of the machine (p. 86) will be found by considering W to be raised one foot; P will then descend one foot.

$$\text{Useful work done on } W = W \times 1.$$

$$\text{Energy supplied to effect this result} = P \times 1.$$

$$\therefore \text{ Efficiency} = \frac{W}{P} \times 100 \text{ per cent.}$$

Plan of procedure.—In experimenting with any machine, about 10 experiments should be made with loads increasing by equal steps up to the maximum the machine can safely carry. If scale pans or hooks are used for attaching P and W, the weights of these should be included in the recorded values of P and W. In recording the results, a sketch showing the mechanism clearly should first be inserted and a description of the machine. The calculation for the velocity ratio of the machine should then be given and its result stated. If suitable, the velocity ratio should also be determined by direct measurement at the places where W and P are applied. Weigh the scale pans or hooks and state their weights separately. Oil the parts of the machine requiring lubrication, make sure that everything is running nicely and then make the experiments. Record the results in the form of a table. Curves should then be plotted on squared paper showing the relations of P to W, of F to W, and of the efficiency of the machine to W. From the first two curves equations showing the connection of P and W and of F and W may be found. The third curve will show the value of the efficiency of the machine for any load; as a rule it will be observed that the efficiency rises rapidly when the loads are small, and tends to become constant as the maximum load is approached.

As an example of the method, the following experiment is worked in full. The machine was first tested prior to oiling; this was done in order to indicate the gain in efficiency after oiling, and need not be carried out by the student. As explained above, the machines should be oiled before starting any experiments.

Expt. 31.—An Experiment on Pulley Blocks.

Pulleys used: A single pulley at A (Fig. 127) and another at C, both hung from an overhead beam; a single movable pulley at B, from which W is suspended.

These pulleys were of galvanised iron, small size, such as are used for ordinary household purposes. A thick cord attached to the pulley A at D passes down through pulley B, up over pulley A, then over pulley C and a scale pan is attached to its end for applying P. Another scale pan attached to pulley B serves for applying W. The pulley C would not be usually employed in practice, its present use is to keep the scale pan for applying P clear of the other scale pan and cords.

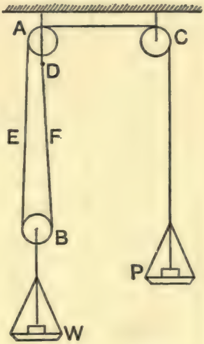


Fig. 127.—Arrangement of pulleys used in the experiment.

Velocity Ratio. If both cords E and F were raised one foot each, W would also be raised one foot. In the actual machine, the cord F does not move until after it passes B, so that if E alone is raised one foot, W will ascend half a foot. The pulleys at A and C merely change the direction of the rope E, so that raising E one foot causes P to descend one foot. The velocity ratio is therefore equal to 2. This was confirmed by actual measurement of the distance moved by P when W was raised one foot.

We may also obtain the velocity ratio by considering the machine as being free from frictional resistances. In this case, as W is suspended by two cords E and F, the pull in each will be $\frac{1}{2}W$; consequently P will be equal to $\frac{1}{2}W$ and the mechanical advantage, neglecting frictional losses, would be equal to 2. In this case (p. 89) the values of the mechanical advantage and velocity ratio are equal, consequently the velocity ratio is 2.

Two sets of experiments were made; I., with the machine unlubricated, after a period of some months' disuse; II., after oiling.

Weight of scale pan for $W = 0{\cdot}68$ lb.
" " " $P = 0{\cdot}68$ lb.

I. *Before oiling.*

Actual Load Raised, including Weight of Pan, W lbs.	Force Applied, including Weight of Pan, P lbs.	Calculated Load, if no Friction, W_1 lbs. $=2P$.	Effect of Friction, $W_1 - W = F$ lbs.	Mechanical Advantage, $\frac{W}{P}$.	Efficiency, $\frac{W}{2P} \times 100$ per cent.
0·68	1·14	2·28	1·6	0·6	29·8
1·68	1·88	3·76	2·08	0·89	44·7
2·68	2·68	5·36	2·68	1·0	50·0
4·68	4·42	8·84	4·16	1·06	53·0
6·68	6·02	12·04	5·36	1·11	55·5
8·68	7·42	14·84	6·76	1·17	58·5
10·68	9·12	18·24	7·56	1·17	58·5
12·68	10·72	21·44	8·76	1·18	59·0
14·68	12·22	24·44	9·76	1·2	60·0
16·68	13·72	27·44	10·76	1·21	60·7

II. *After oiling.*

0·68	0·92	1·84	1·16	0·74	37·0
1·68	1·52	3·04	1·36	1·13	56·3
2·68	2·22	4·44	1·76	1·21	60·4
4·68	3·42	6·84	2·16	1·37	68·5
6·68	4·72	9·44	2·76	1·42	70·9
8·68	5·92	11·84	3·16	1·47	73·3
10·68	7·22	14·44	3·76	1·48	73·9
12·68	8·52	17·04	4·36	1·49	74·4
14·68	9·72	19·44	4·76	1·51	75·5
16·68	11·2	22·4	5·72	1·49	74·5

These results show an increased efficiency, due to oiling, of about 15 per cent. with the highest loads.

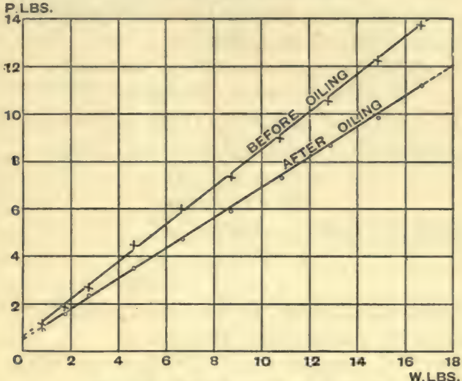


FIG. 128.—Plotted values of *P* and *W*.

On plotting the values of *P* and *W* (Fig. 128), also of *F* and *W* (Fig. 129), the points are found to lie nearly on a straight line. This line is found by stretching a thread on the paper and shifting it about until the plotted points are well divided on either side of it. A mark is then made under each end of the thread and the line drawn through it.

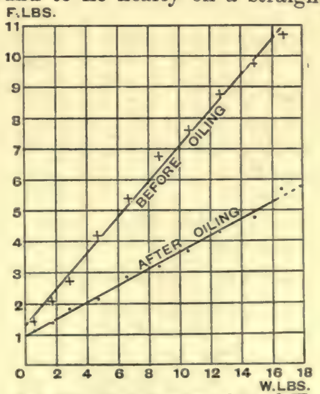


FIG. 129.—Plotted values of *F* and *W*.

Equations for the machine.—In all cases where a straight line is given when the results are plotted, the relation between the results can be expressed by an equation like

$$P = aW + b, \qquad (1)$$

where *P* and *W* are the observed quantities and *a* and *b* are constants.

If the constants are found and inserted in the equation, then

this equation may be used for determining the value of P for any value of W. These constants are found from the straight line on the diagram thus : Taking the diagram showing P and W before oiling, we see from it that

$P=2{\cdot}2$ lbs. when $W=2$ lbs., and
$P=13{\cdot}3$ lbs. when $W=16$ lbs.

Fill in these values in equation (1), giving two simultaneous equations from which a and b will be found. Thus :

$$2{\cdot}2=(a\times\ 2)+b. \quad\text{.........................}(2)$$
$$13{\cdot}3=(a\times 16)+b. \quad\text{.........................}(3)$$

Solving these, we find $a=0{\cdot}79$,
$b=0{\cdot}62$;

so that $$P=0{\cdot}79\,W+0{\cdot}62 \quad\text{......................}(4)$$
gives the required equation showing the connection between P and W, with the machine unlubricated.

Taking now the diagram for P and W after oiling, we see that $P=\ 1{\cdot}7$ lbs. when $W=2$ lbs, and
$P=10{\cdot}7$ lbs. when $W=16$ lbs.

These values inserted in (1) give $1{\cdot}7=(a\times\ 2)+b$..............(5)
$$10{\cdot}7=(a\times 16)+b. \quad\text{............}(6)$$

Solving these gives $a=0{\cdot}64$,
$b=0{\cdot}41$;

so that $$P=0{\cdot}64\,W+0{\cdot}41 \quad\text{......................}(7)$$
gives the relation of P and W after oiling the machine.

Taking now the diagrams showing the relation of F and W, we see that

Before Oiling.	After Oiling.
$F=\ 1{\cdot}3$ lbs. when $W=\ 0$.	$F=0{\cdot}95$ lb. when $W=\ 0$.
$F=10{\cdot}62$ lbs. when $W=16$ lbs.	$F=5{\cdot}24$ lbs. when $W=16$ lbs.
$F=a'W+b'$, ...(1)	$F=a'W+b'$, ...(1)
giving $1{\cdot}3\ =0+b'$, ...(2)	giving $0{\cdot}95=0+b'$, ...(2)
$10{\cdot}62=(a'\times 16)+b'$, (3)	$5{\cdot}24=(a'\times 16)+b'$, (3)
and from (2) and (3)	and from (2) and (3)
$a'=0{\cdot}58$,	$a'=0{\cdot}27$,
$b'=1{\cdot}3$;	$b'=0{\cdot}95$;
$\therefore\ F=0{\cdot}58\,W+1{\cdot}3$. (4)	$\therefore\ F=0{\cdot}27\,W+0{\cdot}95$. (4)

Tabulating these equations :

Before Oiling.	After Oiling.
$P = 0{\cdot}79\,W + 0{\cdot}62.$ $F = 0{\cdot}58\,W + 1{\cdot}3.$	$P = 0{\cdot}64\,W + 0{\cdot}41.$ $F = 0{\cdot}27\,W + 0{\cdot}95.$

The unit used for P, F and W is the pound weight. These equations give all the required information about the machine under the two given conditions.

It should be noted that as the efficiency curve (Fig. 130) is not a straight line, an equation like those given above cannot be

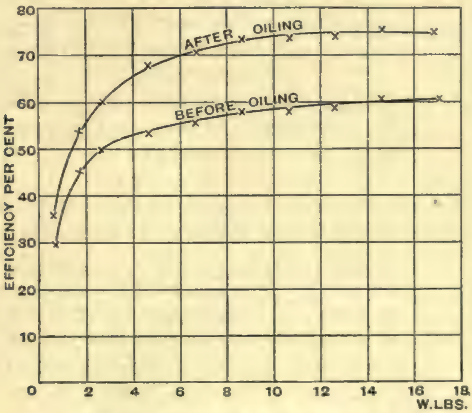


FIG. 130.—Curves of efficiency and load.

applied. Its equation is not so simple, and may be neglected meanwhile by the student.

The above should be taken as showing the method which must be employed in experimenting upon and working out the results for all the simple machines described here. It may be observed, in the example taken, that the pan and weights placed in it are not the only loads raised. The bottom pulley B is also raised, and part of the rope changes its height also. These loads

need not be taken into account, as what we require to know practically is not what parts of the machine itself can be raised by a given force, but what useful load may be raised. Special cases, however, may sometimes arise in which the weights of parts belonging to the machine must be considered.

Another arrangement of pulley blocks.—Keeping the arrangement of ropes the same as in Fig. 127, a higher velocity ratio may be obtained by increasing the number of sheaves, or wheels, in each block. In Fig. 131 there are three sheaves in the upper block and two in the lower. W is now supported by 5 ropes between the upper and lower block, consequently the velocity ratio is 5. Usually the sheaves belonging to each block are mounted side by side on the same spindle.

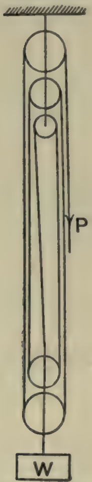


FIG. 131.—Pulley blocks.

Let W lbs. = load actually raised.
P lbs. = pulling force.

Then, if there were no frictional losses, and if the lower block and the rope had no weight, the useful load raised would be

$$W_1 = 5 \times P.$$

Therefore losses in machine $= 5P - W = F$, as before. The mechanical advantage will be $\frac{W}{P}$, and the efficiency $\frac{W}{5P} \times 100$ per cent. It will be found, on experiments being made on a set of actual blocks like this, that the efficiency is not so great as with those first given, even when well oiled. The stiffness of the rope passing so many times around the sheaves accounts for a large loss.

Running down of the load.—It often happens in hoisting tackle that, if P be removed, the load W will run down, reversing the action of the machine. The effect is owing to there being insufficient friction in the machine.

It can be shown that in any machine in which the removal of the pulling force does nothing to alter the magnitude of the frictional resistances, the suspended load will not run down if the efficiency of the machine is less than 50 per cent. If, however, the removal of the force causes alterations in the

frictional resistances, no general statement true for all machines can be made, each case must be investigated separately.

Running down of the load when P is removed, may be prevented by fitting a **pawl** and **ratchet wheel** to the machine

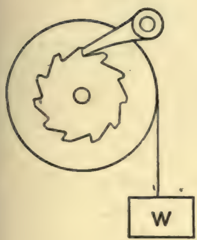


Fig. 132.—Pawl and ratchet wheel.

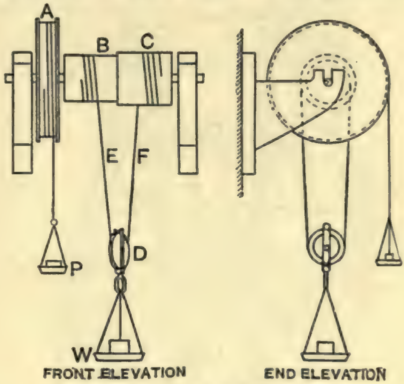


Fig. 133.—Wheel and differential axle.

(Fig. 132). When the pawl engages the teeth, motion can only take place in the direction necessary to raise the load. When lowering the load, the pawl is raised by hand, thereby permitting the required motion to be effected.

The wheel and differential axle (Fig. 133) consists of a drum BC, divided into two portions having different diameters, and mounted on the same spindle as a wheel A. A rope is attached to B, and, after a few turns round B, is led down through a pulley D, then up and turned round C in the opposite direction to the winding on B, its end being made fast to C. A cord made fast to A, passes two or three times round A and carries a load P for working the machine. The load to be raised, W, is carried by the pulley D.

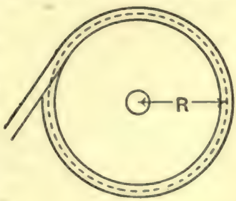


Fig. 134.

Suppose the drum to rotate once. P will be lowered a distance equal to the circumference of the circle in which the cord sustaining P is wrapped. A point on the cord

E will be lowered, and one on the cord F raised, the amounts in each case being equal to the circumference at the drum of the cord circle. It will be noticed that for all three cords, the circumference must be measured, not of the drum, but of the circle at the cord centre; for the inside of the cord, in contact with the drum (Fig. 134), will have a shorter circumference than that of the outside part of the cord, and the actual distance moved by the cord will be the mean of these two, that is, the circumference of the cord centre.

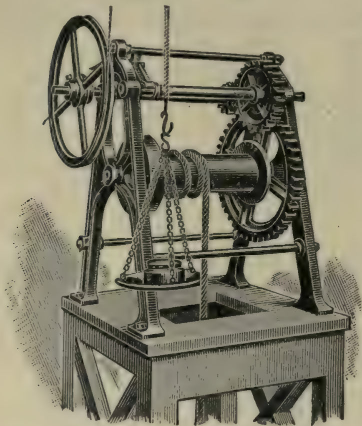


Fig. 135.—Crab, arranged for performing experiments.

To find the velocity ratio, let R_A, R_B, R_C be the radii of A, B and C, in each case measured to the centre of the cord. Let the drum rotate once, then

$$\begin{aligned}\text{Distance moved down by } P &= 2\pi R_A\\ \text{,, \quad ,, \quad down by } E &= 2\pi R_B\\ \text{,, \quad ,, \quad up by } F &= 2\pi R_C.\end{aligned}$$

Consequently, W will be raised a distance equal to

$$\tfrac{1}{2}(2\pi R_C - 2\pi R_B);$$

$$\therefore \text{ Velocity ratio} = \frac{2\pi R_A}{\tfrac{1}{2}(2\pi R_C - 2\pi R_B)}$$

$$= \frac{2R_A}{R_C - R_B}.$$

The actual mechanical advantage will be found by experiment. From the results the effect of friction and the efficiency will be calculated as already explained.

The crab (Fig. 135) is often used along with arrangements of pulley blocks for raising weights. The rope from the pulleys is

wound round a barrel, having a toothed wheel secured to its spindle. A pinion, secured to another spindle parallel to the barrel, gears with the toothed wheel, so that if this second spindle is turned by means of handles, a considerable velocity ratio is obtained. This arrangement is said to be **single geared.** In **double geared** crabs, an additional toothed wheel and pinion are introduced on another spindle, so that a much greater velocity ratio is obtained. The double geared crab is usually so arranged that it can be rapidly converted to single geared ; light loads can then be raised more quickly than by use of the double gear. A band brake is fitted to the drum spindle for use while lowering the load, and a pawl and ratchet wheel on the same spindle prevent the load running down if the handles are released.

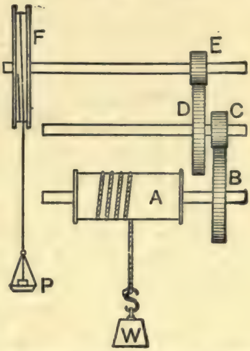


FIG. 136.—Diagram of the mechanism of the crab.

EXPT. 32.—Fig. 135 shows such a double geared crab arranged so that experiments may be performed. The handles have been taken off and a wheel, having a rim grooved to receive a cord, substituted. The pull required to work the machine is supplied by weights placed in a scale pan at the end of this cord, the cord being led over a pulley secured overhead, so as to obtain a considerable vertical travel for the scale pan. Fig. 136 shows a diagram of the mechanism from which the velocity ratio may be calculated.

Let d_1 = the diameter to the centre of the rope on the barrel A ; d_2 = the diameter to the centre of the cord on the wheel F.

Let B, C, D, E represent the numbers of teeth on the wheels as shown.

Then, if A revolves once, F will revolve $\dfrac{B \times D}{C \times E}$ times, and W will be raised a distance equal to πd_1. In one revolution of F, P will be lowered a distance equal to πd_2 ; therefore, for one

revolution of the barrel A, P will be lowered a distance $\frac{B \times D}{C \times E} \times \pi d_2$. Consequently the velocity ratio will be

$$V = \frac{\frac{B \times D}{C \times E} \times \pi d_2}{\pi d_1}$$

$$= \frac{B \times D \times d_2}{C \times E \times d_1}.$$

The experiment is performed and the results reduced in the manner explained in Expt. 31 (p. 109).

Screw jacks are used for heavy loads requiring a small lift only. A hollow case A (Fig. 137) has a hole at its top screwed to receive a strong square threaded screw B. The load is applied at the top of this screw, on C, which is a piece free to rotate on the top of B. B is turned by a *tommy-bar* inserted into holes in the screw head as shown, and as C is free to rotate on B, the load is not turned by the rotation of the screw.

To obtain the velocity ratio:

Let R = radius at which P is applied, inches.

p = pitch of screw, inches.

Then, in one revolution of the screw, P moves a distance tangentially equal to $2\pi R$, and W moves a distance equal to p. Therefore the velocity ratio is

$$V = \frac{2\pi R}{p}.$$

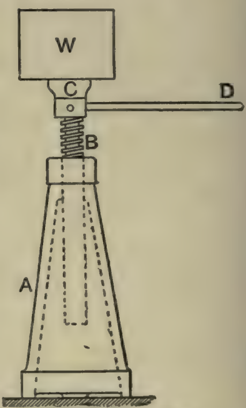


Fig. 137.—Screw jack.

By substituting a wheel with a grooved rim for the tommy bar, and attaching a cord to it, led over a pulley, a load may be hung on equivalent to P, and experiments may be made to find the mechanical advantage, effect of friction and efficiency, in the same manner as for the other machines.

EXERCISES ON CHAP. VIII.

1. An engine driving a line of shafting by belts, has a belt pulley 24″ diameter, that on the line shafting being 26″ diameter. The engine runs at 230 revolutions per minute. A counter-shaft is driven from the line-shaft, the belt running on a pulley 3 feet diameter on the line-shaft and on one 2 feet diameter on the counter-shaft. What are the speeds of revolution of the line-shaft and of the counter-shaft, supposing no slip of belt.

2. A belt, running at a speed of 2000 feet per minute, has a difference of 240 lbs. between the pulls on the tight and slack sides. What horse-power is being transmitted?

3. Give sketches and description of the train of wheels connecting the hour axle with the minute axle in a clock. Give suitable numbers to the teeth.

4. A single geared crab has handles 15″ long and barrel 6″ diameter to the rope centre. The pinion has 20 teeth and the spur wheel 60 teeth. Give an outline sketch and calculate the velocity ratio.

5. A screw, 1″ pitch, is used to raise a load of 5 tons. The screw is turned by means of a toothed wheel 15″ effective radius. Calculate the pressure required on the teeth tangential to the pitch circle, supposing 50 per cent. to be lost in friction.

6. In a wheel and differential axle, the wheel is 24″ diameter and the drum has diameters of 7″ and 6″ respectively. Calculate the velocity ratio.

7. A worm and wormwheel are used for applying the twist to a piece of material under torsion test. The worm is turned by a handwheel, and the test piece is connected to the shaft on which the wormwheel is mounted. If the wormwheel has 90 teeth, how many degrees of twist will be given to the test piece by turning the handwheel through 235 revolutions?

8. A lifting tackle is formed of two blocks, each weighing 15 lbs.; the lower block is a single movable pulley, and the upper or fixed block has two sheaves. The cords are vertical and the fast end is attached to the movable block. Sketch the arrangement and determine what pull on the cord will support 200 lbs. hung from the movable block, and also what will then be the pressure on the point of support of the upper block.

9. Describe either a screw jack (pitch of screw ½″, handle 19″ long) or a simple winch for lifting weights up to 1 ton by one man. What is the mechanical advantage neglecting friction? Describe what sort of trial you would make to find its real mechanical advantage under various loads, and what sort of result would you expect to find?

10. Describe any machine, workable by hand, for lifting weights. Give the rule for its velocity ratio. When is its velocity ratio the same as its mechanical advantage? Describe carefully how you would make tests to determine its real mechanical advantage under various loads.

11. A machine is concealed from sight except that there are two vertical ropes; when one of these is pulled downwards the other rises. How would you find the efficiency of this lifting machine? What do we mean by the velocity ratio, and by the mechanical advantage?

12. In a lifting machine an effort of 26·6 lbs. just raised a load of 2,260 lbs.; what is the mechanical advantage? If the efficiency is 0·755, what is the velocity ratio?

CHAPTER IX.

VELOCITY. ACCELERATION. INERTIA. KINETIC ENERGY. RELATIVE VELOCITY. CHANGE OF VELOCITY. MOMENTUM. MOTION IN A CIRCLE.

Velocity.—The term **Velocity** has been used before when considering machines. The term refers to **the rate at which a body is changing its position relative to other bodies.** Velocity has magnitude, direction, and sense, and, like force, may be represented by a straight line.

Velocity is measured by stating the distance travelled by the body in a given time. Thus, a velocity of 15 feet per second means that in one second the body will travel a distance of 15 feet. When we say that the speed or velocity of a train is 55 miles an hour, we mean that if that velocity were kept up constantly, the train would travel a distance of 55 miles in one hour.

Velocities are seldom constant, but the velocity a body has at any instant may be stated by considering what space the body would travel in unit time if the velocity it has at the given instant were kept constant. Thus, it is found that if a body falls freely from a height, at the end of the first second of the fall its velocity is 32·2 feet per second. This does not mean that the body has fallen during the first second a distance of 32·2 feet, or that it is going to travel that distance during the next second. The meaning is that if the velocity possessed by the body at the end of the first second were kept unaltered, it would travel a distance of 32·2 feet during the next second. Actually it does not remain unaltered, for at the end of the

next second its velocity will be found to be 64·4 feet per second, and at the end of the third second 96·6 feet per second and so on, the speed continually increasing.

Uniform velocity will occur if the body travels over equal distances in equal intervals of time. Velocity which is not uniform is said to be **accelerating**. If the speed is becoming greater the acceleration is said to be *positive*, and if the speed is diminishing, *negative*.

Parallelogram of velocities.—Although it is impossible for a rigid body to be moving in two directions at one and the same time, yet it is often convenient to think of a velocity as made up of two component velocities. Thus, supposing a point A to have component velocities represented by $AB = V_1$ and $AC = V_2$, its resultant velocity may be found by the parallelogram of velocities, which is similar to the parallelogram of forces. On completing the parallelogram $ABDC$ (Fig. 138), $AD = V$ will be the resultant velocity of the point A. Notice that here, as with the parallelogram of forces, both velocities must be either towards A or away from A before applying the parallelogram.

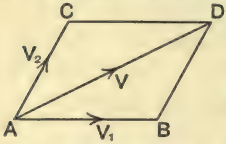


FIG. 138.—Parallelogram of velocities.

Velocity-time diagrams.—The velocity which a body has at any instant can be very conveniently shown in a diagram. Take the following case :

A train leaves Liverpool Street Station at 6.0 p.m.; its speed gradually increases and at 6.2 p.m. is 15 miles an hour, and keeps uniform till 6.4 p.m., when brakes are applied, and at 6.4¼ p.m. the train stops at Bethnal Green Station. After 1 minute stop, the train starts again, and in 1½ minutes its speed is 30 miles an hour and keeps uniform for 9 minutes. Brakes are again applied, and it comes gradually to rest at Stratford Station ½ minute later.

Time being taken for abscissae and velocities for ordinates, the diagram (Fig. 139) shows at a glance all that has occurred to the train's speed.

The average speed of a body during a given journey can be calculated by dividing the total distance by the time taken to

perform the journey. In the time taken should be included the time lost in any stops. Thus, suppose a train takes 12 minutes

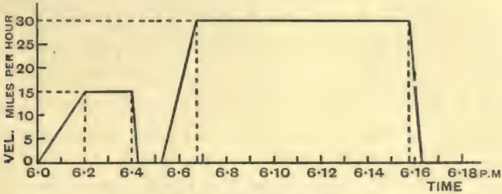


FIG. 139.—Velocity-time diagram.

to perform a journey of 5 miles, its average velocity will be $\frac{5}{12}=0{\cdot}4166$ mile per minute, including stops.

Or, the average velocity may be found from its velocity-time diagram by any of the well-known mensuration rules. In the given diagram (Fig. 140), if the base be divided into 10 equal parts and the sum of the velocities measured by the height of the diagram at the centre of each part be taken, this sum divided by 10 will give the average velocity.

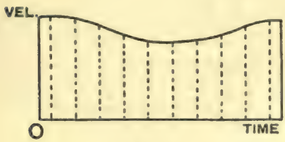


FIG. 140.

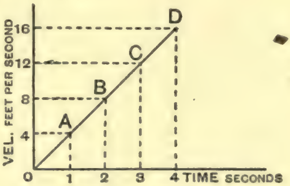


FIG. 141.

Acceleration.—The case of a body gaining speed must now be studied in more detail. Supposing it starts from rest at time 0, and gains speed gradually throughout. Let its velocity at the end of the first second be 4 ft. per second. Its velocity at any instant during the first second will be shown by the height of the diagram at that instant (Fig. 141). During the next second, its velocity will gradually increase again, and at the end of this second will be 8 feet per second, represented by $2B$ in the figure. At the end of the third second its velocity will be

12 feet per second and at the end of the fourth second 16 feet per second, represented by $3C$ and $4D$ respectively. The gain of velocity in any particular second, or positive acceleration, as it is called, will be 4 feet per second. We state this by saying,

Acceleration = 4 feet per second, every second, or,
Acceleration = 4 (feet per second) (per second),

the part of the units in the first bracket referring to the gain of velocity, and in the second bracket to the time in which that change of velocity took place.

Notice, in the diagram, that the total change in velocity in 4 seconds was 16 feet per second. So that we may find the change per second in velocity by dividing the total change by the time in which that change took place. Thus,

Acceleration $= \frac{16}{4} = 4$ (feet per second) (per second).

We must be careful always to state not only the *change in velocity*, but the *time* in which that change took place. Had the body in the above case gone on moving for 10 seconds, its velocity at the end would have been 40 feet per second.

Its acceleration = 40 (feet per second) (per 10 seconds),
or " = 16 (feet per second) (per 4 seconds),
or " = 4 (feet per second) (per second).

Referring to Fig. 141 again, if the body moves for 4 seconds, its velocity at the end is 16 feet per second, its velocity at the start is 0 and the change was gradual and uniform. Its average velocity will therefore be 8 feet per second. The distance travelled may now be calculated from,

$$\begin{aligned} \text{Distance} &= \text{average velocity} \times \text{time} \\ &= 8 \times 4 \\ &= 32 \text{ feet.} \end{aligned}$$

This result can be represented by the area of the triangular diagram $D04$ (Fig. 141), for its area will be the base 04 multiplied by half the height $D4$. Measuring the base in units of time and the height in units of velocity, we get

Distance = area of $D04 = 4 \times \frac{16}{2} = 32$ feet, as before.

Equations of motion.—Taking an example with general terms, let a body start from rest and gain every second a

velocity a feet per second. Let this go on for a time t seconds, and call its velocity, at the end of the time, v feet per second. Let S be the distance travelled in feet. The diagram shown in Fig. 142 represents these conditions.

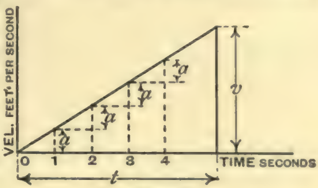


FIG. 142.

The acceleration will be a (feet per second) (per second), and since a feet per second of velocity are gained each second, in t seconds the gain will be $a \times t$, so that

$$\mathbf{v = at.} \quad \text{(1)}$$

The distance travelled, S, will be represented by the area of the diagram, so that

$$\mathbf{S = \tfrac{1}{2}v \cdot t.} \quad \text{(2)}$$

Or, substituting $(a \,.\, t)$ for v in (2),

$$\mathbf{S} = (\tfrac{1}{2}at) \times t = \tfrac{1}{2}\mathbf{at^2}. \quad \text{(3)}$$

Again, from (1) $\quad t = \dfrac{v}{a}.$

Substitute this in (3) giving

$$S = \tfrac{1}{2}a \times \left(\frac{v}{a}\right)^2,$$

$$= \frac{v^2}{2a},$$

or, $$\mathbf{v^2 = 2aS.} \quad \text{(4)}$$

Body falling freely.—When a body falls freely, its velocity increases approximately by 32·2 ft. per second every second. A special symbol, g, is used for the acceleration in this case, so that

$g = 32{\cdot}2$ ft. per second per second.

It should be remembered that the acceleration of a falling body is not always the same. Its magnitude depends on the latitude of the place. Applying the above equations to the case

of a falling body, let h=height fallen in time t seconds, and v=velocity at the end of this time, then

$$\mathbf{v=gt.} \quad \text{.....(5)}$$

$$\mathbf{h=\tfrac{1}{2}vt.} \quad \text{.....(6)}$$

$$\mathbf{h=\tfrac{1}{2}gt^2.} \quad \text{.....(7)}$$

$$\mathbf{v^2=2gh.} \quad \text{.....(8)}$$

In these equations the body is supposed to be falling freely, that is, no atmospheric or other resistances oppose it.

Inertia.—The importance of the study of acceleration lies in the fact that force must act on a body to produce acceleration. If no resultant force acts on a body free to move when it is at rest, it will remain at rest; or, if the body is moving, it will continue to move with uniform velocity in a straight line. This laziness, or **inertia**, as it is called, of matter, must be overcome by a force or forces, if any change in a body's velocity is to be made. Thus, if a machine is at rest and is to be started, the pulls of the belt must not only overcome the frictional resistances, but must also overcome the resistances due to the inertia of the parts which move when the machine is running. When the belt is thrown on the loose pulley to stop the machine, the machine would go on moving uniformly but for the frictional resistances applying forces to the moving parts, thereby producing negative acceleration, and so bringing the machine to rest.

We may estimate how much force is required to produce a given acceleration in a body, by considering again the case of a falling body. If the body has a mass of one pound, there will be a force of one pound weight acting on it. This produces, in all parts, practically, of the British Isles, an acceleration of 32·2 feet per second per second. If we could reduce the weight of the body to $\frac{1}{2}$ lb. without altering its mass, we should find that the acceleration produced would be 16·1 feet per second per second; and if we could reduce the body's weight to $\frac{1}{32 \cdot 2}$ lb. the acceleration produced would be 1 foot per second per second. In each of these cases the resistance due to the inertia of the body will be an upward force of 1 lb., $\frac{1}{2}$ lb., and $\frac{1}{32 \cdot 2}$ lb. respec-

tively. Again if we could increase the mass of the body to 32·2 pounds, still keeping its weight 1 lb., we should find that its acceleration would be one foot per second per second. In fact, the law is, **the force required to produce a given acceleration in a given body is proportional to the product of the body's mass and the required acceleration**, and as we know that a force of 1 lb. weight acting on a mass of 1 pound gives an acceleration of 32·2 feet per second per second, we may calculate the force P lbs. required to give an acceleration a feet per second per second to a mass m pounds from

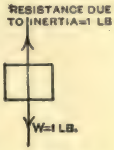


FIG. 143.

$$P=\frac{ma}{32{\cdot}2} \text{ lbs. weight.}$$

If, instead of the pound weight as the unit of force, we were to use a unit of force equal to $\frac{1}{32{\cdot}2}$ lb. weight, or $\frac{1}{g}$ lb. weight, then this unit, acting on the one pound mass, would give an acceleration of one foot per second per second. This unit of force is called an **absolute unit of force**, and in our system of units—the **poundal**. The absolute unit of force for the metric system is the **dyne**, and is of such magnitude that it gives an acceleration of one centimetre per second per second when it acts on a mass of one gram. Using the poundal as the unit of force, the equation stated above may be written

$$\mathbf{P=ma} \text{ poundals.}$$

EXAMPLE. A body has a mass of 150 pounds and we have to give it an acceleration of 100 feet per sec. per sec. Find the force required.

$$P=\frac{ma}{g} \text{ lbs. weight}$$
$$=\frac{150\times 100}{32{\cdot}2}$$
$$=\underline{456} \text{ lbs. weight.}$$

Or in poundals,

$$P=456\times 32{\cdot}2=\underline{15{,}000} \text{ poundals.}$$

Since the acceleration of a body, falling freely under the action of its weight, is g teet per second per second, it follows that we may express its weight, W, in poundals, from

$$\mathbf{W=mg} \text{ poundals.}$$

EXPT. 33.—To verify the law $P=\frac{ma}{g}$, arrange an apparatus as shown in Fig. 144. This consists of two light pulleys attached to a support as high as possible, and having a light cord passing over them, with scale pans at its ends.

FIG. 144.—Apparatus for verifying the law $P=\frac{ma}{g}$.

1. Place equal masses in the pans. It will be found that the pans remain at rest, and that if motion is started by the hand, the frictional resistances in the apparatus rapidly bring it to rest.

2. Increase the mass in one of the pans, until its excess weight enables steady motion to be maintained.

3. An additional mass placed in the same pan will now, by its weight, produce acceleration in the whole of the moving parts of the apparatus. Place a known additional mass in this pan, and elevate it a measured height. Allow it to descend, and note the time of its descent, using a stop-watch for this purpose. From these data, calculate the acceleration of the masses, and also the force required to produce this acceleration, using the equation $P=\frac{ma}{g}$. The result should agree closely with the weight of the additional mass used to produce acceleration in the experiment if the law is true.

In one experiment the following results were obtained :

Mass of each scale pan $=0{\cdot}72$ pound.

Mass placed in A (Fig. 144) $=1$ pound.

Mass placed in B to produce *steady* speed $=1{\cdot}15$ pounds.

Mass placed in B in acceleration experiment $=1{\cdot}2$ pounds.

Excess weight $=$ force producing acceleration $=1{\cdot}2-1{\cdot}15$

$=0{\cdot}05$ lb. weight.

In the acceleration experiment, B was allowed to descend 9 feet, and was found to do so in an average time of $6{\cdot}3$ seconds.

$$s=\tfrac{1}{2}at^2.$$

$$9=\tfrac{1}{2}a(6{\cdot}3)^2=\tfrac{1}{2}a\times 39{\cdot}7$$

$$a=\frac{18}{39{\cdot}7}=\underline{0{\cdot}45}\text{ feet per sec. per sec.}$$

Let m = the whole mass set in motion, neglecting the masses of the pulleys and cord.

$$m = 0{\cdot}72 + 0{\cdot}72 + 1 + 1 + 0{\cdot}2 = 3{\cdot}64 \text{ pounds.}$$

$$P = \frac{ma}{g} = \frac{3{\cdot}64 \times 0{\cdot}45}{32{\cdot}2} = \underline{0{\cdot}0509} \text{ lb. weight,}$$

which agrees closely with the actual P used in the experiment, viz. 0·05 lb. weight.

Measurement of kinetic energy.—When a body falls freely from a given height, its potential energy is gradually changed into kinetic energy of an equal amount. Let m be its mass in pounds, and h the height in feet from which it falls. Then its weight will be $W = mg$ poundals. At the end of its fall, its velocity will be

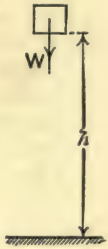


FIG. 145.

$$v = \sqrt{2gh},$$

or $$v^2 = 2gh,$$

giving $$h = \frac{v^2}{2g}.$$

We may therefore write,

$$\text{Potential energy transformed} = mgh \text{ foot-poundals,}$$

$$= mg\frac{v^2}{2g} = \frac{mv^2}{2}$$

$$= \text{kinetic energy gained.}$$

Consequently, the kinetic energy of a body is given by this equation in terms of its mass and velocity. Notice that it makes no difference to the kinetic energy possessed by a body moving with a given velocity, whether it is moving vertically or in any other direction; so that in general,

$$\textbf{Kinetic energy} = \frac{\mathbf{mv^2}}{\mathbf{2}} \text{ foot-poundals}$$

$$= \frac{\mathbf{mv^2}}{\mathbf{2g}} \text{ foot-lbs.}$$

or foot-tons if m is in tons.

Relative velocity.—When we speak of a body being at rest, what meaning do we attach to the statement? Thus, a house appears to be at rest, that is, it is not shifting its position on

the earth; yet being attached to the earth, it possesses the complicated motion of the earth at that place and is therefore not actually at rest. In fact, no body is absolutely at rest. When we speak of rest, we usually mean *at rest relative to the earth*, that is, an observer standing on the earth perceives no motion. If we say that a train has a velocity of 60 miles an hour, we do not mean that this is the absolute velocity of the train, but only its velocity relative to the earth. If two trains are moving side by side with equal speeds, an observer in one of them perceives no motion in the other and therefore says that the relative velocity of the trains is zero. If the train carrying the observer has a velocity of 30 miles an hour, and the other, one of 35 miles an hour, he will see the other train moving past him at a rate of 5 miles an hour, which velocity he would call the relative velocities of the trains. If the second train were going in the opposite direction at 35 miles an hour the relative velocity would be 65 miles an hour. **Relative velocity of two bodies may be defined as the velocity which an observer on one of them would perceive in the other.** Thus, if a stream of water moving at 8 feet per second reaches a water wheel the buckets of which are moving at 6 feet per second, the water will enter the buckets with a relative velocity of 2 feet per second.

If two bodies A and B have velocities as shown at V_1 and V_2 in Fig. 146, their relative velocity can easily be obtained in the following manner. Stop one of the bodies by giving each of them a velocity equal and opposite to the velocity of that body. Then the resultant velocity of the other one will be the relative velocities of the two bodies; thus, giving both A and B velocities equal and opposite to V_1 as shown in Fig. 146, the result will be to bring A to rest, and B's velocity will now be V_R, the resultant of V_1 and V_2 at B. This velocity V_R will be the relative velocity of A and B.

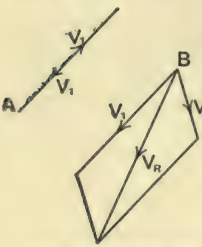


FIG. 146.—Relative velocity of A and B.

As an example of relative velocity, take the case of a person entering a compartment of a railway carriage when in motion.

Suppose that the velocity of the carriage is 6 feet per second. If the person desires to enter without being thrown against the seats, he will contrive matters so that his velocity relative to the carriage is along the line BA (Fig. 147), at 90° to the direction of motion of the carriage. Suppose that this relative velocity is to be 2 feet per second. Stop the carriage by giving both it and the person at P velocities, towards the right, of 6 feet per second. This is shown at P by the line $PD=6$ feet per second. Now $V_R=2$ feet per second, represented by PE, has to be the relative velocity of person and carriage, and hence must be the resultant of his actual velocity along the platform and PD. Completing the parallelogram $PDEF$, we get the velocity of the person along the platform represented by $PF=6{\cdot}32$ feet per second, and if he runs along the platform in the direction PF with this speed, he will enter the compartment without shock along the line BA.

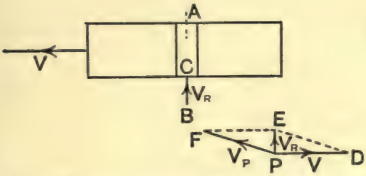


FIG. 147.—Velocity of a person entering a railway carriage.

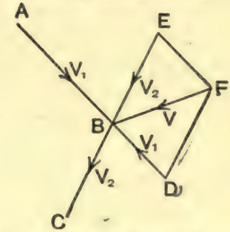


FIG. 148.—Velocity changed from V_1 along AB to V_2 along BC.

Velocity changed in direction.—Supposing a point is moving along a line AB (Fig. 148), with a velocity V_1, and that when it arrives at B, something is done to it, in consequence of which it moves off along BC with a velocity V_2. Let us examine what change must be effected in its velocity to produce this result. First stop the point at B by giving it a velocity equal and opposite to V_1. This is shown by DB in the figure. Then give it a velocity V_2 in the direction BC, this will fulfil the required conditions ; V_2 being represented by EB. To find the resultant change in velocity, find the resultant of V_1 and V_2 by the parallelogram of velocities $DBEF$. Then $FB=V$ is the resultant change in velocity. A moving *point* has been considered instead of a *body*, in order to avoid drawing on the

imagination in following out the process of reasoning, for we have to think of velocities applied suddenly at B, and a sudden change in velocity implies an infinitely great acceleration, and therefore an infinitely large force to be applied to the body at B. This, of course, is impossible, but what actually occurs is a gradual change in velocity causing the body to turn gently into the direction BC along a curve (Fig. 149). The line FB in Fig. 148 shows, however, the total change in the velocity of the body.

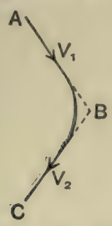


FIG. 149.—The change in velocity must be gradual.

Momentum.—Momentum is possessed by a body when in motion; it is proportional to the mass of the body and to its velocity jointly, and is measured by the product of these.

$$\textbf{Momentum} = \mathbf{mv}.$$

Units of momentum will be stated by giving the units of mass and velocity employed; thus, if the pound is used for the unit of mass, and one foot per second is the unit of velocity, then

$$\text{Momentum} = mv, \text{ pound-foot-seconds.}$$

EXAMPLE. Find the momentum of a body of mass 100 pounds when it has a velocity of 5 feet per second.

$$\text{Momentum} = mv$$
$$= 100 \times 5 = 500 \text{ pound-ft.-secs.}$$

Forces generating momentum.—Suppose a body, of mass m pounds, to be acted on by a force P lbs. weight during a time t seconds, and that the body is at first at rest. An acceleration a feet per second per second will be produced, such that

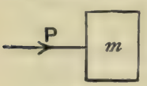


FIG. 150.

$$P = \frac{ma}{g} \text{ lbs. weight. ....(1)}$$

Since P acts for a time t seconds, the velocity of the body at the end of this time will be

$$v = at \text{ feet per second;}$$

$$\therefore \ a = \frac{v}{t} \text{ feet per second per second.}$$

And from (1), by substitution,

$$\mathbf{P} = \frac{\mathbf{mv}}{\mathbf{gt}}. \ \text{..................................(2)}$$

Now mv is the momentum possessed by the body at the end of the time t seconds, consequently $\frac{mv}{t}$ will be the momentum it acquires each second, and **the force generating momentum will be numerically equal to the momentum generated per second divided by** g. This relation is very useful when considering **impulsive forces**, *i.e.* forces which act during a very small interval of time.

Impulsive forces.—Imagine a body in motion to possess a momentum equal to M, which is abstracted by the body encountering a uniform resistance P. If this is accomplished in t seconds, then

$$P=\frac{M}{gt}.$$

It will be noticed that if t becomes very small, P will become very large, and in fact, the force will be **impulsive.** Notice also that, if the resistance encountered is not uniform, we may still find its average value from this equation. This force we may call, in the case of impulsive action, the **average force of the blow.** It is quite impossible, in most cases of impulse, to state exactly what the actual reactions are at any instant, and it is very convenient to be able to calculate, at any rate, their average value.

EXAMPLE. A hammer head, 2 pounds mass, moving with a velocity of 40 feet per second, is arrested in $\frac{1}{200}^{\text{th}}$ second by meeting an obstacle. Calculate the average force of the blow.

$$\begin{aligned}\text{Momentum of hammer} &= 2\times 40\\ &= 80 \text{ pound-ft.-secs.}\\ \text{Momentum changed per second} &= 80\div\frac{1}{200}\\ &= 16{,}000 \text{ pound-ft.-secs.}\end{aligned}$$

and
$$\begin{aligned}\text{average force of blow} = P &= \frac{16{,}000}{32\cdot 2}\\ &= \underline{500} \text{ lbs. weight nearly.}\end{aligned}$$

Change of momentum.—Momentum depends on the velocity of a body ; and, as velocity has direction, momentum will also be a directed quantity. Change of momentum in any given case must therefore be measured by taking the change in the body's velocity. The method of ascertaining this has already been described on p. 131. Having found the change in momentum and its direction, the force required will act in the same line.

EXAMPLE. Suppose a stream of 180 bullets per minute to impinge at 90° to a plate with a velocity of 1000 feet per second, and then to drop vertically downwards. If each bullet has a mass of 1 ounce, what is the reaction of the plate?

The change of velocity in this case will be at 90° to the plate and will be equal and opposite in sense to the velocity of the bullets.

$$\text{Change in velocity} = 1000 \text{ feet per second.}$$

$$\text{Mass reaching plate per second} = \frac{180}{60} \times \frac{1}{16} = \frac{3}{16} \text{ pound.}$$

$$\text{Reaction of plate} = \frac{mv}{g} = \frac{\frac{3}{16} \times 1000}{32 \cdot 2}$$

$$= \underline{5 \cdot 8} \text{ lbs. weight.}$$

If a jet of water be substituted for the bullets, the problem will be similar.

EXAMPLE. Suppose the impinging jet to have a sectional area of $\frac{1}{100}$th square foot and a velocity of 200 feet per second.

$$\text{Mass reaching plate per second} = \frac{1}{100} \times 200 \times 62 \cdot 5$$

$$= 125 \text{ pounds.}$$

$$\text{Reaction of plate} = \frac{mv}{g}$$

$$= \frac{125 \times 200}{32 \cdot 2} = \underline{776} \text{ lbs. weight.}$$

Motion in a circle.—If a body be attached to one end of a string, the other end held in the hand, and the body whirled round in a circle, it will be noticed that a pull along the string has to be resisted. Suppose the body m to be at A (Fig. 151); the natural tendency is for it to move in the line AB, tangential to the circle; and the pull P, applied to the string AC by the hand at C, is required to overcome this tendency and to cause the body to move in the circular path. The inertia of the body causes it to resist this pull with an opposite force F equal to P. P is called the **Central Force** (sometimes **Centripetal Force**), and F is called the **Centrifugal Force**. Evidently, if the string is let go, these forces would cease to exist, and the body would move off in a path tangential to the circle.

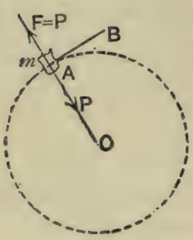


FIG. 151.

Since a central force is required to overcome the inertia of the body, it follows that there must be an acceleration caused by it, towards the centre of the circle.

Let v = the velocity of the body in its circular path, in feet per second,

r = the radius of the circle, feet,

then it can be shown that the acceleration towards the centre of the circle is given by

$$\mathbf{a}=\frac{\mathbf{v}^2}{\mathbf{r}} \text{ feet per second per second.} \quad \text{..................(1)}$$

Again, $$P=\frac{ma}{g};$$

$$\therefore \ \mathbf{P}=\frac{\mathbf{mv}^2}{\mathbf{gr}}, \quad \text{..(2)}$$

the units being pounds, or tons, weight depending on the unit of mass used for measuring m.

This equation (2) may be used to calculate the central force, or the centrifugal force, acting on a rotating body.

Example. Calculate the central force required to cause a body of mass 100 pounds to whirl in a circle 4 feet diameter 120 times per minute.

Here
$$v=\frac{120}{60}\times\pi d$$
$$=2\times\pi\times4=8\pi \text{ ft. per sec.}$$
$$P=\frac{mv^2}{gr}$$
$$=\frac{100\times(8\pi)^2}{32{\cdot}2\times2}$$
$$=\underline{982} \text{ lbs. weight.}$$

Simple pendulum.—A simple pendulum may be constructed by attaching a small heavy body, such as a lead bullet, to one end of a fine thread and fastening the other end to a fixed support. If left alone, the thread will hang vertical; small vibrations under the action of gravity and the pull of the thread may be produced by pulling the bob to one side slightly and then letting go. It may be shown that such a pendulum will execute one complete vibration, starting from the end of a

swing and coming back to the same place again, in a time given by

$$\mathbf{t}=2\pi\sqrt{\frac{\mathbf{l}}{\mathbf{g}}}\text{ seconds,}$$

where l = the length of the thread in feet, measured to the centre of the bob,

g = the acceleration of gravity, in feet per second per second.

Expt. 34.—The simple pendulum may be used for roughly determining the value of g. Thus, arrange a small bullet to swing through a small angle at the end of a fine thread 3 or 4 feet long; take the time of, say, 100 vibrations; call this T seconds; then time of one vibration

$$t=\frac{T}{100}\text{ seconds.}$$

g can now be calculated, from

$$t=2\pi\sqrt{\frac{l}{g}},$$

or,

$$t^2=4\pi^2\frac{l}{g},$$

and

$$g=\frac{4\pi^2 . l}{t^2}.$$

EXERCISES ON CHAP. IX.

1. What distance will be travelled in 5 seconds by a train running at 60 miles per hour?

2. A train is observed to pass two points 480 ft. apart in 10 seconds. What is its speed in miles per hour?

3. A ship is moving due north with a speed of 10 knots. A person crosses the deck from port to starboard, a distance of 40 feet, in 10 seconds. What is his actual velocity relative to the earth? Give a diagram.

4. What is the average speed of a train which travels from London to Edinburgh, a distance of 400 miles, in 8½ hours?

5. A ship steadily acquires a speed of 15 knots, from rest, in 5 minutes. What has been its acceleration in foot and second units?

6. A body falling freely passes two points, the vertical distance between which is 120 feet, in two seconds. From what height above the higher point was it dropped?

7. A train the mass of which is 300 tons is started from rest and gains a speed of 30 miles an hour in 4 minutes. Calculate the force required, additional to that utilised in overcoming frictional resistances, to overcome the inertia of the train.

8. The moving parts of a steam hammer have a mass of 500 lbs., and are raised a height of 3 feet above the work before each blow. What is the kinetic energy of these parts when the hammer head is just reaching the work, assuming no frictional losses, and that steam is used for lifting the hammer only?

9. What is the kinetic energy possessed by a hammer head, mass 2 lbs., moving with a velocity of 40 feet per second?

10. What kinetic energy has a ship of 15,000 tons mass when its speed is 20 knots?

11. What exactly does a man mean when he says "this train is going at 30 miles an hour"? Suppose you have a watch with a seconds hand, and know that the telegraph posts are 200′ apart, how can you approximately find the speed of the train?

12. A body is moving towards the north at 40 feet per second. In two seconds later, we find it moving towards the north at 50 feet per second. What velocity has been added in these two seconds?

13. A body is moving towards the north at 50 feet per second. In two seconds afterwards we find that it is moving towards the north-east at 60 feet per second. Find by drawing what is the added velocity. State the magnitude and direction of the added velocity.

14. A man weighing 160 lbs. is in a lift which starts to descend with an acceleration of 2 feet per second per second. What force is exerted by the man upon the floor of the lift? What would the force be if the lift were descending at a uniform speed?

15. A truck, mass 10 tons, moving with a velocity of 4 feet per second, comes into collision with fixed buffers and is stopped in $\frac{1}{4}$ second. What is the average force of the blow?

16. The mass of the moving parts of a steam hammer is 1000 pounds, and the hammer head reaches the work with a velocity of 20 feet per second and is brought to rest in $\frac{1}{100}$ second. Calculate the average force of the blow.

17. A hammer head of $2\frac{1}{2}$ pounds moving with a velocity of 50 ft. per second, is stopped in 0·001 second. What is the average force of the blow?

18. A ship of 2000 tons, moving at 3 knots, is stopped in one minute; what is the average retarding force? Neglect the motion of the water. One knot is 6080 feet per hour.

19. If a gun delivers 400 bullets per minute, each having a mass of 0·5 oz. with 2000 feet per second horizontal velocity; neglecting the momentum of the gases, what is the average force exerted upon the gun?

✓**20.** A body, whose mass is 20 lbs., rotates round an axis at a radius of 9″, with a velocity of 40 feet per second. Calculate the pull on the axis.

21. A disc rotates on a shaft 120 times per minute. A wrought-iron pin, mass 5 lbs., projects from the disc, its radius being 12 inches. Find the mass required to balance the pin at a radius of 4 inches.

22. Find the length of a simple pendulum to beat seconds. Take $g = 32 \cdot 2$.

23. A railway coach, mass 20 tons, runs round a curve of 1,600 feet radius at a speed of 45 miles per hour. Calculate the centrifugal force.

24. A motor car moves in a horizontal circle of 300 feet radius at 30 miles per hour, what is the ratio of its centrifugal force to its weight? This is the tangent of the angle at which the track ought to be inclined sideways to the horizontal if there is to be absolutely no tendency to side slip; find this angle.

CHAPTER X.

HYDRAULICS. WATER PRESSURE. FLOATING BODIES. SPECIFIC GRAVITY.

Some properties of fluids.—Fluids are substances which are not able to offer permanent resistance to any forces, however small, which tend to change their shape. **Fluids** are either **liquid** or **gaseous**; gases possess the property of indefinite expansion, liquids do not. Thus, a small quantity of gas introduced into a perfectly empty vessel will at once expand and occupy the whole of the vessel, while a small quantity of liquid in the same circumstances will simply lie at the bottom of the vessel. Gases exist either as *vapours*, or as so-called *perfect gases.* The perfect gas was supposed to exist under all conditions as a gas, but it is now well known that all gases can be liquefied by great pressure and cold. **A vapour may be defined as a gas near its liquefying point, and a perfect gas as the same substance far removed from its liquefying point.**

Some liquids are more easily able to change their shapes than others. Liquids which change their shapes with difficulty are said to be the more viscous, the property being called **viscosity.** *Mobile* liquids change their shape very easily; thus, chloroform is used for delicate spirit levels on account of the extreme ease with which the bubble can change its position, chloroform being very mobile. Other liquids, such as cylinder oils, treacle, pitch, shoemakers' wax, are very viscous, but all change their shape if given sufficient time. Change of shape is always produced by shearing forces, *i.e.* forces acting along the surface. If equal compressive stresses are applied to all the faces of a cube,

the body will become smaller, but will remain cubical; but if shear stresses be applied, the shape changes. It follows, therefore, that if shearing stresses be applied to a fluid, it will not remain at rest, but will change its shape, and therefore, **if the fluid is at rest, there can be none but normal stresses acting anywhere on or in it.**

Stress on horizontal immersed surfaces.—Since there can be no shearing stress in a fluid at rest, and since friction is always brought about as a shearing stress, it follows that when a liquid such as water is at rest, there can be no frictional forces contributing to preserve the equilibrium of any portion of it. Suppose, in a tank of water (Fig. 152), we think of the equilibrium of a vertical column of it standing on a horizontal base of one foot square. The forces acting will be the weight of the column, which, if the depth is D feet, will be equal to the volume of the column multiplied by $62\frac{1}{2}$, the weight of a cubic foot of water nearly, so that

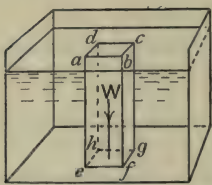


FIG. 152.—Equilibrium of a column of water.

$$W = D \times 1 \times 1 \times 62\tfrac{1}{2} = 62{\cdot}5 \times D \text{ lbs.}$$

There will also be stresses on each vertical side of the column, everywhere directed perpendicular to the sides, these being due to the pressures from the surrounding water, but as there can be no friction between the surrounding water and the sides of the column, these stresses merely serve to keep the column in shape and do not help in any way to balance the vertical force W. W is balanced by the upward stresses on the base of the column, due to the pressure from the bottom of the tank. Consequently, the total force on the base, which is one square foot in area, will be equal to $62{\cdot}5 \,.\, D$ lbs. Any other horizontal square foot at the same depth will have a similar and equal pressure on it. If, therefore, we have a horizontal area, A square feet, at a vertical depth D in water, the total pressure on it will be found by multiplying the pressure per square foot by the area A, or

$$P = 62{\cdot}5 \times D \times A \text{ lbs.}$$

If the liquid is not water, but some other which weighs w lbs. per cubic foot, then

$$P = w \times D \times A.$$

We notice from this that **the pressure on a horizontal area depends directly on its depth in the liquid and is proportional to it**; at double the depth the pressure on a given horizontal area will be doubled, and so on. It will be observed that the shape of a tank does not influence the pressure on its bottom. So long as the area of the bottom and the depth of liquid are kept the same, the pressure will be unaltered. The student should keep clear of the error of supposing that the weight of water in the tank gives the pressure on the bottom. This is not the case, as may be seen in the three examples shown in Figs. 153-5.

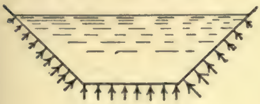


FIG. 153.

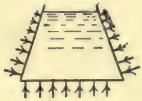


FIG. 154.

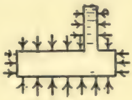


FIG. 155.

As a consequence of the pressures of the sides on the liquid being everywhere perpendicular to the sides, in the first case shown, the vertical upward components of these pressures sustain some of the weight of the contained water; in the second case, the vertical downward components of these pressures give more pressure to the bottom and so compensate for the diminished weight of water in the tank; in the third case, the pressure of the closed top has to be allowed for. In every case, the total pressure on the bottom will be found as already stated, by finding first the pressure on a square foot at the given depth and multiplying this by the actual horizontal area.

EXPT. 35.—To prove the law that the pressure on a horizontal surface is proportional to the depth of the liquid and to its density, arrange apparatus as shown in Fig. 156. A is a wide glass tube having its lower edge ground flat and square to the axis of the tube. A disc B closes the lower end of the tube and is supported by means of a cord C from one pan of a balance. A scale D serves to show the level of the liquid. First place a weight in the other scale pan sufficient to balance the disc and

cord. Then put an additional weight W_1 in the same scale pan, which will pull the disc upwards against the bottom of A. Pour water in carefully and slowly until the disc begins to separate from the tube. Note the height H_1 of the surface of the liquid above the disc. Repeat the experiment with different weights W_2, W_3, etc. and note H_2, H_3, etc. for each. Plot W and H on squared paper; a straight line graph will indicate that the pressure on the disc is proportional to the depth.

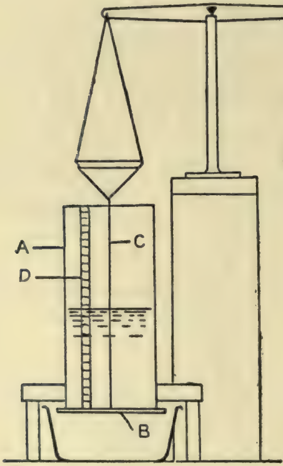


FIG. 156.—Pressure of a liquid on a horizontal surface.

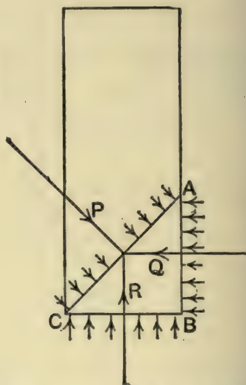


FIG. 157.—Equilibrium of the wedge ABC.

Repeat the experiment, using an oil having a density of about 0·8. Compare the depths at which the disc separated with water and with oil when equal loads were in the scale pan. Thus show that the pressure is proportional to the density.

Stress on inclined immersed surfaces.—Suppose, now, we think of a column of liquid as standing not on a horizontal, but on an *inclined* base, forming the sloping upper face of a wedge ABC (Fig. 157), making, for simplicity, an angle of 45° with the horizontal. The pressures on this wedge will be, as before, perpendicular to its faces. Let P be the resultant pressure on

the face AC, R the upward pressure on the bottom. For equilibrium of the wedge a third force Q, is required, which will act perpendicular to the face AB and, if we neglect the weight of the wedge itself, will be equal to R, by the triangle of forces. That is to say, at a given depth in a liquid, the stress on a vertical is the same as on a horizontal area. It can be shown that this is true for all areas sloping at any angle. This fact is usually stated as the principle that **fluids transmit stresses equally in all directions.** It should be noted in the case of liquids in tanks subjected to pressures due to their own weight, that as the stress varies directly as the depth, a vertical square foot anywhere will not have uniform stress, but one which varies from the top to the bottom. The stress at any point on the square foot will be that due to the vertical column of water above it, and is stated as the pressure which would be exerted on a square foot embracing that point, if the stresses were uniform. Thus, the stress at a point 4 ft. deep in water will be $4 \times 62{\cdot}5 = 250$ lbs. per square foot on any plane—horizontal, vertical, or inclined.

Pressures on the sides of a tank.—This principle enables us to calculate the pressures on the *sides of a tank.* Taking a rectangular tank as shown (Fig. 158) the stress on the bottom at a depth AB feet will be, for water,

$AB \times 62{\cdot}5$ lbs. per square foot.

The stress at the surface level will be zero, and as it uniformly increases as we descend, we may represent the stress at any depth by the breadth of a triangular diagram ABK at that point, the base BK being made equal to $AB \times 62{\cdot}5$. The average stress on any of the vertical sides will obviously be one half of this maximum stress, and we may find the total pressure on any of the sides by multiplying the area of that side by this average stress. It may be shown that the average stress on any immersed area is the stress at the depth of its centre of area.

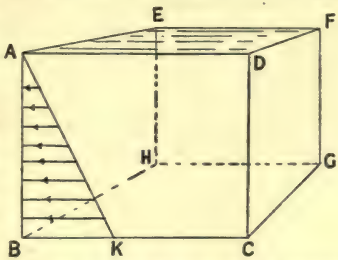


FIG. 158.—Stress diagram for the vertical side of a tank full of water.

EXAMPLE. A rectangular tank, 10 feet long, 6 feet broad, 4 feet deep, is full of fresh water. Calculate the resultant water pressure (a) on the bottom, (b) on one end, (c) on one side.

(a) Resultant pressure on bottom = average stress × area of bottom
$= (62{\cdot}5 \times 4) \times (10 \times 6)$
$= \underline{15{,}000}$ lbs.

(b) Resultant pressure on one end = average stress × area of end
$= (62{\cdot}5 \times 2) \times (6 \times 4)$
$= \underline{3000}$ lbs.

(c) Resultant pressure on one side = average stress × area of side
$= (62{\cdot}5 \times 2) \times (10 \times 4)$
$= \underline{5000}$ lbs.

It should be noted that no allowance has been made for the pressure exerted by the atmosphere in the above calculations. This amounts to about 15 lbs. per square inch and is uniformly distributed over both the inner and outer walls of the tank. Evidently the atmospheric pressure acting on the inside and outside of any side or of the bottom of the tank will balance and will produce no bulging effects on the tank sides or bottom. Hence, these are disregarded in practice and are neglected in subsequent similar examples unless in special cases where the vessel is not open to the atmosphere both externally and internally.

EXAMPLE. Find the total pressure on the inside of the bottom of the above tank, taking the atmospheric pressure as 15 lbs. per square inch.

Total atmospheric pressure = 15 × area of bottom in square inches
$= 15 \times 10 \times 6 \times 144$
$= \underline{129{,}600}$ lbs.

Resultant water pressure = 15,000 lbs.

Total pressure on the inside of the tank bottom = $\underline{144{,}600}$ lbs.

Centre of pressure.—The resultant pressure on an immersed horizontal area acts at its centre of area, and in the case of a vertical rectangular area, having one edge in the surface, at two-thirds the depth of water from the surface. The case of other more complicated areas cannot be dealt with here. The point at

which the resultant pressure on a surface acts is called the **Centre of Pressure.**

Head of water.—Water under pressure is often spoken of as being under a **head**. Head is the height from the point considered in the water to the surface level. The connection between head and stress is easily seen from the principles already discussed; thus, if H is the head in feet from the mouth of the pipe at A (Fig. 159) to a pump at B, then the fluid stress on the pump piston will be $62 \cdot 5\, H$ lbs. per square foot. In general, if w is the weight of the liquid in pounds per cubic foot, H the head in feet, and p the fluid stress in pounds per square foot, then

$$\mathbf{p = wH.}$$

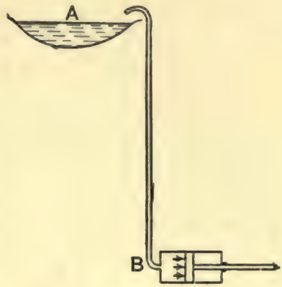


FIG. 159.

The total force on the pump piston will be found by multiplying its area in square feet by p.

Retaining wall for water.—The overthrowing action of water pressure on a retaining wall may now be examined.

It is usual to consider a portion of the wall one foot long; on the assumption that whatever happens to it will equally happen to every other portion. This being so, the resultant pressure P (Fig. 160) will be given by

$$P = \text{average pressure} \times \text{wetted area}$$
$$= (62 \cdot 5 \times \tfrac{1}{2} H) \times (H \times 1)$$
$$= \tfrac{1}{2} \times 62 \cdot 5 \times H^2 \text{ lbs.}$$

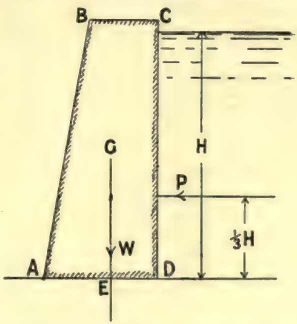


FIG. 160.—Section of retaining wall.

P will act at a distance $\dfrac{H}{3}$ from the bottom, and will tend to make the wall turn about A, its moment about A being P

multiplied by $\frac{H}{3}$ lb.-feet. This moment will be resisted by the weight, W lbs., of the portion of the wall under consideration, acting at its centre of gravity G. The moment of W about A will be opposite to that of P, and will be W multiplied by AE lb.-feet. If the moment of P is less than the moment of W, the wall will not be overthrown. For safety the overthrowing moment will always be considerably less in practice than the maximum resisting moment.

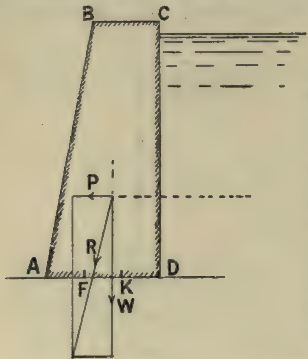


FIG. 161.—"Middle third" test for safety.

It is usual to test in this way. Draw a section of the wall to scale, and show P and W in their proper positions. Find the resultant R, of P and W, by the parallelogram of forces. Divide the base AD (Fig. 161) into three equal parts at F and K. If R passes within the middle part FK, the wall is safe, and if outside, it is not strong enough.

Lock gates.—In Fig. 162 is shown a lock in plan and section; by means of this arrangement, a boat may pass from the lower

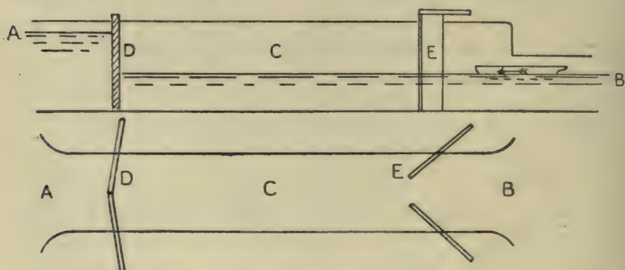


FIG. 162.—Section and plan of a lock.

level B of a canal to a higher level A. C is the lock chamber, and may be closed by means of gates at D and E. The gates are furnished with holes, or sluices, closed by valves which may

be opened by hand from above. It is clear that the gates at D could only be opened with great difficulty against the excess head of water in A over C, while those at E may be opened easily, as the water is at the same level in both B and C. A boat floats into the chamber C, through the open gates E, and the gates at E are then closed. The sluices in D are opened, and water pours into the chamber C from A, raising the water level and the boat in C, until the level in C is the same as that in A. The gates at D can then be opened easily, and the boat may emerge into the higher level canal A. Transference of a boat from the higher to the lower level is performed by reversing the operation.

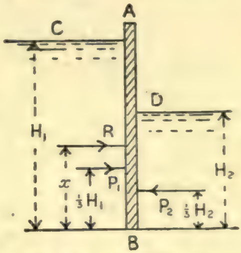


FIG. 163.—Pressures on a lock gate.

The water pressures acting on the gate are shown in Fig. 163 in which AB is a vertical section of the gate; the water level is at C on one side and at D on the other side of the gate. H_1 and H_2 are the heads in feet. If the width of the gate is L feet, we have:

$$
\begin{aligned}
P_1 &= \text{average pressure} \times \text{wetted area}, \\
&= (62{\cdot}5 \times \tfrac{1}{2}H_1) \times (H_1 \times L) \\
&= \tfrac{1}{2} \times 62{\cdot}5 \times H_1^2 \times L \text{ lbs.},
\end{aligned}
$$

and acts at $\frac{1}{3}H_1$ from B. In the same way:

$$P_2 = \tfrac{1}{2} \times 62{\cdot}5 \times H_2^2 \times L \text{ lbs.},$$

and acts at $\frac{1}{3}H_2$ from B.

P_1 and P_2 being of opposite sense, the resultant force R, will be obtained by taking the difference, and will have the same sense as the larger force P_1; its position may be found by taking moments about B. Thus:

$$
\begin{aligned}
Rx &= (P_1 \times \tfrac{1}{3}H_1) - (P_2 \times \tfrac{1}{3}H_2), \\
x &= \frac{P_1H_1 - P_2H_2}{3R}, \\
&= \frac{P_1H_1 - P_2H_2}{3(P_1 - P_2)} \text{ feet.}
\end{aligned}
$$

Forces acting on a floating body.—When a body, such as a ship, is floating in water, it is subjected to two resultant forces—its weight and the resultant water pressure on its sides and bottom. Consider the ship as floating at rest in still water. Its weight will be a downward vertical force W (Fig. 164), acting through G, the centre of gravity of the ship. The resultant water pressure balances W, and therefore must be an upward vertical force $R = W$, and in the same straight line with W. This force R, due to the buoyant effect of the water, is called the **buoyancy.**

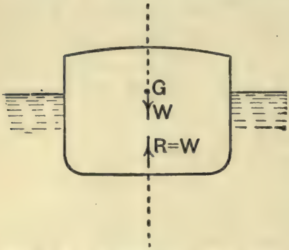


FIG. 164.—Equilibrium of a body floating at rest in still water.

Imagine for a moment that the surrounding water becomes solid, and so can preserve its shape, and let the vessel be lifted out, leaving a hole in the water which it exactly fits. Pour water into this hole until it is full to the surface level, and then let the surrounding water become liquid again (Fig. 165). The pressures of the surrounding water on the water poured in will be exactly the same as when the vessel occupied the hole, and their effect is similar—the weight of the water poured in is balanced. This being the case, we see that the weight of the water poured in and the weight of the vessel must be equal to one another, as each is equal to R, the resultant pressure from the surrounding water. Further, R must act through the centre of gravity of the water poured in, in order to support its weight. This point is called the **centre of buoyancy**—the point through which the resultant pressure of the water acts. Since R acts through G when the vessel is in the hole, and through B when water is in, it follows that G and B must be in the same vertical line. We may state, therefore, that **when a vessel is floating at rest in still water, the weight of the vessel is equal to the weight of the water displaced, and that the centres of**

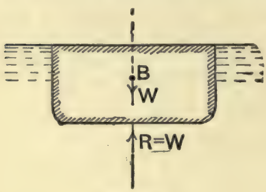


FIG. 165.

gravity of the vessel and of the displaced water are both in the same vertical line. This law is generally referred to as the **Principle of Archimedes.**

It will be evident that the same reasoning may be applied to a body wholly immersed in a liquid and that the same law holds good. Thus, the upward effort, or buoyancy acting on a piece of lead lying at the bottom of a tank of water will be equal to the weight of the water displaced by the lead. Hence, the force which the lead exerts on the tank bottom will be the difference between its weight and the buoyancy.

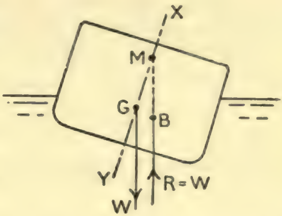


FIG. 166.—**Stability of a floating body.**

The stability of a floating vessel may be determined by slightly inclining it (Fig. 166) so that the original vertical centre line now occupies the position XY. The weight W acts through the centre of gravity G; the resultant water pressure acts vertically upwards through the centre of buoyancy B. It will be noted that, as more water is now displaced on the right hand side of XY, the tendency is to move B a little to the right of its first position. In Fig. 166 R and W form a couple tending to restore the vessel to its original position, hence the equilibrium is stable. If R be produced upwards, it will cut XY in M. It will be clear that, if M is above G, the sense of the couple is such as to restore the vessel to the original position; if M coincides with G (as in the case of a floating rubber ball) the lines of R and W coincide and the equilibrium will be neutral; if M falls below G, the couple has an upsetting tendency and the original equilibrium was unstable. M is called the **metacentre.**

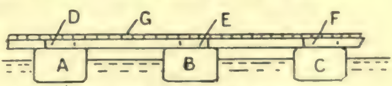


FIG. 167.—**A pontoon bridge.**

Pontoons are closed, or partially open, vessels used for supporting weights by reason of their buoyancy. Fig. 167 shows a method of constructing a floating bridge by use of pontoons A, B, C, etc. Timber beams DE, EF are laid across these,

and planks, G, laid on the top of the beams and at right angles to them form the roadway. In Fig. 168 is shown another application of pontoons to raising a sunken body. Two pontoons, A and B, support a staging CD, having hoisting tackle at E and F. Slings are placed round the sunken body G, which may thus be raised from the bottom. In Fig. 169 the pontoon A is used for

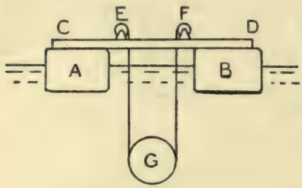


FIG. 168.—Pontoons raising a sunken body.

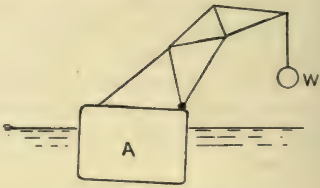


FIG. 169.—A floating crane.

supporting a crane hoisting a load W, the arrangement forming a floating crane. In all these cases, the additional depression of the pontoon, when the load comes on it, must not be sufficient to cause submergence.

Taking the case of a rectangular pontoon (Fig. 170) of length L feet and breadth B feet, its area as seen in the plan will be

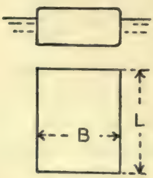


FIG. 170.

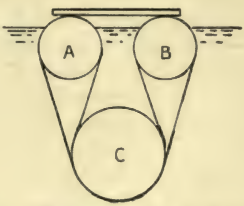


FIG. 171.—Camels raising a sunken body.

LB square feet. If depressed one foot deeper in the water, the additional displacement will be $LB \times 1 = LB$ cubic feet. The additional upward pressure of the water will be 62·5 LB lbs. (or 64 LB lbs. if in sea water) and this will be equal to the load which would produce an increase of one foot in the draught of the pontoon.

Camels are used for raising sunken vessels, or for diminishing the draught of a boat when sailing in shallow waters or if required to cross a river bar. The camel is an entirely closed box, A and B (Fig. 171), which may be submerged by permitting some water to flow into it through a valve, thus diminishing the volume of water displaced by the box, and hence diminishing the buoyancy. Slings are then passed round the sunken body C and round the camels. The valves are closed and the water is pumped out of the camels, thus lightening them sufficiently to enable the buoyancy of the water to bring the camels to the surface and thus to raise the sunken vessel from the bottom. Fig. 172 shows two camels attached to the sides of a vessel C for the purpose of diminishing its draught. These are applied in the same way as above described.

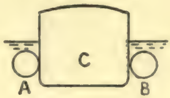


Fig. 172.—Camels for diminishing draught.

Another application of the same principle is to be found in the modern floating dock. In Fig. 173 (a) A is the dock, consisting of a large vessel which may be sunk to the position shown by the admission of water into internal tanks. The ship B may then float in as shown in Fig. 173 (a). On pumping the water out of the tanks, the dock rises slowly in the water, the ship rests on its floor and is shored by means of props between its sides and the dock sides. Ultimately the position shown in Fig. 173 (b) is attained, in which the ship is entirely out of the water. Repairs may now be carried out on any part of the outer skin of the ship.

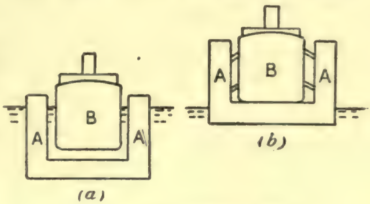


Fig. 173.—A floating dock.

Submarine boats provide another example of variable displacement. When cruising, the water surface is AB (Fig. 174), and a considerable portion of the vessel is above water. In action, the vessel may be sunk lower in the water by admitting water into internal tanks. The water surface may then be at

CD, or even higher. Pumps are provided in the interior for emptying the water tanks, and thus bringing the vessel again to

FIG. 174.—Submarine boat.

its original level. When the boat is in motion, diving may be accomplished by use of horizontal rudders which cause the centre line of the boat to become inclined to the horizontal. It will be noted (Fig. 175) that, if a submarine is entirely submerged and then inclined transversely, there is no change in the displacement and therefore no alteration in the position of the centre of buoyancy *B*. Hence, for stability, the centre of gravity *G* must fall below *B*.

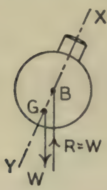


FIG. 175.—Stability of a submarine boat.

Specific gravity by experiment.—Since specific gravity is defined as the weight of a substance compared with the weight of an equal volume of water, it may be seen now how to determine experimentally the specific gravity of a body heavier than water.

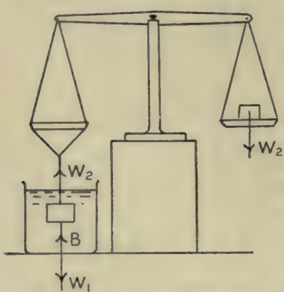


FIG. 176.—Specific gravity by weighing in water.

Expt. 36.—First weigh the body in the pan of an ordinary balance; let this be W_1. Then suspend it from the balance beam by a fine thread, the body being immersed in water at a temperature of 60° F. (Fig. 176). Restore balance by means of weights W_2 in the right-hand pan. The forces acting on the body will be its weight W_1 acting downwards, the buoyancy *B* acting upwards, and the pull of the thread, also acting upwards and equal to the weight W_2. These forces are in

equilibrium, hence :

$$B + W_2 = W_1,$$
$$B = W_1 - W_2,$$
$$= \text{the weight of an equal volume of water.}$$

Hence, Specific gravity $= \dfrac{W_1}{W_1 - W_2}$.

EXAMPLE 1. An iron rivet weighs 0·365 lbs. when weighed in the balance pan and 0·320 lbs. when weighed in water at 60° F. ;

$$\text{the loss of weight} = 0{\cdot}365 - 0{\cdot}320$$
$$= 0{\cdot}045 \text{ lbs.},$$

and this is the weight of an equal volume of water. The specific gravity is therefore $\dfrac{0{\cdot}365}{0{\cdot}045} = \underline{8{\cdot}1}.$

EXAMPLE 2. A vessel A (Fig. 177), floating in fresh water weighs 100 tons and supports by means of a chain an immersed iron body C which weighs 20 tons and has a specific gravity of 8. Find the pull in the chain and what weight of water is displaced by the vessel A.

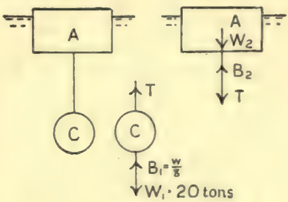


FIG. 177.

The forces acting on C are shown separately, and are the pull in the chain T, the buoyancy B_1, and the weight W_1; these are in equilibrium ; hence

$$T + B_1 = W_1,$$
$$T = W_1 - B_1,$$
$$= 20 - \tfrac{20}{8},$$
$$= \underline{17{\cdot}5} \text{ tons weight.}$$

The forces acting on A are also shown separately ; these are the pull of the chain T, the buoyancy B_2 and the weight W_2. For balance, we have $B_2 = W_2 + T$

$$= 100 + 17{\cdot}5$$
$$= \underline{117{\cdot}5} \text{ tons weight.}$$

The weight of water displaced by A will therefore be 117·5 tons.

The specific gravity of a liquid may be found by means of a **hydrometer** such as is illustrated in Fig. 178. The instrument is made of glass and is weighted at its lower end with mercury; a long graduated stem is attached to the upper part of the bulb. Since the weight of the instrument is constant, and is always equal to the weight of liquid displaced, it follows that the surface level AB will cut different divisions on the stem depending on the weight of the liquid per cubic unit. Deeper immersion will occur with lighter liquids. The stem is usually graduated to indicate specific gravities direct at 60° F.

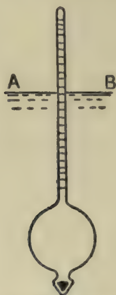


FIG. 178.—Hydrometer.

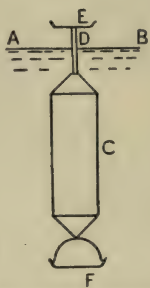


FIG. 179.—Nicholson's hydrometer.

The Nicholson hydrometer (Fig. 179) is an instrument of constant immersion, and consists of a hollow vessel C having a wire stem D and scale pans at E and F. A scratch on the stem D determines the depth of immersion, and the level of the surface of the liquid should cut this mark. A solid body may be weighed in air by first ascertaining what weight W_1 must be placed in the pan E in order to bring the instrument to the standard mark in water. W_1 is then removed and the body to be weighed is placed in the pan E; a weight W_2 is added to the pan E, sufficient to bring the instrument again to the standard mark. Then

$$\text{Weight of the body in air} = W = W_1 - W_2.$$

The weight of the body in water may now be ascertained by placing it in the pan F (use some fine cotton to tie it down if the body floats in water); again find the weight W_3, which

must be placed in the top pan in order to bring the instrument to the standard mark. Then,

$$\text{Weight of the body in water} = W' = W_1 - W_3.$$

The difference of these weights in air and in water will be equal to the weight of the water displaced by the body, viz.

$$\begin{aligned}\text{Weight of water displaced} &= W - W' \\ &= (W_1 - W_2) - (W_1 - W_3) \\ &= W_3 - W_2.\end{aligned}$$

Hence, $$\text{Specific gravity} = \frac{W_1 - W_2}{W_3 - W_2}.$$

EXAMPLE. A Nicholson's hydrometer weighs 100 grams, and requires 15 grams weight to sink it to the standard mark in water. What weight will sink it to the mark in a liquid of specific gravity 2·5?

The total downward force when in water is 115 grams, hence this is equal to the weight of the water displaced. When in the other liquid, the volume displaced will be the same, but the weight of this will be 2·5 times the former weight, viz. $2{\cdot}5 \times 115 = 287{\cdot}5$ grams. The upward pressure of the liquid is therefore 287·5 grams, and the weight required will be this amount less the weight of the instrument.

$$\begin{aligned}\text{Weight required} &= 287{\cdot}5 - 100 \\ &= 187{\cdot}5 \text{ grams.}\end{aligned}$$

EXPT. 37.—You are supplied with (*a*) a small piece of metal, (*b*) a piece of paraffin wax. Find the specific gravity of each body by use of the Nicholson's hydrometer.

EXPT. 38.—You are supplied with some turpentine, a piece of brass and a means of weighing. Find the specific gravity of the turpentine. The method to be followed is indicated by the following experimental record:

Weight of brass in air	= 126 grams.
„ „ water	= 110 grams.
„ „ turpentine	= 112 grams.
Weight of water displaced by the brass	= 16 grams.
„ turpentine „ „	= 14 grams.

The volumes of water and turpentine displaced are equal, hence:

$$\text{Specific gravity} = \frac{\text{weight of turpentine displaced}}{\text{weight of water displaced}}$$
$$= \tfrac{14}{16} = \underline{0{\cdot}875}.$$

EXERCISES ON CHAP. X.

1. A tank, 12 feet long, 8 feet broad, and 5 feet deep, is full of sea water (64 lbs. per cubic foot). Calculate the resultant pressures on the bottom, on one side and on one end of the tank.

2. A rectangular tank, 3 feet long, 3 feet wide, and 2 feet deep, is full of oil which weighs 8 lbs. to a gallon. Find the resultant pressures on the bottom and on one side.

3. Fresh water stands to a depth of 6 feet on one vertical side of a wall 20 feet long. Calculate the resultant water pressure on the wall and its overthrowing moment.

4. If the section of the wall in Question 3 is rectangular, material 120 lbs. per cubic foot, and the height of the wall is 7 feet, what should be its thickness in order to have a righting moment of twice the overthrowing moment?

5. Find the head of water in feet corresponding to a pressure of 60 lbs. weight per square inch.

6. A lock gate is 12 feet wide and has water standing on one side to a depth of 9 feet, and on the other side to a depth of 6 feet. Find the resultant pressure on the gate and its position.

7. A ship weighs 10,000 tons. What will be the volume of fresh water, in cubic feet, displaced by the vessel? Answer the question if the vessel sails into sea water of 64 lbs. weight per cubic foot.

8. A block of lead of weight 20 lbs. and specific gravity 11·4 is thrown into a tank of fresh water. Find the pressure which it exerts on the bottom.

9. Answer Question 9 if the tank contains oil having a specific gravity 0·8.

10. A rectangular pontoon is required to carry a weight of 4 tons and the additional depression when the load is applied is not to exceed 6 inches in fresh water. Find the horizontal area of the pontoon in square feet.

11. A plank of wood is 6 feet long, 9 inches wide, and 3 inches deep. If the specific gravity is 0·6, how many cubic inches will be below the surface when the plank is floating in fresh water? What vertical force would be required in order to immerse the plank entirely?

12. A camel weighs 5000 lbs. and consists of a cylindrical vessel 6 feet in diameter and 15 feet long. What weight of submerged iron, specific gravity 8, can it support by a chain in fresh water if the camel floats with the axis of the cylinder in the plane of the water surface?

13. Why does the admission of water into internal tanks in a submarine boat cause the boat to sink in the water? Explain fully.

14. Explain the principle of (*a*) a variable immersion hydrometer, (*b*) Nicholson's hydrometer.

15. A piece of zinc is found to weigh 42 grams in air and 37·8 grams when immersed in an oil having a specific gravity 0·7. Find the specific gravity of the zinc.

16. A piece of brass of specific gravity 8·5 weighs 2 lbs. in air. What will be the pull in the suspending cord when the brass is immersed in a liquid having a specific gravity of 0·82?

CHAPTER XI.

HYDRAULIC MACHINES.

Energy transmitted by water.—In Fig. 180 is shown a pipe *ABCD* of uniform bore and full of water up to the levels *E* and *F*. Pistons are fitted at *E* and *F*, and these support equal loads W_1 and W_2 lbs. weight. The weights of the loads will produce pressures on the contained water. If W_1 be allowed to descend through a given height *H*, W_2 will ascend through an equal height. This is owing to the fact that water is practically incompressible, hence if a certain volume of water is pushed out of *AB* by the descending piston *E*, an equal volume of water must find accommodation in *CD* by the piston *F* ascending; as the bore of the pipe is uniform, the heights will be equal. It is clear that W_1 has parted with potential energy to the amount of W_1H foot-lbs., and that W_2 has had its potential energy increased by an equal amount. This transference of energy has taken place by the agency of the water under pressure flowing along the pipe, and will evidently continue so long as the flow of water is maintained. Frictional waste has been disregarded in the above explanation.

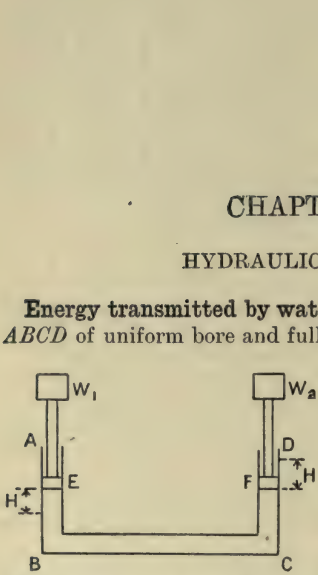


FIG. 180.—Hydraulic transmission of energy.

The pressure energy of water may be understood by reference to Fig. 181, in which water is entering a pipe at *A* under a pressure *p* lbs. per square foot and pushes before it a piston *C*

against a resistance R. Suppose that the stream in the pipe has an area of one square foot and that one cubic foot of water enters the pipe. The piston will be driven through a distance of one foot from C to D, and the work done by the cubic foot of water admitted will be given by:

Work done per cubic foot of water

$$= \text{total pressure on piston} \times \text{travel of piston,}$$

$$= (p \times 1) \times 1$$

$$= p \text{ foot-lbs.}$$

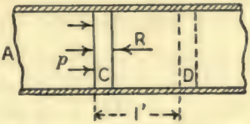


FIG. 181.—Pressure energy of water.

Let w be the weight in lbs. of a cubic foot of water, then:

$$\text{Work done per lb. weight of water} = \frac{p}{w} \text{ foot-lbs.}$$

This quantity expresses the store of energy which a pound weight of water possesses by virtue of its pressure. Water is supplied in the mains of hydraulic power supply companies at pressures of 700 or 800 lbs. per square inch. At the latter pressure the energy supplied per pound weight of water will be:

$$\text{Pressure energy per lb. weight of water} = \frac{800 \times 144}{62 \cdot 5},$$

$$= \underline{1840} \text{ foot-lbs.}$$

It will thus be understood that hydraulic methods afford a means of supplying energy in a very compact form over a district.

The water flowing in the pipe also possesses kinetic energy, but this, at ordinary practical speeds of flow is very small; at 5 feet per second, the kinetic energy of one pound of water would be about 0·8 foot-lb.

Water flowing along a horizontal pipe will preserve unaltered the potential energy with which it entered the pipe. If the pipe slopes downward, the potential energy of one pound of the flowing water will diminish to the extent of one foot-lb. for every foot of fall vertically in the pipe. The potential energy thus disappearing may be transformed into kinetic energy if

the pipe is not running full of water; if the pipe is running quite full and if the bore is uniform there can be no change in velocity and hence no change in kinetic energy; in this case the potential energy transformed is utilised partly in overcoming frictional resistances and partly in increasing the pressure of the water and hence increasing its pressure energy.

EXAMPLE. 1000 gallons of water under a pressure of 700 lbs. per square inch are delivered from hydraulic power mains in 5 minutes. What horse-power is being supplied?

$$\text{Weight of water delivered per minute} = \frac{1000 \times 10}{5},$$

$$= 2000 \text{ lbs.}$$

$$\text{Pressure energy per lb. weight of water} = \frac{p}{w},$$

$$= \frac{700 \times 144}{62 \cdot 5},$$

$$= 1610 \text{ foot-lbs.}$$

$$\text{Energy supplied per minute} = 2000 \times 1610,$$

$$= 3{,}220{,}000 \text{ foot-lbs.}$$

$$\text{Horse-power} = \frac{3{,}220{,}000}{33{,}000},$$

$$= \underline{97 \cdot 5.}$$

Hydraulic pumps are used for supplying water under high

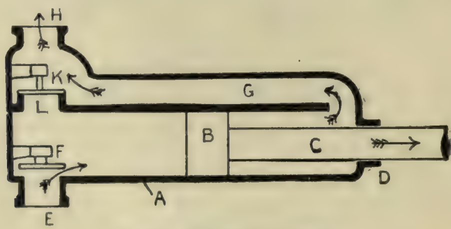


FIG. 182.—Hydraulic pump.

pressure for power purposes. They may be either belt-driven from a line shaft, or direct-coupled to a steam engine, or other

source of power. The object is to deliver a steady stream of water to the pipes at high pressure. A common type of pump is shown in outline in Fig. 182. A cylinder A is fitted with a piston B which may be pushed to and fro by means of a rod C driven by the engine. An arrangement is supplied at D for rendering water-tight the hole through which the rod passes. Valves F and K which open upwards are supplied so as to close the passages E and L. The piston is shown moving towards the right, and water is flowing into the cylinder from E past

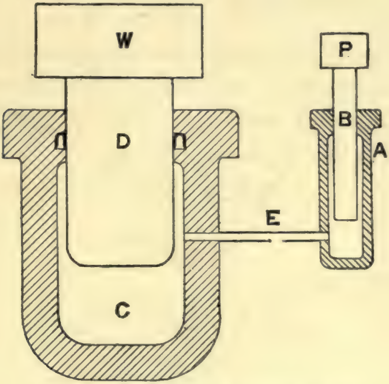


FIG. 183.—Diagram of the hydraulic press.

the open valve F. At the same time water on the right-hand side of the piston is being expelled through a passage G into H, and so into the delivery mains. When the piston is moving towards the left, F drops and closes E; the water on the left-hand side of the piston is then forced through L past the valve K (which has now lifted), and so partly into the delivery mains at H, and partly through the passage G into the right-hand side of the cylinder. The pump is thus double-acting, *i.e.* it delivers water during each stroke.

The hydraulic press.—The **hydraulic press** depends for its action on the fact that water transmits stress equally in every direction and is also practically incompressible. In Fig. 183, in

which all details have been omitted, A is a pump having a plunger B, say d_1 inches diameter, and carrying a load P. C is a large cylinder with a ram, or cylindrical piston D, say d_2 inches diameter, carrying a load W. The pump cylinder and the large cylinder are connected by a pipe at E. Due to the load P on the plunger B, a stress will be produced on the contained water which occupies the whole of the space in the cylinders not taken up by the plunger and the ram, this stress being

$$\frac{P}{\text{area of plunger}}=\frac{P}{\frac{\pi d_1^2}{4}}=\frac{4P}{\pi d_1^2}=p \text{ say.}$$

The stress p will be transmitted to all parts of the water, and will exert a pressure on the bottom of the ram D, tending to raise it, the resultant pressure being p multiplied by $\frac{\pi d_2^2}{4}$. W will be equal to this, neglecting friction.

$$W=p\times\frac{\pi d_2^2}{4},$$

or

$$p=\frac{4W}{\pi d_2^2}.$$

We see, therefore, that

$$p=\frac{4P}{\pi d_1^2}, \text{ and also}=\frac{4W}{\pi d_2^2},$$

or

$$\frac{W}{P}=\frac{d_2^2}{d_1^2}.$$

The mechanical advantage (without friction) of the arrangement is therefore equal to the ratio of the squares of the diameters of the ram and the pump plunger. For example, if d_1 is 1 inch, and d_2 10 inches, then if P is 1 ton, W would be 100 tons.

It will be observed, also, that if P is allowed to descend, W will be raised a much smaller distance. Suppose, for example, that the area of the pump plunger section is 1 square inch, and that the ram sectional area is 100 square inches ; then, if P descends 1″ it will deliver 1 cubic inch of water to the other cylinder. This cubic inch spread over the area of 100 square inches, will give a movement of $\frac{1}{100}$″ to the ram. So we see that the velocity ratio of the arrangement will be 100.

The hydraulic accumulator is used in connection with all hydraulic power plants. Its functions are to absorb the work done by the pumps when the presses, cranes, lifts, or other machines are at rest, and therefore taking no water, and also to prevent the water pressure exceeding a given maximum. It consists of a *hydraulic cylinder* (Fig. 184) placed upright and connected to both *pumps and machines* to be driven. The *ram* is loaded with heavy weights, and rises, when the water pressure is applied, against the resistance of these. When the ram approaches the top of its stroke it works a *tappet arrangement* connected to the throttle valve of the pump engine, or to the belt striking gear in a belt-driven pump, and so stops the pumps. The maximum working pressure of water which can be obtained is determined by the weights placed on the ram. Let W be this weight in tons and d the diameter of the accumulator ram in inches; then the water pressure

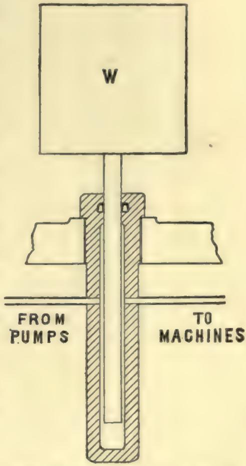


FIG. 184.—Diagram of the hydraulic accumulator.

$$p=\frac{W}{\frac{\pi d^2}{4}}=\frac{4 . W}{\pi d^2} \text{ tons per square inch,}$$

neglecting the loss by friction of the ram leathers. The ram will not begin to rise until the water pressure attains this value. Suppose the accumulator ram is *up* at the top of its stroke, and that the rise has been H feet. The work done by the pumps in raising it will be, neglecting friction, WH foot-tons. If now one of the hydraulic machines, such as a crane, be started, it draws its water supply at first from the accumulator, the weights of which in consequence descend, giving up some of their stored energy to the crane. Soon after starting, the descending weights release the tappet arrangement, and the pumps start off again, delivering water to the machine direct

until it is stopped, when the water from the pumps again goes into the accumulator and raises the ram. The arrangement, as will be seen from the above description, prevents any damage being done through stopping the hydraulic machines while the pumps are still working. Without the accumulator such stoppage, as water is practically incompressible, would have to be accompanied by a simultaneous stoppage of the pumps, which could not easily be accomplished.

FIG. 185.—Simple hydraulic lift.

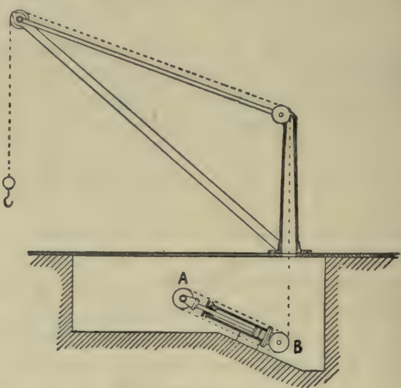


FIG. 186.—Hydraulic crane.

Hydraulic lift.—Fig. 185 shows a simple form of hydraulic lift. The cage is secured direct to the top of the ram of a vertical hydraulic cylinder. Water entering the cylinder raises the ram and so also the load. If d is the diameter of the ram in inches, and p the water pressure in pounds per square inch, then the total load which can be lifted, neglecting friction will be $p \times \frac{\pi d^2}{4}$ lbs. weight.

Hydraulic cranes are much used. Their action will be understood from Fig. 186. The chain sustaining the load passes along the tie, and down the interior of the hollow post to a hydraulic

cylinder situated in a pit. This cylinder has a piston ram with a chain wheel *A* at its outer end, and another chain wheel *B* is mounted on the base of the cylinder. The chain is secured to the side of the cylinder, passes over *A*, then back over *B* and thence up the post. The object is to obtain a larger travel of the chain for a small movement of the ram; in the present example, the chain raising the load will move twice as fast as the ram. The cylinder of a hydraulic crane may be placed in any convenient position and the chain led to it. Usually another cylinder is fitted for revolving the post, so as to swing the jib to any convenient position for raising a load.

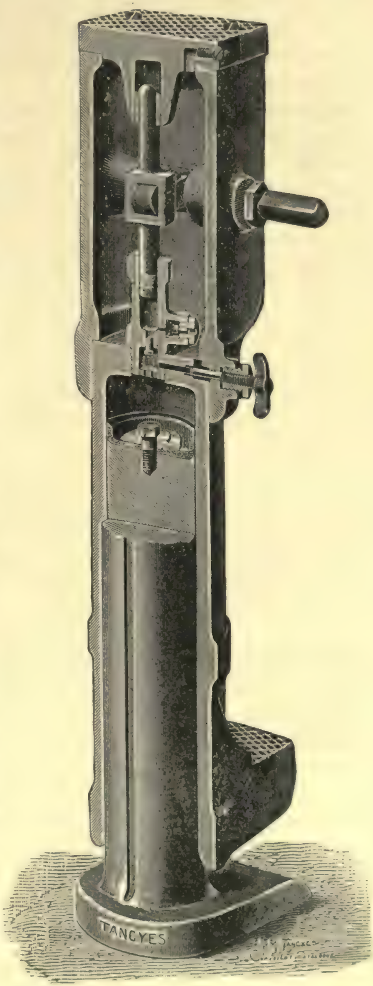


Fig. 187.—Hydraulic jack.

The hydraulic jack, Fig. 187, consists of a hydraulic cylinder inverted and working on a stationary ram. The cylinder contains a small pump operated by a lever outside. Water contained in the upper portion of the cylinder is forced, on working the outside lever,

into the lower part, and so raises the cylinder and any load which may be placed on its top. These machines are very convenient for raising heavy loads through a short distance; a large velocity ratio is possible.

Hydraulic engine.—In Fig. 188 is shown in outline a common type of hydraulic engine in which the pressure of water is used in order to rotate a shaft. There are three cylinders, A, B, and C arranged at angles of 120°, and each fitted with a piston—that at A is shown in section. Each piston is connected by means of a rod to a crank DE, which is fixed to a shaft rotating about D. The water acts on the outer sides of the pistons only,

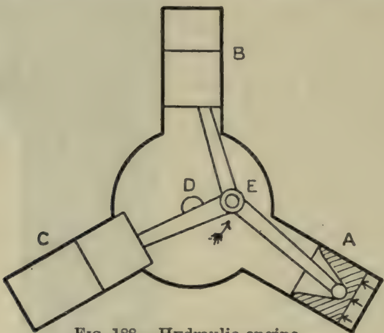


Fig. 188.—Hydraulic engine.

and is admitted and exhausted by means of valves not shown in Fig. 188. The piston in A has just commenced to move outwards, and is doing work on the crank; that in C is just finishing, and the piston in B is moving outwards, the water in the latter cylinder is flowing out into the exhaust pipe. It will be noticed that there will always be at least one piston acting, thus ensuring continuous rotation of the shaft.

Let p = the water pressure in lbs. per square inch.
d = the diameter of each cylinder in inches.
L = the length of stroke of the piston in feet.
N = the revolutions per minute of the shaft.

Then,

$$\text{Total pressure on one piston} = p \times \frac{\pi d^2}{4} \text{ lbs.}$$

$$\text{Work done per stroke} = p\frac{\pi d^2}{4} \times L \text{ foot-lbs.}$$

As there are three pistons, there will be $3N$ strokes per minute, during each of which work will be done ; hence :

$$\text{Work done per minute} = p\frac{\pi d^2}{4}L \times 3N \text{ foot-lbs.}$$

$$\text{Horse-power} = \frac{3p\pi d^2 LN}{4 \times 33000}.$$

This makes no allowance for frictional resistances, hence the mechanical efficiency has been assumed to be unity.

Conversion of the energy of water.—Let us now consider what happens when water falls from a height into a pond of water at rest. Suppose one pound of water to overflow from a cistern A (Fig. 189), of surface level H feet above the surface level of a pond B, into which the water falls. In A, the pound of water possesses potential energy equal to H ft.-lbs. This energy is gradually changed into kinetic energy during the fall, until at the surface level of B the whole of the potential energy has been transformed into kinetic energy, which, if the velocity of the water is v feet per second there, will be equal to $\frac{v^2}{2g}$ foot-pounds.

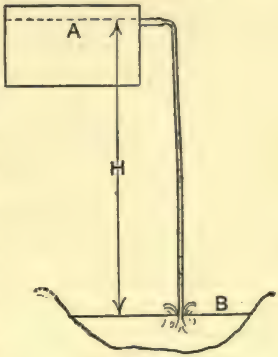


FIG. 189.

Potential energy lost = kinetic energy gained, or $H = \frac{v^2}{2g}$.

The water in B will be disturbed by the water entering it, but presently quietens down again, that is, the whole of the kinetic energy of the pound of water has been dissipated in creating disturbances in B, and none has been utilised in producing useful work. Useful work may be derived from the H foot-pounds of potential energy available by permitting the water to descend through a pipe, thereby producing pressure energy at the level of B, which may be converted into mechanical work by driving the pistons of a hydraulic engine. Or, the energy available may be utilised by means of a water-wheel of which there are three varieties—over-shot, breast-shot, and under-shot.

In the **over-shot wheel** (Fig. 190) water is brought to the top of the wheel, which has buckets fastened all round its rim; the water enters these buckets and remains in them until the wheel, turned by the extra weight of water on one side, has brought the buckets into such a position that the water is spilled out. In large wheels such as this it is usual to have teeth secured to the inner rim of the wheel and the power is taken from the wheel by a shaft carrying a pinion gearing with these teeth. This is to prevent the large stresses which would be introduced into the arms of the wheel if the power were taken from the shaft on which the wheel turns.

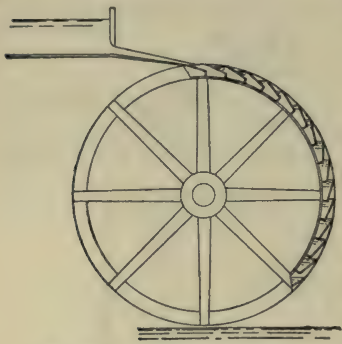


FIG. 190.—Over-shot water wheel.

In **breast-shot wheels** the water enters the buckets half-way up and remains in them until the bottom of the wheel is nearly reached, when it is spilled out.

In **under-shot wheels** the water is allowed to acquire as much velocity as possible before reaching the wheel and is then allowed to impinge on the blades. In this last case the change is from kinetic energy to mechanical work, in the others the mechanical work is done directly by the gravitational effort on the water in the buckets.

Turbines are machines used for converting the energy of water coming from a height into mechanical work. These are of two kinds—one in which the energy of the water is partly pressure and partly kinetic in passing through the machine, these being called **reaction turbines**; and another kind in which the energy of the water is wholly kinetic on reaching the machine, these being called **impulse turbines**.

The turbine consists of a wheel having blades running in a casing furnished with guide blades. The entering water is guided by these blades so as to have tangential velocity and

consequently tangential momentum. This momentum is abstracted during the passage through the wheel by the action of the curved blades on the wheel. Consequently, pressure is exerted on the rotating wheel, and work is done thereby. Reaction wheels, in which the water has its energy partly in the pressure form, must run full of water; in impulse wheels, on the other hand, the pressure of the water is atmospheric or nearly so, and the water slides along the blades in comparatively thin streams.

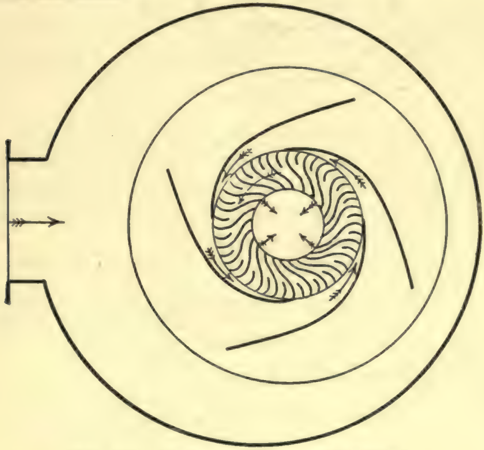


FIG. 191.—Diagram of Thomson's turbine.

In Thomson's turbine (Fig. 191) the water enters the wheel at its outer circumference, being guided by four blades which may be adjusted to suit varying quantities of water passing; it then passes through the wheel, moving inwards, and is discharged at the inner circumference. The shape of the guide blades and of the wheel blades at the outer circumference is such that *the velocity of the entering water relative to the wheel is along the wheel blade*; consequently the water enters without shock. At the inner circumference the shape of the blade is such that *the water leaves with radial velocity only.*

Horse-power of wheel.—Assuming that the entering velocity V_w is known, and that the exit velocity is radial, we may easily

find the momentum changed by passage through the wheel. Thus, let BA, equal to V_w, be the velocity of the entering water in Fig. 192,

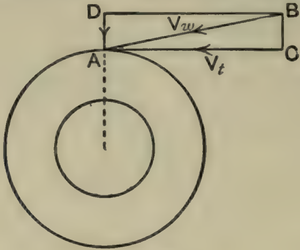


FIG. 192.

resolving this into tangential and radial components, we find the tangential velocity V_t represented by CA. This disappears on passage through the wheel, consequently the tangential momentum changed per pound of water is equal to $1 \times V_t$. Let m lbs. of water pass per second, then

$$\text{Change of momentum per second} = m V_t, \text{ and}$$

$$\text{Force given to wheel at } A \text{ in consequence of this} = \frac{m V_t}{g} \text{ lbs.}$$

Therefore, Work done per second $= \dfrac{m V_t}{g} . V$ ft.-lbs., where V is the velocity of the wheel at A, and

$$\text{Horse-power} = \frac{m V_t}{g} . V . \frac{60}{33{,}000}.$$

EXAMPLE. Suppose 500 lbs. of water per second to be delivered to a wheel with a tangential velocity of 40 feet per second. The velocity of the wheel rim is 35 feet per second. The water leaves the wheel radially. What horse-power can be developed?

$$\text{Pressure on wheel} = \frac{500 \times 40}{32 \cdot 2} \text{ lbs.}$$

$$\text{Work per second} = \frac{500 \times 40}{32 \cdot 2} \times 35 \text{ ft.-lbs.}$$

$$\text{H.P.} = \frac{500 \times 40}{32 \cdot 2} \times 35 \times \frac{60}{33{,}000} = \underline{\underline{39 \cdot 5}}.$$

Fig. 193 shows in outline a Jonval reaction water turbine. Water is supplied from A and passes through a ring of guide passages B, B, which are so shaped as to give the water a whirling velocity. The water then passes through the passages

C, *C*, of a revolving wheel which is attached to a vertical shaft *DD*, and is discharged into a tail-race *E*. The action of this turbine is similar to that in the Thomson turbine; whirling

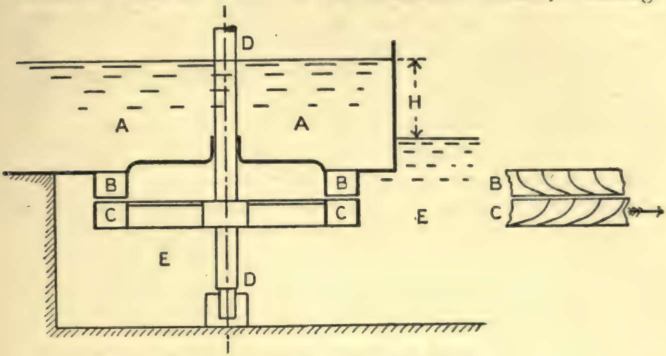


Fig. 193.—Jonval reaction water turbine.

velocity is given to the water in the guide passages, and is removed by the revolving wheel; the water is discharged from the wheel vertically without whirling velocity.

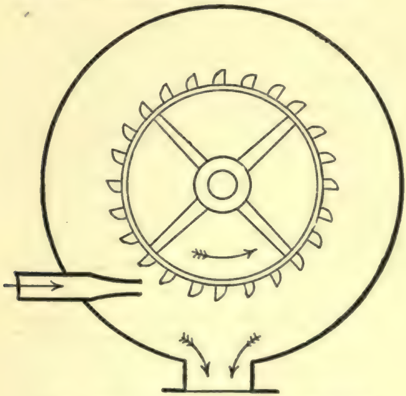


Fig. 194.—Diagram of the Pelton wheel.

The **Pelton wheel** is an example of an impulse wheel. It consists of a wheel running in an outer casing (Fig. 194), and

having blades or buckets arranged round its rim. A jet of water impinges on the buckets, gives up its momentum to the wheel, and escapes by a pipe from the lower part of the casing. The shape of the buckets is shown in plan in Fig. 195; the jet impinges centrally, divides and circles round the curved bucket, and is then discharged. The shape is semi-circular in plan, and is such that the maximum amount of momentum is abstracted from the water. If the bucket were at rest, the water would be directed backwards with a velocity equal to its original one, v_1. The whole change of momentum per pound of water would be $2 \times v_1$, and the pressure on the bucket due to this would be $\frac{2v_1}{g}$ lbs. If the wheel had such a speed of rotation that the velocity of the bucket was equal to that of the jet, no momentum would be changed, and the pressure would be zero. In either case, no work would be done. At a speed of rotation such that the buckets have a velocity half that of the jet, these conditions giving the theoretical maximum efficiency, the water would leave the bucket with little or no velocity relative to the earth, and consequently would have a maximum quantity of energy abstracted from it. The whole momentum $1 \times v_1$ of a pound of water in the jet would be changed. The efficiency of such a wheel would be 100 per cent., only the imperfect action of the water reaching the buckets, due to their different inclinations caused by the rotation of the wheel, and the interference of one bucket just entering the jet with the supply going to another, prevent this. An efficiency of about 80 per cent. can be attained.

FIG. 195.—Plan of bucket.

In Fig. 196 is shown an efficiency curve obtained in testing a small Pelton wheel. V_1 is the velocity of the jet and was kept constant; V_2 is the velocity of the wheel buckets and was varied during the experiments. It will be noticed that the maximum efficiency occurs when the ratio of V_2 to V_1 is about 0·4.

A Girard impulse turbine is shown in outline in Fig. 197. The general arrangement is similar to that of the Jonval

turbine (p. 171). The water leaves the guide blades B, B, in a ring of jets under atmospheric pressure and passes in thin layers

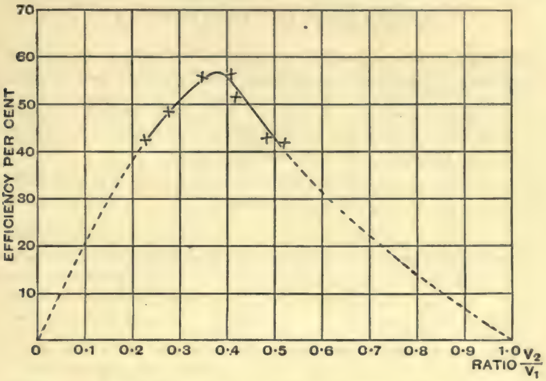


FIG. 196.—Test on a Pelton wheel; curve of efficiency for ratios $\frac{V_2}{V_1}$.

over the wheel blades C, C. The wheel is ventilated by means of small side passages, one at the back of each blade, in order to maintain the pressure approximately that of the atmosphere.

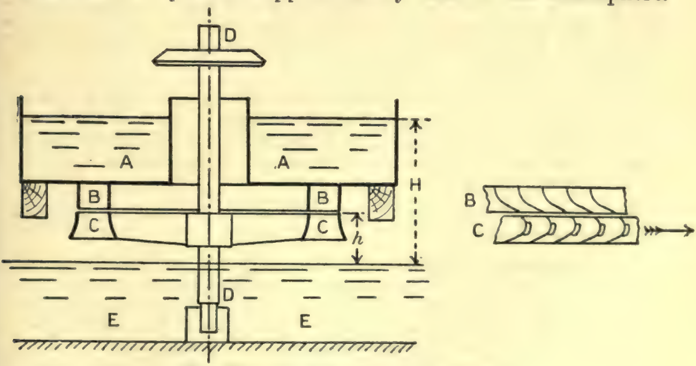


FIG. 197.—Girard impulse water turbine.

The whirling velocity attained in the guide passages is removed by the wheel blades and the water is discharged vertically into the tail-race E.

EXERCISES ON CHAPTER XI.

1. Find the pressure energy in foot-lbs. of one pound of water flowing along a pipe under a pressure of 1200 lbs. per square inch.

2. How many gallons of water, supplied under the conditions stated in Question 1, must be delivered per hour in order to maintain a rate of working of one horse-power?

3. Give a sketch and describe the working of a pump suitable for supplying water under high pressure.

4. A hydraulic press is operated by a pump having a plunger 1·5 inches in diameter. If the water pressure is 800 lbs. per square inch, what force is required to operate the plunger, neglecting friction? The ram of the press is 12 inches in diameter; find what total force it will exert.

5. Sketch and describe the construction and working of any hydraulic accumulator with which you are acquainted. If an accumulator has a ram 20″ diam. with a lift of 15′, and the gross weight of the load lifted is 130 tons, what is the pressure of water per square inch and the maximum energy in ft.-lbs. stored in the accumulator, neglecting friction?

6. A hydraulic crane is supplied with water at a pressure of 700 lbs. per sq. inch, and uses 2 cubic feet of water in order to lift 4 tons through a height of 12 feet. How much energy has been supplied to the crane, and how much has been converted into useful work?

7. Water at 750 lbs. per square inch pressure acts on a piston 1 square foot in area, through a stroke of 1 foot; what is the work that such water does per cubic foot and per gallon? If a hydraulic company charges 18 pence for a thousand gallons of such water, how much work is given for each penny?

8. Distinguish between the velocity ratio and the mechanical advantages of a machine.

In a hydraulic lifting-jack the ram is 6″ in diameter, the pump plunger is $\frac{7}{8}$″ diameter; the leverage for working the pump is 10 to 1. What is the velocity ratio of the machine? Experimentally we find that a force of 20 lbs. applied at the end of the lever lifts a weight of 8500 lbs. on the end of the ram. What is the mechanical advantage of the machine? What is the efficiency of the machine?

9. Give sketches and describe the working of a simple hydraulic lift, or a hydraulic crane.

10. A single-acting hydraulic engine has three rams, each of 3 inches diam.; common crank, 3 inches long; pressure of water

above that of exhaust, 100 lbs. per sq. inch ; 100 revolutions per minute ; no slip of water. What is the horse-power? If this engine does 2·15 horse-power usefully by means of a rope, what is the efficiency?

11. Four cubic feet of water per second enter an overshot wheel whose diameter is 40 feet. Taking an efficiency of 65 per cent., what horse-power can be obtained from the wheel?

12. Give sketches and explain the working of a Thomson turbine or any other form of reaction turbine.

13. Sketch and describe the mode of action of a Pelton wheel or other form of impulse turbine.

14. A Pelton wheel receives a jet of water 3 inches in diameter and having a velocity of 200 feet per second. What energy is being supplied per second? Supposing the efficiency to be 80 per cent., what horse-power will the machine develop?

15. Ten cubic feet of water per second enters a turbine wheel with a tangential velocity of 50 feet per second ; it enters without shock, the velocity of the rim of the wheel being 50 feet per second ; the water leaves the centre of the wheel with only a radial velocity ; what energy does the water give to the wheel per second?

PART II.

HEAT.

CHAPTER XII.

TEMPERATURE. EXPANSION

Temperature.—A person, on touching different bodies, may perceive that some of them are hot and others cold. The hotter bodies are said to be at a higher **temperature** than the colder. We may say, in fact, that **temperature means the hotness of a body as compared with some standard temperature.** Our sense of hotness often enables us to form an opinion regarding the temperature of a body, but it is not always trustworthy. Hence, some form of instrument is required for measuring temperatures, and such instruments are known as **thermometers** or temperature measurers.

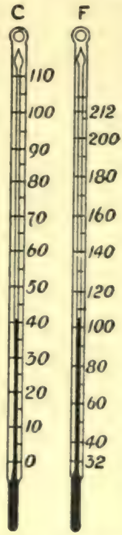


FIG. 198.—Centigrade and Fahrenheit mercury thermometers.

Thermometers.—Almost all substances expand on being warmed and contract when cooling. Advantage is taken of this property in the commonest form of thermometer, in which temperatures are measured by the amount of the expansion, or contraction, on change of temperature of mercury contained in a glass tube. The mercury thermometer consists of a fine glass tube (a capillary tube), at one end of which a bulb of either spherical or cylindrical shape is formed (Fig. 198). Mercury is introduced into the bulb and boiled to drive off any air it may contain. The other end of the tube is then sealed, so that

the only contents consist as nearly as possible of mercury and vapour of mercury. The glass walls of the bulb are blown very thin in order that the contained mercury may come quickly to the temperature of any body to which the thermometer may be applied. It will be found that the level of the mercury in the stem of the thermometer rises when the bulb is brought into contact with a hot body, and falls when touching a cold body, this result being due to the expansion or contraction of the mercury in the bulb. The level of the mercury surface may be taken as an indication of the temperature of the body with which the thermometer is in contact.

Graduation of thermometer.—In order that a thermometer may be used for comparing temperatures, a **scale of temperature** must be engraved along its stem. Two **fixed points** are first marked, and the interval between them is then divided into a number of parts called **degrees.** The fixed points are :

(*a*) the level of the mercury surface when the bulb and the part of the stem containing mercury are surrounded with melting ice ; this level being called **freezing point** ;

(*b*) the level of the mercury surface when the bulb and the part of the stem containing mercury are surrounded with steam coming from water boiling under standard atmospheric pressure ; this level being called **boiling point.** As the temperature at which water boils varies greatly with the pressure to which it is subjected, it is necessary to take a standard pressure in order to secure a definite boiling point. Standard atmospheric pressure is 760 mm., or 30 inches, of mercury, as shown by the barometer at sea-level.

Scales of temperature.—The **Fahrenheit scale** (named after its inventor) has the freezing point of water marked 32° and the boiling point 212°. The interval between these fixed points is divided into 180 degrees. Zero on this scale is 32° below the freezing point.

The **Centigrade scale** has the freezing point of water marked 0° and the boiling point 100° ; the interval between is divided into 100 degrees.

In the **Réaumur scale**, the freezing point of water is marked 0° and the boiling point 80°.

Temperatures below zero on all these scales are indicated by the negative sign. Thus $-10°$ F. means 42 Fahrenheit degrees below the freezing point of water.

Of these scales, the first two mentioned are largely used; the Réaumur is used only in some parts of the Continent. Engineers in this country use, for the most part, the Fahrenheit scale; physicists and chemists employ the Centigrade scale. It would be better if all were to use the latter, but in existing circumstances the engineering student must be familiar with both, and should be able to convert with facility temperatures stated on one scale into the corresponding temperatures on the other.

Conversion of temperatures.—The error which students are most liable to make in converting temperatures from the Fahrenheit to the Centigrade scale, or *vice versâ*, is due to the same temperature—the freezing point of water—being marked 32° on one and 0° on the other, rendering it necessary sometimes to add and sometimes to subtract 32°. To avoid risk of error, the following method may be used. Sketch two thermometers side by side, as shown in Fig. 199. Mark one F. and the other C. Mark the fixed points on each diagram, putting corresponding points opposite one another. Mark the given temperature on the proper thermometer. Suppose this to be 60° F. It will be seen by inspection that this temperature is $(60-32)=$ 28 Fahrenheit degrees above freezing point. Now since 180 Fahrenheit degrees correspond with 100 Centigrade degrees, we may find the number of Centigrade degrees corresponding to 28 Fahrenheit degrees from

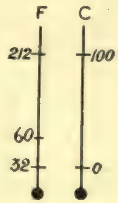


FIG. 199.—Conversion of thermometric scales.

$$\text{Cent. degrees} : 28 = 100 : 180,$$

or,

$$\text{Cent. degrees} = 28 \times \tfrac{100}{180}$$
$$= 28 \times \tfrac{5}{9} = \underline{15°\cdot5.}$$

The given temperature, viz. 60° F., therefore corresponds with 15°·5 C.

EXAMPLE i. Find the temperature on the Fahrenheit scale corresponding to 10° C.

Cent. degrees above freezing point $= 10$ (Fig. 200.)

Fah. ,, ,, ,, ,, $= 10 \times \frac{180}{100}$

$= 10 \times \frac{9}{5} = \underline{18°}.$

∴ Required temperature $= 18 + 32 = \underline{50°}$ F.

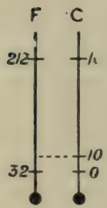


FIG. 200.

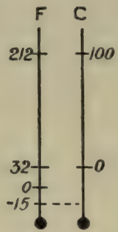


FIG. 201.

EXAMPLE ii. Find the temperature on the Centigrade scale corresponding to $-15°$ F.

Fah. degrees below freezing point $= 15 + 32 = 47$ (Fig. 201).

Cent. ,, ,, ,, ,, $= 47 \times \frac{5}{9}$

$= \underline{26°{\cdot}1}.$

∴ Required temperature $= -26°{\cdot}1$ C.

Testing thermometers—The following experiments should be performed carefully.

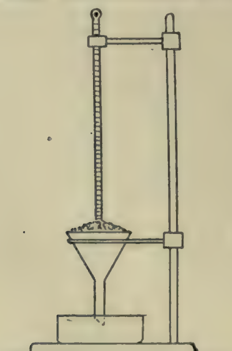


FIG. 202.—Apparatus for determining the freezing point of a thermometer.

EXPT. 39.—Arrange a funnel and beaker on a retort stand, as shown in Fig. 202. With a chisel remove some shavings from a block of ice, and put them into the funnel. Insert a thermometer, and pack the ice shavings closely round the bulb and stem as far up as the freezing point graduation. Bring the eye to the same level as the top of the mercury column, and take readings at intervals. Note the final

steady reading; this may be taken as the true freezing point of water. The freezing point error of the instrument will be the difference between the final steady reading and 32° if a Fahrenheit thermometer has been used, or 0° if it is a Centigrade thermometer. The error should be noted with its proper sign, + or −, attached.

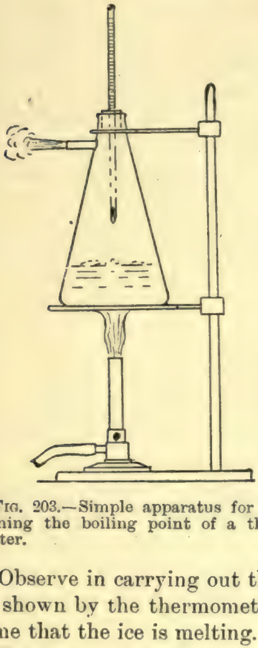


FIG. 203.—Simple apparatus for determining the boiling point of a thermometer.

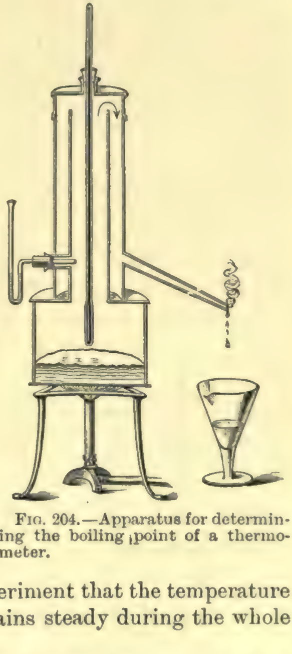


FIG. 204.—Apparatus for determining the boiling point of a thermometer.

Observe in carrying out this experiment that the temperature as shown by the thermometer remains steady during the whole time that the ice is melting.

EXPT. 40.—Bring some water to boiling temperature in a flask fitted with a side branch through which the steam evolved may be discharged (Fig. 203). By means of a cork fit a thermometer to the mouth of the flask. Notice that the temperature remains steady at or near the boiling point graduation during the whole time that the water is boiling.

A better form of apparatus for this experiment is shown in Fig. 204. This consists of a small copper boiler, to the cover of

which a double copper tube is attached. The thermometer under test is placed in the inner tube, and is surrounded by steam coming from the boiling water. This steam passes up the inner tube, then down the outer tube, being finally discharged at the bottom. The object of this arrangement is to **steam-jacket** the tube containing the thermometer, thereby ensuring that the tube shall be at the same temperature as the steam. A small glass U-gauge containing water is connected to the outer tube. When the water stands at the same level in both limbs of this gauge, the pressure of steam inside the apparatus is equal to that of the atmosphere outside. In using this apparatus, the thermometer is pushed through a cork, and placed in position so that its boiling point graduation is just visible above the cork. The water is then brought to boiling, and after steam has been given off freely during a few minutes, readings of the thermometer are taken.

Should the reading in this experiment differ from the boiling point graduation of the thermometer, it does not necessarily follow that the thermometer is in error. It will be remembered (p. 180) that the water must be boiling under standard atmospheric pressure, and this may not be the pressure of the atmosphere at the time the test is carried out. To obtain the pressure of the atmosphere, readings of a standard barometer should be taken while the experiment is going on, and the temperature at which water boils when subjected to this pressure will then be found from the Table, p. 367. The error may be stated approximately as the difference between the observed boiling temperature and that shown in the Table, and should be noted with its proper sign attached, + or −.

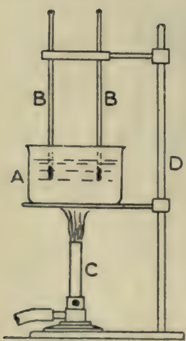


FIG. 205.—Apparatus for comparing the scale of a thermometer with that of a standard thermometer.

Should the bore of the stem of a thermometer not be uniform, equal increases of volume of the mercury will not be indicated by equal differences in the level of the mercury column. The bore, of course, ought to be as uniform as possible, and this is tested in ordinary

physical work by careful measurement, and any inequalities found allowed for. For our purpose, the following simple experiment will suffice for testing the accuracy of the graduations between the freezing and boiling points.

EXPT. 41.—In Fig. 205, *B*, *B*, are two thermometers, one of which is a standard thermometer, *i.e.* one in which the graduation errors are known, and the other is a thermometer to be tested. Both are suspended with their bulbs immersed in water contained in a beaker *A*. Gradually raise the temperature of the water, and take simultaneous readings of the thermometers at intervals of say 5°, being careful to stir the water well before taking the readings. Note these readings thus:

Standard thermometer.		Thermometer under test.	
Observed temp.	True temp.	Observed temp.	Error.

Columns 1 and 3 are filled in from the observations; column 2 from the known errors of the standard thermometer; column 4, obtained by taking the differences of columns 2 and 3, shows the errors of the thermometer under test at various parts of the scale.

Care and use of thermometers.—Thermometers should be handled carefully; no attempt should be made to force the thin-walled bulbs through corks; thermometers are liable to be injured if subjected suddenly to great changes of temperature. Do not use any thermometer in a place where there is risk of its being subjected to a temperature higher than that to which it is graduated, otherwise the bulb may be burst by the pressure of the expanding mercury. This danger may be guarded against partially by using thermometers having a safety bulb blown at the top of the stem (Fig. 198).

In obtaining the temperature of water or steam under pressure, a metal cup closed at its inner end may be secured to the containing vessel (Fig. 206). Some oil, or mercury, is

poured into the cup and comes quickly to the temperature of the contents of the vessel, and this may be measured by inserting a thermometer in the liquid. This arrangement prevents any risk of the thermometer bulb being collapsed by the pressure inside the vessel.

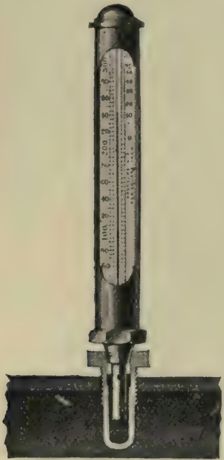


FIG. 206.—Hopkinson's pressure cup for thermometers.

In many cases where differences in temperature at two parts of a pipe are required, it suffices to secure the thermometers with their stems lying along the pipe, and to wrap cotton waste or flannel round the pipe over the bulbs, so as to ensure that the mercury comes to the same temperature, approximately, as the pipe. As both thermometers are under similar conditions, the difference in their readings will very nearly equal the difference in the temperatures of the contents of the pipe at the two places.

Measurement of high temperatures.—Under ordinary atmospheric pressure, mercury boils at 357° C., consequently ordinary mercury thermometers cannot be used for measuring temperatures higher than this. High temperatures may sometimes be stated with sufficient exactness by reference to the known melting temperatures of certain substances. Thus, we may say that the temperature of a body is about that of melting lead (617° F.), if the temperature be such that a small piece of lead placed in contact with the body just melts. Paraffin, sulphur, tin may be used in the same way. The method is useful for roughly determining the temperature of furnaces. Substances used in this way are called **thermoscopes.**

Pyrometers are instruments used for determining high temperatures with considerable accuracy. The temperature of a flue or furnace may be found by inserting a piece of platinum, copper, or other substance, allowing it to remain some time so as to come to the temperature of the furnace, then removing and plunging it into water. From the known weights of the

substance and of the water used, and the temperature of the water before and after, the temperature of the flue or furnace may be calculated by a method which will be explained later (Expt. 47, p. 199).

The electrical resistance of platinum wire at varying temperatures is used in some pyrometers to indicate the temperature to which the wire is exposed. In other pyrometers advantage is taken of the varying strength of electric current set up in an

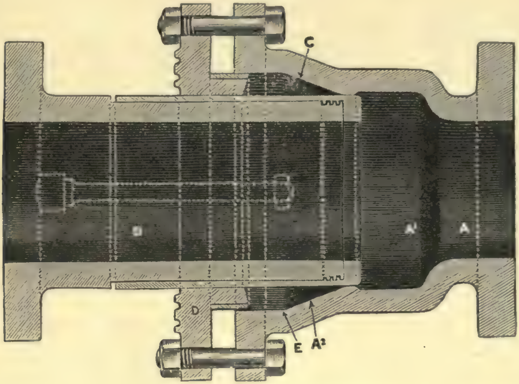


FIG. 207.—Hopkinson's arrangement for permitting a line of pipes to expand on elevation of temperature.

outer circuit when two dissimilar metals, such as platinum and iridium, in contact with one another, and in the circuit, are exposed to different temperatures. Other electrical and optical methods are in use, but the details of all of them are beyond the scope of this book, although their use in practice has been rendered exceedingly simple in modern instruments of appropriate design.

Expansion.—The expansion of metals as a consequence of elevation of temperature is well known. Advantage is taken of the property in the execution of work requiring **shrinking**. One piece of material, such as a wheel tyre, or a gun tube, has to fit tightly on to another piece. The outer piece is bored out

too small to go on when cold ; but on being heated, it expands, and may then be slid into place. As cooling goes on, the outer piece shrinks, and binds itself tightly to the inner piece of material.

In other classes of work, such as steel bridges, rails, steam pipes, boiler furnace tubes, etc., the expansion due to heating is a nuisance which has to be provided for.

Fig. 207 shows an arrangement for giving freedom to expand in the case of steam pipes. In this arrangement, the end *B* of one part of the pipe may slide inside the other part *A*, which is formed at *A′* to receive it. The joint is made by means of packing *E*, A_2, inserted in the circular box *C*, and forced down tight by the gland *D*. Two long studs, one alone being shown by dotted lines, are screwed into the flange of *B* ; these studs pass through holes in the flange of *A*, and prevent any danger of the pipes becoming separated by the internal pressure.

Expt. 42.—Apparatus by means of which an experiment useful for showing the expansion of a metal tube when heated

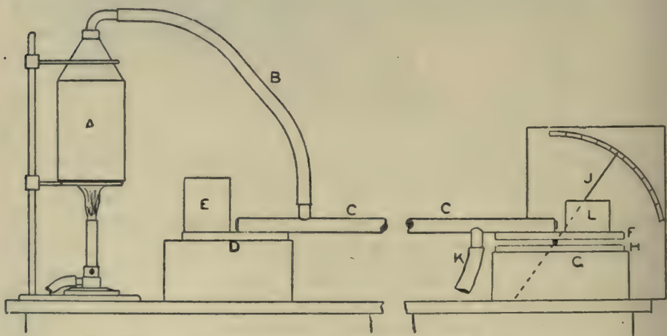


Fig. 208.—Apparatus for showing the expansion of a metal tube.

is shown in Fig. 208. *A* is a small boiler connected by rubber tubing *B* to a copper tube *C*. The copper tube is about 3 feet long and is plugged at both ends ; a branch is soldered near each end on opposite sides of the tube. Steam from the boiler enters the tube through *B* and is discharged freely through *K*. Two brass plates, *D* and *F*, each about $3\frac{1}{2}''$ long, $1''$ wide, and

$\frac{1}{4}''$ thick are soldered to the tube. The tube is supported by a block at *D* and held down by a weight *E* placed on the brass plate. The brass plate at the other end of the tube rests on a small roller made of thin drill steel which may roll on a brass plate *H* secured to a supporting block *G*. Any movement horizontally of this end of the copper tube will cause the roller to rotate, and this rotation will be rendered evident by the pointer *J* travelling over the graduated scale. The pointer may be made of a narrow strip of card secured to the roller by means of sealing-wax. On passing steam from the boiler through the copper tube, the expansion will be shown clearly by the movements of the pointer.

EXPT. 43.—The expansion of water when heated may be shown by using a small flask (Fig. 209) fitted with a rubber stopper and having a glass tube inserted. A paper scale is secured to the glass tube. Water is introduced to the flask and stands, when cold, two or three inches up the tube. The water is best coloured with some red ink. On placing the flask into a vessel containing hot water, the water in the flask will be warmed gradually and its consequent expansion will be shown by the level rising in the glass tube.

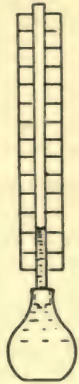


FIG. 209.—Apparatus for showing the expansion of water.

Coefficient of linear expansion.—All metals do not expand to the same extent on being heated through the same range of temperature. Thus, copper and brass expand more than iron for the same increase of temperature. This fact may be illustrated by means of two flat bars, one of copper, the other of iron, riveted together and heated. If straight to begin with, the composite bar will be found to be bent after heating, the copper bar being on the convex side of the bend, showing that it has expanded more than the iron.

The coefficient of linear expansion of a substance is the increase in length which a bar of unit length undergoes when its temperature rises through one degree.

Let e = coefficient of linear expansion,
L = original length of bar,
t = elevation of temperature.

Then, the increase in length of a bar of unit length, heated through t degrees $= t \times e$; and for a bar of length L, the increase in length $= L \times t \times e$;

$$\therefore \text{ length of bar after heating} = L + Lte = L(1 + te).$$

EXAMPLE. Steel rails, each 20 feet in length, are laid when the atmospheric temperature is 50° F. It is intended that the ends should touch if the temperature reaches 120° F. What space should be left between the ends when laying the rails? Take the coefficient of linear expansion = 0·0000067 per degree F.

$$\text{Increase in temperature} = (120 - 50) = 70° \text{ F.}$$

$$\begin{aligned} \text{Increase in a length of 20 feet} &= Lte \\ &= 20 \times 70 \times 0{\cdot}0000067 \\ &= 0{\cdot}00938 \text{ foot} \\ &= \underline{0{\cdot}113} \text{ inch.} \end{aligned}$$

The space left between the ends in order to allow for this increase in length must be 0·113 inch.

Coefficient of superficial expansion.—This coefficient may be defined as the **increase in area which a plate of unit area undergoes when its temperature rises through one degree.**

The value of this coefficient for a given substance may be shown to be **double that of the linear coefficient of expansion** for the same substance.

The **coefficient of cubical expansion** of a substance may be defined as the **increase in volume which unit volume will undergo when its temperature rises through one degree.**

The value of this coefficient may be shown to be **three times that of the linear coefficient of expansion of the substance.**

In using the numerical values of the above coefficients, care should be taken to ascertain whether the values given in the Table from which they are extracted are for the Fahrenheit or Centigrade scale. In books on engineering subjects they are generally stated for the Fahrenheit scale (such a Table will be found on p. 367), and in physical text books, for the Centigrade scale.

EXERCISES ON CHAPTER XII.

1. Explain the terms "temperature," "scale of temperature," "freezing point," "boiling point."

2. Find the temperature F. corresponding to 140° C.

3. Find the temperature C. corresponding to −40° F.

4. Find the temperature F. corresponding to −273° C.

5. A bar of brass measures 34″ in length at a temperature of 60° F. What will be its length at a temperature of 200° F.? Take the coefficient of linear expansion = 0·0000105 per degree F.

6. A crank has to be shrunk into its position on the shaft. The hole in it is 12·02″ in diameter at 60° F. Calculate to what temperature it must be raised in order that the diameter of the hole may be 12·05″. Take the coefficient of linear expansion = 0·0000067 per degree F.

7. Give sketches and description of any means for providing for the expansion of a long line of steam pipes.

8. Calculate the change in length of a line of wrought iron steam pipes, 65 feet long, when the temperature is raised from 50° F. to 338° F. Take the coefficient of linear expansion = 0·0000067 per degree F.

CHAPTER XIII.

HEAT AND ITS MEASUREMENT.

Quantity of heat.—When a hot and a cold body are brought into contact, both will come ultimately to the same temperature, which will lie between the temperatures originally possessed by the bodies.

EXPT. 44.—Take two vessels, one, A, containing about 2 pints of water at about 60° F., the other, B, containing about $\frac{1}{2}$ pint of water at about 150° F. Place a thermometer in each vessel, stir the water and take the temperatures. Now pour the water from B into A, stir again and read the temperature. With the quantities as mentioned above this will probably be about 70° F. As the water originally in A has been warmed about 10° F., while that in B has been cooled about 80° F., it is evident that heat has been transferred from the hotter to the colder water.

Notice that temperature and heat are not the same. As the temperature of the water in A has not been increased to the same extent that the temperature of the water in B has been lowered, it is evident that it has not been temperature that has been transferred. Moreover, a comparatively cold body may contain a great quantity of heat while a very hot body may possess a small quantity of heat only. A vessel of water set to boil over a Bunsen burner receives a great quantity of heat, as estimated by the time taken, while its final temperature of 212° F. is comparatively low. A wire held in a flame comes to a very high temperature almost immediately, and evidently can only contain a small quantity of heat.

Units of heat.—Quantities of heat are measured by comparison with that quantity required to elevate the temperature

of unit mass of water through 1°. In the metric system, the **gram-degree-Centigrade** is the unit of heat; this unit is **the quantity of heat required to raise the temperature of one gram of water through 1° C.** This unit is sometimes called a **therm**, or a **gram-calorie.** Frequently a unit of heat 1000 times as large as the therm is used, this being called a **calorie**, or sometimes a **major-calorie** or **great calorie.**

In Britain, two heat units are in common use as well as the metric units. These are:

(*a*) **the quantity of heat required to raise the temperature of 1 lb. of water through 1° C.**;

(*b*) **the quantity of heat required to raise the temperature of 1 lb. of water through 1° F.**

The former of these units is called the **pound-degree-Centigrade unit**, or, briefly, the **Centigrade unit** of heat; the latter is called the **pound-degree-Fahrenheit unit**, or the **Fahrenheit unit**, or, more generally, the **British Thermal Unit**, written B.T.U.

Engineers use most frequently the British thermal unit, although recently in several important papers on engineering subjects the pound-degree Centigrade unit has been employed. Students should make themselves familiar with all three units.

EXAMPLE i. Calculate the quantity of heat in British thermal units required to raise the temperature of one pound of water through 1° C.

Since 1 B.T.U. can raise the temperature of 1 lb. of water through 1° F., and since $\frac{9}{5}$ degree F. is equivalent to 1 degree C., it follows that $\frac{9}{5}$ B.T.U. will be required.

From this example it will be seen that

1 lb.-degree-Cent. unit $= \frac{9}{5}$ B.T.U.

1 B.T.U. $= \frac{5}{9}$ lb.-degree-Cent. unit.

These values enable us to convert from one system to the other.

EXAMPLE ii. What factor must be employed to convert a given quantity of heat stated in gram-calories into B.T.U.?

1 gram-calorie can raise the temp. of 1 gram water through 1° C.
453·6 gram-calories „ „ „ 453·6 grams „ „ 1° C.

And, since 1 lb. $=453 \cdot 6$ grams, this statement may be written,

453·6 gram-calories can raise the temp. of 1 lb. water through 1°C.

($\frac{5}{9} \times 453 \cdot 6$) ,, ,, ,, ,, ,, ,, 1°F.

$$1 \text{ B.T.U.} = \tfrac{5}{9} \times 453 \cdot 6$$
$$= \underline{252} \text{ gram-calories.}$$

Hence, to convert from gram-calories to B.T.U. multiply the gram-calories by $\frac{1}{252} = 0 \cdot 003968$.

To convert from B.T.U. to gram-calories, multiply the B.T.U.'s by 252.

Specific heat.—Suppose 1 lb. each of several different substances to be experimented on with a view to ascertain what quantity of heat is required to raise the temperature of each through 1°, it would, in general, be found that the quantities required differ. Water requires most, metals require a much smaller quantity of heat. The **specific heat** of a substance may be defined as **the quantity of heat required to raise the temperature of unit mass of the substance through one degree.** The specific heat of water by this definition will be 1. The specific heat of iron is about $\frac{1}{9}$, from which we infer that $\frac{1}{9}$ B.T.U. when imparted to 1 lb. of iron will raise its temperature through 1° F. It will also be seen that since 1 lb.-degree-Cent. unit can raise the temperature of 1 lb. of water through 1° C., $\frac{1}{9}$ lb.-degree-Cent. unit will raise the temperature of 1 lb. of iron through 1° C. In the same way, $\frac{1}{9}$ gram-calorie unit can raise the temperature of 1 gram of iron through 1° C. It will therefore be understood that the number giving the specific heat of a substance is the same for all three systems of units.

A Table giving the specific heats of some common substances will be found on p. 367.

Water equivalent of a body.—By this expression is meant that weight of water which requires the same quantity of heat to raise its temperature through one degree as the body would require in order to elevate its temperature to the same extent. Thus, the water equivalent of a piece of iron weighing 1 lb. is $\frac{1}{9}$ lb.; and of an iron cylinder weighing 2 cwts. is $\frac{224}{9} = 25$ lbs. water nearly. This conception is useful in simplifying the calculations required in reducing data obtained in experiments on transference of heat.

Let W = the weight, in lbs., of a body,

s = the specific heat of the material,

Then, Water equivalent of the body = Ws lbs.

Transference of heat.—Heat may be transferred from one body to another by (*a*) **conduction**, (*b*) **convection**, (*c*) **radiation**.

In **conduction**, the particles of a body nearest to a source of heat are warmed first; these then heat the neighbouring particles, which in turn pass on heat to the others; heat is thus transferred through chains of particles until the whole body has been heated. Solids are heated by conduction.

In **convection**, particles nearest to the source of heat are warmed and then pass off to some other part of the body, making room for other particles to approach the source of heat to be warmed and pass off in turn. The whole body ultimately becomes warmed by these currents of particles. Liquids and gases are heated by convection.

In **radiation**, heat is transferred by a kind of wave motion in the ether, a medium which, it is assumed, fills all interstellar space and the space between the molecules of material bodies. The heat waves arriving at a body and being absorbed produce the ordinary effects of heat.

All these methods of transference of heat occur in steam boilers. The furnace plates receive heat radiated from the fire; heat passes through the plates into the water by conduction; and the water becomes heated by convection, the resulting **convection currents** setting the water into circulation.

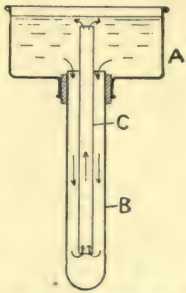


FIG. 210.—Apparatus for showing the circulation of water due to convection currents.

EXPT. 45.—Fig. 210 shows an apparatus by means of which convection currents in water may be studied. *A* is a vessel with a hole in its bottom made to receive a glass tube *B*. *B* is sealed at its lower end, and its upper end is about flush with the bottom of *A*. Another smaller bore glass tube *C*, open at both ends, is suspended centrally in *B*, its lower end being about 2″ from the bottom of *B*, and its upper end being a little below the surface of water which is contained by

A, and fills both tubes. On applying a Bunsen flame gently at the lower end of B, the water there will be warmed and convection currents will be set up, the current flowing up through C to the surface of the water in A, while colder water descends from A, through the space between the two tubes, to be heated in turn.

This device is used in certain types of steam boilers for the purpose of enabling free circulation of the water in the boiler to take place.

Hot water supply.—The means of supplying hot water to the lavatory taps in a house depend on convection currents for successful working. In Fig. 211, A is an open cold water tank from which the general supply of cold water is obtained, and is connected to an entirely closed hot water storage tank C, by a pipe B which enters C near the bottom. The tank C is connected by a pipe D to a boiler E, which is placed usually at the back of the kitchen fireplace and is heated by the kitchen fire. The pipe D is connected to both C and E as low down as possible. Another pipe F is connected to the boiler near the top and leads to the top of the storage tank C. The pipes D and F are called circulating pipes. The pipe G leads from the upper part of C to the bath tap at H, and may have branches leading to other taps at different parts of the house. K is a pipe for returning to A any water which may be thrown upwards from C by reason of ebullition or other causes, and also to get rid of air from the system.

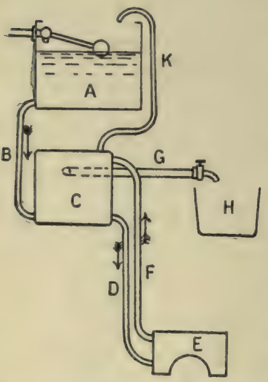


FIG. 211.—Arrangement for hot water supply.

In working, the water in the boiler becomes heated and rises to the top, ascends the pipe F and enters C; meanwhile a further supply of cold water travels downwards from C through D and enters the boiler to be heated in turn. After a few minutes working, it will be found that the water in the upper

part of C has become hot. The colder, heavier water accumulates at the bottom of C, hence the reason for the cold water supply pipe B entering C at the bottom and also for the tap supply pipe being connected near the top of C. If the tap at H be opened, hot water will be drawn from the upper part of C and an equal quantity of cold water will flow from A through B into the lower part of C.

A somewhat similar device is used in arrangements for heating buildings (Fig. 212) by hot water circulation. A is a boiler completely filled with water and generally situated in the basement. A pipe B leads the heated water into the room or rooms to be heated, where it travels along the pipe CD, then back along EF, giving up part of its heat to the air in the room. The water is finally returned to the lower colder part of the boiler at G. H is an air pipe leading up to the roof, and allows air to escape from the system; a small automatic air valve may be substituted for this pipe; the valve closes as the pipe CD becomes hot. Further supplies of cold water may be admitted as required into the boiler by means of the valve at K. Cold air may be brought into the room through openings in the walls behind the pipes CD and EF; convection currents will then carry the air so heated throughout the room. The heating surface may be made greater by connecting to CD, at intervals, groups of short vertical pipes; these are called **radiators.**

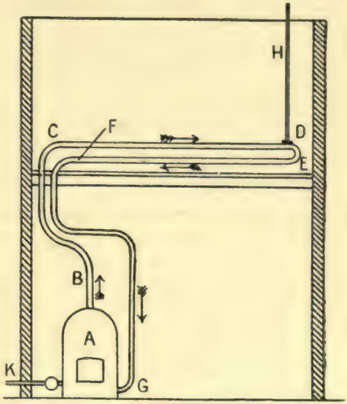


FIG. 212.—Heating by circulation of water.

Calculation regarding heat transference.—In making calculations of the ultimate temperature attained when heat is transferred from one body to another, we may assume in the first instance that no heat is wasted in raising the temperature

of any body other than the colder one considered. Corrections may then be estimated and applied for any heat known to be wasted. Calling the two bodies A and B, we may state as an approximate solution:

Heat passing from A = heat entering B.

EXPT. 46.—Weigh about 4 lbs. of cold water in a vessel, preferably made of copper, and about 1 lb. of hot water in another vessel. Take the temperatures, using two thermometers, one in each vessel. Then pour the hot water into the first vessel, stir up well, and note the final steady temperature.

Compare this temperature with the value calculated as follows:

Let W_A = weight in lbs. of the cold water.
W_B = " " " " hot "
$t°_A$ = temperature of the cold water.
$t°_B$ = " " hot "
$t°$ = calculated final steady temperature.

The temperatures are to be stated all in the same scale.

Heat passing from $B = W_B \times$ fall in temperature from $t°_B$ to $t°$.

Heat entering $A = W_A \times$ rise in temperature from $t°_A$ to $t°$.

Assume

Heat passing from B = heat entering A.

$$W_B(t_B - t) = W_A(t - t_A) \quad \text{.......... (1)}$$

$$W_Bt_B - W_Bt = W_At - W_At_A,$$

$$W_Bt_B + W_At_A = W_At + W_Bt = t(W_A + W_B),$$

$$t = \frac{W_At_A + W_Bt_B}{W_A + W_B} \quad \text{.......... (2)}$$

Inserting the experimental values on the right hand side of this equation, the calculated value of t will be found. This value will be found to be somewhat higher than that found for t experimentally, the difference representing the total correction which must be applied to the calculated value in order to obtain the true one.

Setting aside errors in measuring the temperature accurately, and the possible error due to some of the hot water being left in its original vessel, the principal source of error lies in the fact that the vessel in which the mixture is effected has its temperature raised as well as the water which it contains. To allow for

this, the water equivalent of the vessel should be calculated and added to the weight of cold water taken.

Let W = weight of vessel,

s = specific heat of its material.

Then, Ws = water equivalent of vessel.

Equation (1) above now becomes

$$W_B(t_B - t) = (W_A + Ws)(t - t_A) \quad \text{.......(3)}$$

giving

$$t = \frac{(W_A + Ws)t_A + W_B t_B}{W_A + Ws + W_B} \quad \text{.......(4)}$$

Try if the final temperature as calculated from equation (4) agrees more nearly with that found experimentally.

Vessels, such as that used in Expt. 46, in which the heat transference takes place, are called **calorimeters**. We may define a calorimeter as **an instrument used for measuring quantities of heat.**

EXPT. 47.—The specific heat of a solid may be determined by a simple method of mixtures. By means of a piece of thread suspend a piece of iron, copper, brass, or other metal in a beaker of boiling water. Keep the water boiling for several minutes. Have ready a copper calorimeter containing a quantity of water, and adjust the temperature of this water so that it is only slightly below that of the room. This may be effected by adding hot or cold water as required. When ready, note the temperature t_1 of the water in the calorimeter ; rapidly transfer the metal from the beaker to the calorimeter, taking as little boiling water as possible in doing so, and keep it moving about in the calorimeter. Note the highest temperature attained by the water in the calorimeter, calling this t_2. Remove the metal and weigh the calorimeter and water, deduct from this the weight of the calorimeter, the result being the weight of the water alone. Weigh the metal.

Let W = weight of metal.

W_w = „ „ water.

$W_c s_c$ = water equivalent of the calorimeter.

$$\left.\begin{array}{l} 212^\circ = \text{initial temp. of metal,} \\ t_1 = \text{„ „ „ water,} \\ t_2 = \text{final „ „ „ and metal.} \end{array}\right\} \text{F}^\circ$$

s = specific heat of metal under test.

Heat passing from metal $= W \times$ drop in temp. of metal $\times s$.
Heat entering calorimeter

$$= (W_w + W_c s_c) \times \text{rise in temp. of calorimeter.}$$

$$\therefore W(212 - t_2)s = (W_w + W_c s_c)(t_2 - t_1),$$

$$s = \frac{(W_w + W_c s_c)(t_2 - t_1)}{W(212 - t_2)}.$$

From which s may be found.

In carrying out Expt. 47, if the initial temperature of the water in the calorimeter be above that of the air in the room, heat will pass into the atmosphere from the calorimeter throughout the experiment. On the other hand, if the water temperature be initially considerably below that of the room, heat will pass from the atmosphere into the calorimeter throughout the test. In either case, a correction must be applied. The necessity for this correction disappears if the initial temperature of the water is as much below the temperature of the room as the final temperature is above it. With a little trouble in adjusting the initial temperature, these conditions may be approximately secured.

It may be noted here that heat cannot flow from a cold body into a body at higher temperature unless some special means of aiding it to do so are supplied. Equality in the temperatures of two bodies, A and B, implies that there is no resultant heat transference from one to the other going on, *i.e.* if any heat flows from A into B it is immediately balanced by an equal quantity of heat being transferred from B to A.

Specific heat of a liquid.—The specific heat of a given liquid may be found by the method described above for the specific heat of a solid, using a metal the specific heat of which is known, and having the liquid in the calorimeter instead of water.

Let W_1 = weight of metal.
W_2 = „ „ liquid.
$W_c s_c$ = water equivalent of the calorimeter.

$$\left.\begin{array}{l} 212^\circ = \text{initial temp. of metal,} \\ t_1 = \text{„ „ „ liquid,} \\ t_2 = \text{final „ „ „ and metal.} \end{array}\right\} \text{F}^\circ.$$

s_1 = specific heat of metal.
s_2 = „ „ „ liquid.

Then :

Heat entering calorimeter = heat passing from metal ;

$$\therefore\ (W_2s_2 + W_cs_c)(t_2 - t_1) = W_1(212 - t_2)s_1\ ;$$

$$\therefore\ s_2 = \frac{W_1s_1}{W_2}\left(\frac{212 - t_2}{t_2 - t_1}\right) - \frac{W_cs_c}{W_2}.$$

EXPT. 48.—Find the specific heat of the given liquid by the method described above.

The apparatus illustrated in Fig. 213 is used for experiments on rates of cooling of a liquid when its surroundings are kept as uniform as possible. The liquid is contained in a test tube A, fitted with a cork and a thermometer B. A wire stirrer C, having a loop at the bottom, enables the liquid to be stirred prior to its temperature being observed. D and E are metal cans, one placed inside the other and having ice packed between ; the test tube is suspended so as not to touch the inner can, and the temperature of the air space, which will remain fairly constant, may be observed by means of a thermometer F. The cans are closed at the top by means of a cover G.

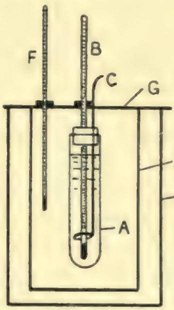


FIG. 213.—Apparatus for experiments on cooling.

EXPT. 49.—Put a measured volume of hot water in the test tube and complete the arrangement of the apparatus. Observe the temperatures of the water and of the air space at one minute intervals during 20 or 30 minutes. Plot a graph showing the temperature of the water as ordinates and time as abscissae. Show the temperature of the air space on the graph. Repeat the experiment, using the same volume of some liquid other than water. Plot a similar graph.

One of the graphs is sketched on Fig. 214, AB being the cooling curve of the liquid and CD the temperature line of the air space. Select equal intervals of time FH and HL. The fall in temperature of the liquid during the interval FH is EP, and during HL the fall is GQ. During the interval FH, the mean difference in temperature of the liquid and the air space is

$\frac{1}{2}(EM+GN)$, and, during the interval HL, the mean difference in temperature is $\frac{1}{2}(GN+KO)$. Evaluate the ratios of fall in temperature to mean temperature difference for both intervals, viz.

$$EP \div \tfrac{1}{2}(EM+GN)$$
$$GQ \div \tfrac{1}{2}(GN+KO).$$

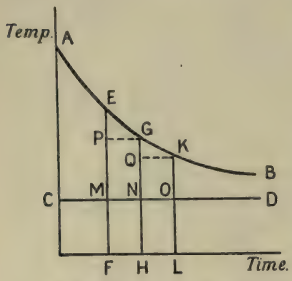


FIG. 214.—Cooling curve.

It will be found that these are practically equal, showing that the rate of cooling of the liquid is proportional at any instant to the difference in temperature between the liquid and its surroundings.

It may be inferred from this result that the quantity of heat passing from the liquid per unit of time is proportional to the temperature difference between the liquid and its surroundings, a law which is generally known as **Newton's law of cooling.**

Taking the two graphs for water and for the other liquid, obtain from them the time in each case to cool through the same range of temperature from t_1 to t_2. Let

T_1 = the time in minutes for the water to cool through the given temperature range,

T_2 = the time in minutes for the other liquid to cool through the given temperature range,

W_1 = the weight of the water,

W_2 = the weight of an equal volume of the liquid,

S = the specific heat of the liquid.

Then, from the above law,

$$\frac{\text{Heat lost by the liquid}}{\text{Heat lost by the water}} = \frac{W_2 S(t_1 - t_2)}{W_1(t_1 - t_2)} = \frac{T_2}{T_1}.$$

$$\therefore\ S = \frac{W_1 T_2}{W_2 T_1}.$$

The cooling experiment therefore enables the specific heat of the liquid to be obtained.

It will be noted that equal volumes of water and of the liquid are used in order that the area of the wetted interior of the test tube may be the same for both; this precaution, together

with the practical uniformity of the air-space temperature secured by the ice, ensures that the surrounding conditions are the same in both experiments.

If Experiment 49 be carried out using an ordinary copper calorimeter instead of a test tube to hold the liquid under test, it will be necessary to apply corrections for the heat abstracted from the metal of the calorimeter during cooling. If rough results only are required, the outer cans and ice-jacket may be omitted. If W_cS_c be the capacity of the calorimeter, we have

$$\frac{\text{Heat lost by the liquid}}{\text{Heat lost by the water}}=\frac{W_2S(t_1-t_2)+W_cS_c(t_1-t_2)}{W_1(t_1-t_2)+W_cS_c(t_1-t_2)}=\frac{T_2}{T_1},$$

or

$$\frac{W_2S+W_cS_c}{W_1+W_cS_c}=\frac{T_2}{T_1}.$$

$$\therefore\ S=\frac{W_1T_2+W_cS_c(T_2-T_1)}{W_2T_1}.$$

Expt. 50.—Procure a number of half-gallon tin cans without handles, and fitted with lids. Have a short piece of brass tube about $\frac{3}{4}''$ diam. soldered centrally to each lid; this will permit of a thermometer being inserted through a cork fitted to the tube. Leave one can with its surface bright; coat the whole of the outside of a second with lamp black; cover another, lid included, with cotton wool; another with hair felt; another with asbestos or any other boiler or pipe covering composition available. Set these prepared cans in a row on the bench and pour equal quantities of hot water into each through the tube, using a glass funnel and being careful that no water is spilled over the outside of the covering material. Insert thermometers and read their temperatures at intervals of 5 minutes. Tabulate thus:

Time.	Temperatures of water in				
	Can *A* bright tin.	Can *B* lamp black.	Can *C* cotton wool.	Can *D* hair felt.	Can *E* asbestos.

Plot these temperature readings and times on a single sheet of squared paper. The plotted curves will enable us to infer the rate at which heat passes from each can by conduction through the material and by radiation from the surface. A rough comparison of the value of each of the coverings used as a non-conductor of heat may now be made. Pay special attention to cans *A* and *B*. Notice that can *B* loses heat more rapidly than can *A*, from which we may infer that if we are compelled to use a bare metal surface for a vessel containing a substance, the temperature of which is to be preserved as nearly constant as possible, the surface should be brightly polished.

Heat is a form of energy.—At one time it was supposed that heat was a material substance, capable of being soaked in or squeezed out, as it were, by a body. Rumford showed by his

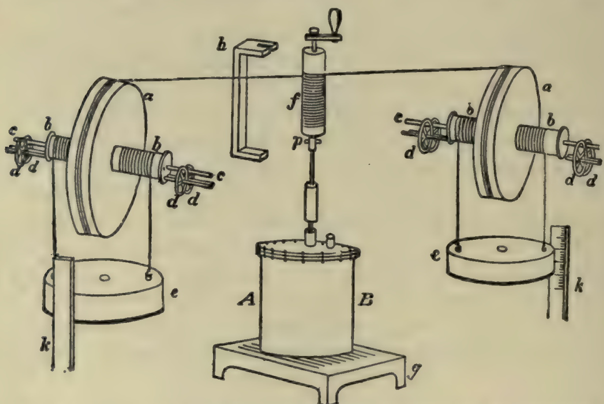


FIG. 215.—Apparatus used by Joule in his experiments on the mechanical equivalent of heat.

cannon-boring experiment, in which a blunt boring tool was used, that sufficient heat was evolved to boil a large quantity of water, while only a very small quantity of material was removed by the tool. He concluded it was impossible that the large quantity of heat given to the water could have been contained by, and squeezed out from, the small amount of material removed, and that therefore heat must be a form of motion.

Davy further confirmed this view by rubbing two blocks of ice together, taking precautions to ensure that no heat could be communicated to them from outside sources. He found that in a short time the ice was melted. Apparently an unlimited quantity of heat can be produced by the simple process of rubbing two bodies together, and therefore it is impossible that heat can be a material substance contained by the rubbing bodies.

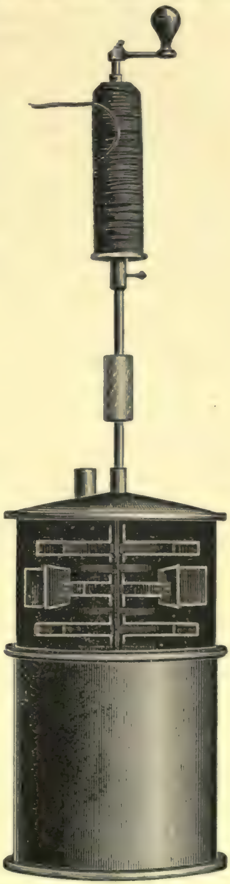


FIG. 216.—Joule's calorimeter.

Nature of heat.—It is now believed that heat is really the energy of the molecules of which any body is constructed. The molecules in a solid do not move about inside the body, that is, do not alter their relative positions, but are in a state of vibration. Heat imparted to a solid increases the molecular vibrations and so increases the energy of the molecules.

In a liquid, the molecules are not only in vibration, but may also move relatively to one another with comparative freedom. Heat imparted to a liquid may increase the energy of vibration and at the same time produce currents of molecules from one part of the liquid to another.

In substances in the gaseous state, the molecules are in rapid motion, continually colliding with one another and with the walls of the containing vessel. This continual bombardment produces pressure on the walls. Heat imparted to the gas increases the speed of the molecules, thereby increasing their kinetic energy and the pressure on the walls of the vessel.

Joules's mechanical equivalent.—Joule investigated the question of how much mechanical work must be done for the production of a given quantity of heat. In his experiments, falling weights *ee* (Fig. 215) were used to drive a paddle revolving inside a vessel *AB* fitted with baffle plates and containing water. The vessel is shown separately in Fig. 216. The work done by gravity on the falling weights was thus converted into heat by stirring the water against the resistance offered by the baffle plates. From the known weights and the height fallen, the mechanical work done was estimated, due allowance being made for mechanical losses. The rise in temperature of the measured quantity of water in the vessel enabled the heat produced to be calculated, corrections for wasted heat being applied. The result of these very careful experiments was that 772 foot-pounds of mechanical energy disappear in the production of one British Thermal Unit.

Later experiments by Rowland, Osborne Reynolds and Griffiths give 774 and 778 as more correct numbers. As the difference between 772 and 778 does not amount to 1 per cent., it matters little in ordinary calculations which of these numbers is used, but considerable confusion has occurred through different numbers having been used in calculating quantities required for insertion in tables giving the properties of steam.

EXERCISES ON CHAPTER XIII.

1. Distinguish clearly between heat and temperature.

2. Express 42·4 B.T.U. in lb.-degree-Cent. units. Express the same quantity of heat in gram-calorie units.

3. Define "specific heat of a substance." A copper vessel weighs $4\frac{1}{2}$ lbs. Calculate the quantity of heat required to raise its temperature 80° F. Take the specific heat of copper to be 0·092.

4. Explain the different ways in which heat may be transferred. Give examples.

5. 5 gallons of water at a temperature of 180° F. are poured into a tank containing 30 gallons of water at 60° F. Calculate the final temperature of the water on the assumption that no heat is lost.

6. A piece of copper weighing 2 lbs. is brought to a temperature of 212° F. and then dropped into a vessel containing 3 lbs. of water

at 55° F. Neglecting any sources of loss, what will be the final temperature?

7. Answer Exercise 6 on the supposition that the water equivalent of the vessel is 0·5 lb.

8. Give a brief explanation of our reasons for believing that heat is a form of energy.

9. Taking Joule's mechanical equivalent of heat to be 778 ft.-lbs., calculate the mechanical work equivalent to the heat which must be imparted to a pound of water in order to raise its temperature from freezing point to boiling point.

10. A piece of iron weighing 100 grams is allowed to remain in a current of hot gas for a few minutes, and is then dropped into a calorimeter containing 500 c.c. of water at 15° C. The water equivalent of the calorimeter is 40 grams. The final steady temperature was observed to be 22°·5 C. Calculate the temperature of the gas. Take the specific heat of iron to be 0·1098.

11. One pound of coal when completely burned can give out 15,000 B.T.U.; one pound of petroleum, 20,500 B.T.U.; one cubic foot of lighting gas, 600 B.T.U. Express these quantities of heat in foot-lbs. Take $J = 778$ ft.-lbs.

12. The metal of a certain boiler weighs 13 tons and the boiler contains 11 tons of water. Calculate the quantity of heat in B.T.U. which must be supplied in order to raise the temperature of the metal and the water from 60° F. to 340° F. Take the specific heat of the metal to be 0·1098.

13. Describe with sketches the ordinary method of supplying hot water to lavatory taps in a house.

14. Show how a large room may be heated by the circulation of hot water in pipes. Give sketches.

15. Explain how to find the specifie heat of a given liquid by the method of cooling.

16. 15 grams of water contained in a copper calorimeter weighing 22 grams are found to cool from 80° F. to 70° F. in 3 minutes. An equal volume of another liquid weighing 14 grams cools from 80° F. to 70° F. in the same calorimeter in 110 seconds. The specific heat of copper is 0·09. Find the specific heat of the liquid.

CHAPTER XIV.

PROPERTIES OF GASES.

The gaseous state.—A substance in the gaseous state possesses the property of **indefinite expansion.** A small quantity of gas introduced into a perfectly empty vessel will at once expand and occupy the whole of the interior. Gases may exist either as **vapours**, or as so-called **perfect gases.** The perfect gas was supposed to exist under all conditions of pressure and temperature as a gas, but it is now well known that all gases can be liquefied by great pressure and cold. A vapour may be defined as a gas near its liquefying point, and a perfect gas as the same substance far removed from its liquefying point. Gases, such as oxygen, hydrogen, nitrogen, and atmospheric air (which is a mixture of oxygen and nitrogen) behave as perfect gases under ordinary atmospheric conditions of pressure and temperature. Steam as it comes from boiling water is a vapour, but, if heated to a high temperature after leaving the water, it behaves more like a perfect gas. If the temperature of any gas be raised, keeping the pressure constant, the volume will be increased; and, if the volume be kept constant, the pressure will be increased as the temperature is raised.

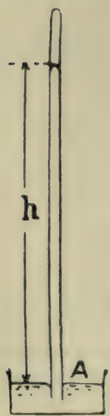


FIG. 217.—Apparatus for showing the principle of the barometer.

Pressure of the atmosphere.—The pressure of the atmosphere will be made evident and may be measured roughly by the following experiment :

EXPT. 51.—Take a glass tube closed at one end and open at the other, about 36″ long, and fill it with mercury. Close the open end by means of a finger, and invert the tube several times so as to collect into one bubble any air contained in the tube. Let this air escape and add mercury so as to fill the tube to the top. Close the end again with a finger and invert the tube, placing the open end in a cup of mercury. On removing the finger, the tube being held vertically, the level of the mercury inside the tube will fall until it stands at a height h inches above that in the cup (Fig. 217).

FIG. 218.—Fortin's barometer.

At A, the pressure inside the tube is that due to a column of mercury h inches high, and is equal to $(w \times h)$ lbs. per square inch, w being the weight of a cubic inch of mercury. The pressure of the atmosphere on the surface of the mercury in the cup will be equal to this. The average height of the mercury column is 30 inches, and as mercury weighs nearly 0·49 pound per cubic inch, this represents a pressure of 0·49 multiplied by 30, or 14·7 pounds per square inch.

The apparatus constitutes the **common barometer.** It is useful to remember that every inch of mercury height in a barometer corresponds nearly to a pressure of $\frac{1}{2}$ lb. per square inch.

EXPT. 52.—Measure in your rough barometer, the height of the mercury column in inches and calculate from this the atmospheric pressure in lbs. per square inch. Compare also the height measured with that shown by a standard barometer at the same time.

Other forms of barometer.—A standard barometer is much more elaborately fitted than the rough one of Expt. 51. In Fig. 218, **Fortin's barometer** is illustrated. This instrument has a screw A fitted to the mercury cup by means

of which the level of the mercury in the cup *R* may be brought to coincide with a point *P*. The upper part of the case is furnished with a scale and a sliding vernier, operated by means of a thumb screw *B*. Mirrors are fitted to the back board behind the cup *R* and the scale *S*. To read this instrument, first adjust the level of the mercury in the cup *R*, using the screw *A*. Bring the eye to the level of the top of the mercury column (the mirror aids this) and operate the screw *B* until the top of the vernier

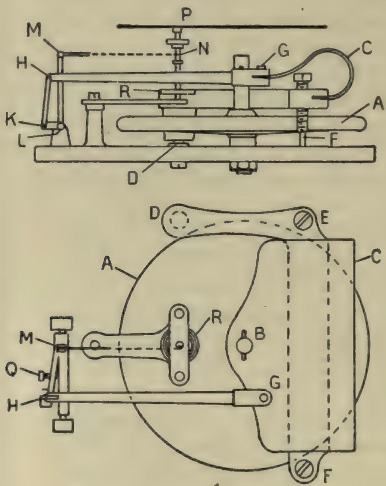


FIG. 219.—Aneroid barometer.

coincides with the top of the mercury column. Readings of the scale and vernier are then taken.

In the **aneroid barometer** (Fig. 219) the action depends on the movements of the flexible top and bottom of a circular box *A* under the changes of atmospheric pressure. *A* is securely fixed to the base-plate of the instrument, and is exhausted of air as thoroughly as possible; its top side is connected at *B* to a powerful spring *C*. *C* is held in a bracket which is secured to the base-plate at *D*, and has two bearing screws at *E* and *F*. A rod *GH*, fixed to the spring at *G*, communicates any rise or

fall of the spring through the lever system HK, KL, LM, to a fine chain MN, which is wrapped round the spindle carrying a pointer P. P moves over a graduated dial divided to show inches or centimetres of barometric height, corresponding to the readings of a mercury barometer. A hair-spring at R keeps the light chain tight. Q is an adjusting screw whereby the effective length of the lever arm KL may be altered. E and F also serve as adjusting screws. Aneroid barometers are liable to changes by reason of alterations in the elastic qualities of the metal of which the box A is made. It is necessary to check them at frequent intervals by comparison with a mercury barometer.

Measurement of pressure of a gas.—Engineers in this country usually measure gaseous pressures in pounds per square inch ; or, if the pressure be very high, in atmospheres, one atmosphere being a gaseous pressure of 14·7 lbs. per square inch. For low pressures of steam, such as are found in the condenser of a steam engine, the pressure is usually stated in inches height of mercury column, as this facilitates comparison with the barometric pressure. The metric unit of gaseous pressure is usually one kilogram per square centimetre. Taking 1 square inch $=6{\cdot}45$ square cms., and 1 kilogram $=2{\cdot}205$ lbs., a pressure of 1 kilogram per square centimetre will be equivalent to $(6{\cdot}45 \times 2{\cdot}205)=14{\cdot}22$ lbs. per square inch. For rough conversion from the metric to the British system, it is convenient to remember that a pressure of one kilogram per square centimetre is approximately equal to a pressure of one atmosphere (*i.e.* 14·7 lbs. per square inch).

Chimney draught, which is the difference in the gaseous pressures inside and outside at the base of a chimney, is usually measured in inches of water. A column of water 144 feet high and 1 square inch in section (*i.e.* one cubic foot in volume) gives a pressure at its base of 62·4 lbs. Hence a column 2·3 feet high gives 1 lb. per square inch pressure. 1 inch water pressure $=$ 0·036 lb. per square inch $=5{\cdot}2$ lbs. per square foot.

The type of gauge in common use is shown in Fig. 220, and consists of a bent glass U tube A containing some water. This tube is connected to an iron pipe BC, which passes into the interior of the chimney. In action, the superior pressure of the atmosphere

causes a change in the surface levels of the tube as shown. The difference in level, h inches, is the chimney draught, and is observed by means of a scale of inches attached to the U tube.

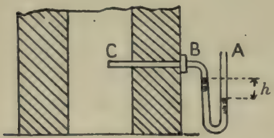


FIG. 220.—Chimney draught gauge.

There are two zeros of gaseous pressure from which other pressures may be measured; these are:

(*a*) atmospheric pressure, other pressures being stated as so much above or below this;

(*b*) perfect vacuum, that is the condition of pressure which exists in a space perfectly empty of gas, which, of course, will be devoid of all gaseous pressure.

Pressures stated above perfect vacuum are said to be **absolute pressures**; those stated from atmospheric pressure are said to be **gauge** or **bursting pressures.** The meaning of the last term will be rendered evident by considering a vessel containing gas under a pressure of say 100 lbs. per square inch above atmospheric pressure. The interior walls of the vessel will actually be subjected to a pressure of $(100+14{\cdot}7)=114{\cdot}7$ lbs. per square inch (absolute pressure), which pressure, tending to force the walls of the vessel outwards, is partially counteracted by the external pressure of the atmosphere, 14·7 lbs. per square inch, tending to crush or collapse the vessel. The net pressure tending to burst the vessel will be the difference, *i.e.* 100 lbs. per square inch, which is therefore called the bursting pressure. The name gauge pressure given to the same pressure is due to the fact that gauges used for indicating the gaseous pressure in a closed vessel show, not the absolute pressure, but the difference between the absolute pressure inside the vessel and the atmospheric pressure outside.

Pressure gauges for steam boilers are usually of the **Bourdon type.** The action depends on the tendency which a curved, partially flattened tube has to become straight when subjected to internal pressure.

EXPT. 53.—This effect can be demonstrated easily by attaching a piece of rubber tube about a yard long to a water tap, closing its outer end by a clip, and then bending the tube into a curve lying on the table. On opening the water tap, the rubber tube,

which has been slightly flattened by the bending, will distinctly show movement in the attempt to straighten itself.

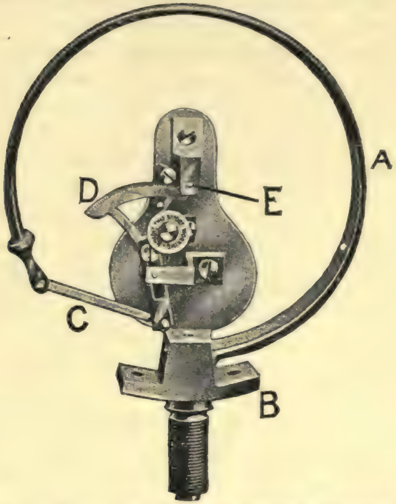


FIG. 221.—Interior parts of a Bourdon steam pressure gauge.

In Fig. 221 is shown the interior parts of a steam gauge constructed by Messrs. Hopkinson; Fig. 222 illustrates the exterior. Referring to Fig. 221, *A* is a flattened tube of hard, solid-drawn phosphor bronze, secured to a bracket *B* which has passages in it forming the steam inlet to the tube. The free end of the tube is closed, and is connected by means of a short link *C* to a small toothed sector *D*. The sector gears with a pinion on the spindle *E* carrying the outside pointer. The whole of the mechanism is carried by the bracket *B*. Injury

FIG. 222.—Bourdon steam pressure gauge.

to the mechanism due to straining of the outer case is thus avoided.

Boyle's Law for perfect gases.—The experiments of Boyle and others, on the connection between pressure and volume of gases, show that **the absolute pressure varies inversely as the volume, provided the temperature remains unaltered.** Taking a given mass of gas under conditions of pressure and volume p_1 and v_1, let these be changed, without alteration of temperature, to any other conditions p_2 and v_2, then

$$p_1 : p_2 = v_2 : v_1,$$

or, $$p_1 v_1 = p_2 v_2.$$

Since any other conditions of pressure and volume may be taken, we may write the law as

$$pv = \text{a constant.}$$

This law has been proved experimentally to be followed closely by such gases as air, hydrogen, oxygen, nitrogen and others when not very far removed from ordinary atmospheric conditions of pressure and temperature, but it is not followed by steam and other vapours which are near to their liquefying points.

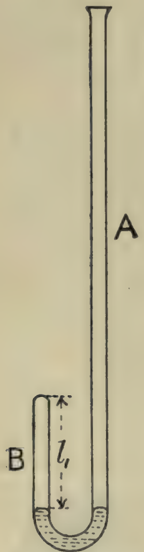


FIG. 223.—Tube used in verifying Boyle's Law.

FIG. 224.

A perfect gas is sometimes defined as one which closely obeys Boyle's law.

EXPT. 54.—To verify Boyle's law roughly, the apparatus shown in Fig. 223 may be used. A vertical glass tube is bent as shown, the long limb A being left open and the short limb B closed. Mercury is introduced and adjusted so that it stands at the same level in both limbs. The air enclosed at B will then be at the atmospheric pressure shown by a barometer. Read the barometer and let its height be h_1 inches of mercury. The volume of the enclosed air may be stated with sufficient

accuracy by measuring the length of tube occupied by air and taking this length to represent the volume. Let this be l_1. On pouring more mercury into A, the level in the long limb will be found to be higher than that in B (Fig. 224). Let h_2 be the difference in levels. Then the pressure of the enclosed air will be (h_1+h_2), and its volume will be represented by l_2. Repeat the experiment several times, in each case waiting a minute or so after adding more mercury in order to allow the compressed air to cool to the temperature of the room, and tabulate thus:

Pressure, inches of mercury.	Volume, length of tube B occupied by air.	Product of pressure and volume.

It will be found that the products in the last column are very nearly equal to one another and to the first product h_1l_1. Plot columns 1 and 2, thus drawing a curve to illustrate Boyle's law.

Mathematicians call this curve a **rectangular hyperbola**. As it is of considerable importance, two methods of finding points in the curve from given particulars will now be explained.

Example. 2 cubic feet of air at an absolute pressure of 100 lbs. per square inch are expanded at uniform temperature until the volume occupied is 8 cubic feet. Plot a curve showing the expansion.

Method 1. From Boyle's law,

$$p_1v_1=p_2v_2=p_3v_3=\text{etc.} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Take volumes differing by 1 cubic foot, and **calculate** corresponding pressures, using equations (1) which will take the form

$$p_2=\frac{p_1v_1}{v_2}\,; \quad p_3=\frac{p_1v_1}{v_3}, \text{ etc.}$$

Arrange the results in a table, thus:

Volumes, cubic feet.	Pressures, lbs. per sq. inch.	Volumes, cubic feet.	Pressures, lbs. per sq. inch.
2	100	6	33·3
3	66·6	7	28·6
4	50	8	25
5	40		

Plot these as shown in Fig. 225.

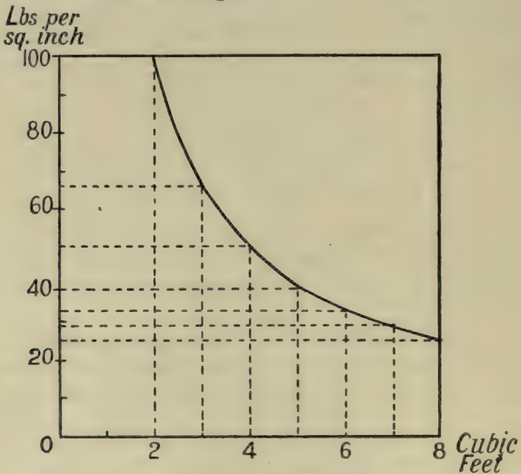


Fig. 225.—Curve illustrating Boyle's Law.

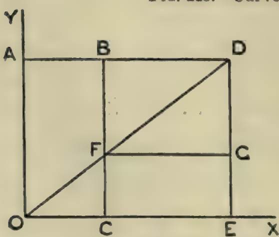


Fig. 226.

Method 2. Take two axes OX, OY (Fig. 226). Set off OA along OY to represent p_1, and OC along OX to represent v_1 to convenient scales of pressure and volume. Make $OE = v_2$. Complete the rectangles $OABC$ and $OADE$. Join OD cutting CB in F. Draw FG parallel to OX. Then EG

is equal to p_2, so that G will be a point on the expansion curve.

Proof. The triangles OCF and OED are similar, therefore

$$FC : OC = DE : OE,$$

or,

$$EG : OC = BC : OE.$$

But

$$OC = v_1, \quad BC = p_1, \quad \text{and} \quad OE = v_2.$$

$$\therefore \ EG : v_1 = p_1 : v_2,$$

or,

$$EG = \frac{p_1 v_1}{v_2},$$

and therefore, by Boyle's law, $EG = p_2$.

Other points may be found in a similar fashion. Fig. 227 shows the complete curve obtained by the use of this constructional method.

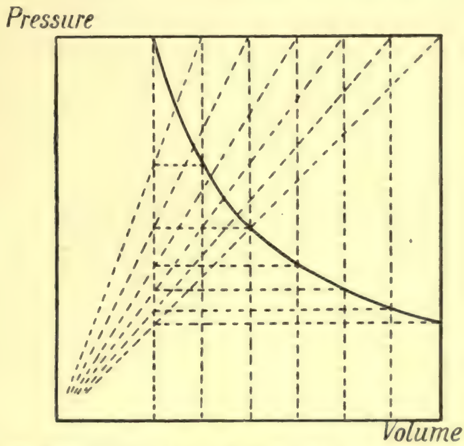


FIG. 227.—Boyle's Law curve drawn by means of a geometrical construction.

EXAMPLE. In a mercury barometer, the top of the tube is at a height of 33 inches above the surface of the mercury in the trough and the barometer reads 30 inches. Some air is admitted to the space above the mercury, and in consequence of its pressure the mercury falls to 28 inches height. Suppose the barometer, now containing air, were to read 27 inches on another occasion when the temperature is the same, what would be the true reading?

Referring to Fig. 228 (a), showing the conditions when the air is first admitted, it will be clear that the pressure p_1 of the enclosed air must be equivalent to a head of 2 inches of mercury; the volume of the air, v_1, assuming uniformity of bore of the tube, will be proportional to the length of 5 inches which it occupies. In Fig. 228 (b), showing the final conditions, the pressure of the enclosed air is p_2 in inches of mercury, and the volume v_2 is proportional to 6 inches length of the tube. As there is no change of temperature, Boyle's law may be applied to the enclosed air, giving

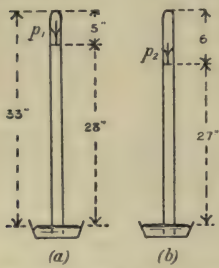


FIG. 228.—Defective barometer.

$$p_1v_1=p_2v_2,$$

$$p_2=\frac{p_1v_1}{v_2},$$

$$=\frac{2\times 5}{6}=1{\cdot}667 \text{ inches of mercury.}$$

Adding this to the observed height of 27 inches in Fig. 228 (b), we obtain:

$$\text{True reading}=27+1{\cdot}667$$
$$=28{\cdot}667 \text{ inches of mercury.}$$

Charles's Law.—This law, first enunciated by Charles and Gay Lussac, states that **all perfect gases expand by the same fraction of the volume they occupy at freezing temperature (0° C. or 32° F.) when their temperature is raised one degree, provided the pressure remains unaltered.**

Using the Centigrade scale of temperature, the fraction has been found experimentally to be $\frac{1}{273{\cdot}7}$, or $\left(\frac{1}{273{\cdot}7}\times\frac{5}{9}\right)=\frac{1}{493}$ if the Fahrenheit scale be employed.

Let V_0 = volume of a given mass of gas at freezing temperature;
V = its volume at any other temperature t, the pressure being unaltered.

Then,

$$V=V_0+\frac{1}{273{\cdot}7}V_0t, \text{ where } t \text{ is the Centigrade temperature,}$$

or, $V=V_0+\frac{1}{493}V_0(t-32)$, where t is the Fahrenheit temperature.

These equations are not convenient for calculation, as usually

the volume stated is given at some other temperature than freezing. To use the above equations, the volume occupied at freezing temperature would have to be calculated first, before the volume V occupied at some stated temperature t could be found.

Absolute scale of temperature.—By use of the absolute scale of temperature, calculations on the changes of volume and temperature of a gas become much simpler. Suppose we take 493 cubic feet of a gas at freezing temperature, 32° F., and, at constant pressure, raise the temperature to 33° F. The volume will become

$$\left\{493+\left(\frac{1}{493}\times 493\right)\right\}=494 \text{ cubic feet.}$$

Raising the temperature to 34° F. will give a volume of

$$\left\{493+\left(\frac{2}{493}\times 493\right)\right\}=495 \text{ cubic feet.}$$

At 35° F. the volume would become 496 cubic feet and so on, each degree F. of increase in the temperature producing an increase of 1 cubic foot in the volume. Regraduate the Fahrenheit thermometer by marking freezing point 493° instead of 32° and boiling point 673° instead of 212°, and it is easy to see that now the volumes occupied by the gas will be proportional to the temperatures on the regraduated scale. Such a scale is called an **absolute scale of temperature.** Had a Centigrade thermometer been employed, we should have placed 273·7° at freezing point and 373·7° at boiling point. These scales are referred to as the **absolute temperature Fahrenheit**, and the **absolute temperature Centigrade** respectively.

We shall use the letter t for ordinary temperature, Fahrenheit or Centigrade, and the Greek letter τ for absolute temperatures. Zero on the ordinary Fahrenheit scale is marked $(493-32)=461°$ on the absolute scale.

To convert from the ordinary scales to the absolute scales we have:

$$\tau=461+t° \text{ F. for the F. scale.}$$
$$\tau=273{\cdot}7+t° \text{ C. for the C. scale.}$$

Charles's law may now be stated thus:

The volumes of all perfect gases are proportional to the absolute temperatures at which they are measured, provided the pressure remains unaltered.

Let V_1 = volume of a given mass of gas at absolute temperature τ_1.

V_2 = volume of the same mass at absolute temperature τ_2, and at the same pressure.

Then $$V_1 : V_2 = \tau_1 : \tau_2,$$

or, $$V_1\tau_2 = V_2\tau_1,$$

or, $$\frac{V_1}{\tau_1} = \frac{V_2}{\tau_2}.$$

This equation is suitable for use in calculations.

Example i. One pound weight of air at 0° C. and 14·7 lbs. per square inch pressure occupies a volume of 12·4 cubic feet. Find the volume of the same weight of air when the temperature is 60° F., the pressure being unaltered.

This question may be worked using the absolute F. scale.

$$V_1 = 12{\cdot}4 \text{ cubic feet.}$$

$$\tau_1 = 32 + 461 = 493° \text{ F. abs.}$$

$$\tau_2 = 60 + 461 = 521° \text{ F. abs.}$$

$$\frac{V_1}{\tau_1} = \frac{V_2}{\tau_2}.$$

$$\therefore\ V_2 = \frac{V_1\tau_2}{\tau_1}$$

$$= \frac{12{\cdot}4 \times 521}{493}$$

$$= \underline{13{\cdot}1} \text{ cubic feet.}$$

It is convenient to remember that the volume of one pound weight of air under ordinary conditions of atmospheric pressure and temperature is 13 cubic feet nearly.

Example ii. In a certain boiler test it was found that 300 cubic feet of air at a pressure of one atmosphere and 60° F. temperature entered the furnace per lb. of coal burned. What will be the weight of air admitted per lb. of coal?

Using the approximate number found in Example i. :

$$\left.\begin{array}{l}\text{Weight of air}\\ \text{per lb. of coal}\end{array}\right\} = \frac{300}{13} = \underline{23 \text{ lbs.}} \text{ nearly.}$$

Example iii. A room measures 50 feet × 30 feet × 25 feet. If the

air in it be heated from 40° F. to 60° F. what percentage of the contained air will be expelled?

$$V_1 = 50 \times 30 \times 25 = 37{,}500 \text{ cubic feet.}$$
$$\tau_1 = 461 + 40 \quad = 501° \text{ F. abs.}$$
$$\tau_2 = 461 + 60 \quad = 521° \text{ F. abs.}$$

Volume of the original air in the room if heated to 60° F. $= V_2 = \dfrac{V_1\tau_2}{\tau_1}$

$$= \frac{37{,}500 \times 521}{501}$$
$$= \underline{38{,}997} \text{ cubic feet.}$$

Volume of air expelled $= 38{,}997 - 37{,}500$

$$= \underline{1497} \text{ cubic feet.}$$

Notice that the volume of air expelled is stated at a temperature of 60° F.

$$\text{Percentage expelled} = \frac{1497}{38{,}997} \times 100$$
$$= \underline{3{\cdot}8.}$$

Combination of Boyle's and Charles's Laws.—It has now been seen that p is inversely proportional to v when t is constant, and that v is directly proportional to τ when p is constant. A law must now be found which applies when all three conditions, p, v, τ, vary simultaneously. Such a law might be written down at once from the algebraic rules of variation, but we proceed to find it in the following manner.

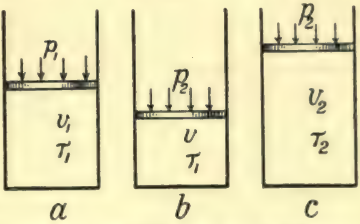


Fig. 229.—Diagram showing changes occurring in the pressure, volume, and temperature of a gas.

Let a mass of gas be enclosed in a cylinder, Fig. 229 (a), under given conditions p_1, v_1, τ_1. Suppose first that p_1 and v_1 are changed at constant temperature τ_1 until a pressure of p_2 and a volume v are obtained.

Applying Boyle's law, $\quad p_1v_1 = p_2v$,

$$v = \frac{p_1v_1}{p_2}. \quad \text{..............................(1)}$$

The gas will now be shown as (b) in Fig. 229.

Now change the temperature from τ_1 to τ_2 keeping the pressure constant at p_2. The volume will change from v to v_2 as shown at (c) in Fig. 229.

Applying Charles's law, $\frac{v}{\tau_1}=\frac{v_2}{\tau_2}$;

$$\therefore\ v=\frac{v_2\tau_1}{\tau_2}. \qquad (2)$$

The results obtained in (1) and (2) being now equated, we obtain

$$\frac{p_1v_1}{p_2}=\frac{v_2\tau_1}{\tau_2},$$

or,

$$\frac{p_1v_1}{\tau_1}=\frac{p_2v_2}{\tau_2}. \qquad (3)$$

Evidently had the conditions been varied simultaneously instead of in the step by step manner adopted above, we should have obtained the same result.

Writing it in the proportional form

$$p_1v_1 : p_2v_2=\tau_1 : \tau_2, \qquad (4)$$

which may be stated thus: When any operation is performed on a given mass of gas involving changes in the pressure, volume and temperature, **the product of the absolute pressure and the volume is proportional to the absolute temperature**.

Isothermal and adiabatic expansion.—A gas expanding or being compressed at constant temperature, is said to do so **isothermally**, that is, at equal or constant temperature. Any operation conducted at constant temperature is said to be an **isothermal operation.** A curve such as is plotted in Fig. 225 in illustration of Boyle's law (where the temperature is kept constant) is called an **isothermal curve.**

A gas expanding or being compressed in such a manner that no heat is allowed to enter it or escape from it while the operation is being conducted is said to be undergoing **adiabatic expansion** or **adiabatic compression.** Such an operation may be imagined if conducted in a cylinder, the walls and piston of which have no capacity for heat, *i.e.* are unable to absorb any heat, and are also perfect non-conductors. Needless to say, such a cylinder cannot be constructed, and consequently such expansion cannot be realised in a cylinder. A near approach to

adiabatic expansion is obtained when steam is allowed to flow freely through a nozzle, expanding as it does so.

An isothermal operation may be imagined in the following manner. Suppose we have a mass of air inclosed in a cylinder the walls of which are good conductors. Maintain the temperature of the room constant throughout, and allow the piston to move out very slowly, taking, say a day or a week, to perform a stroke of one foot length. There is thus plenty of time for heat to flow through the cylinder walls and so maintain the temperature of the enclosed air constantly at that of the air in the room.

Fig. 230 shows a cylinder in which a gas may be compressed by pushing in a piston B by exertion of a force P, or the gas may be expanded by permitting the piston to move outwards. The curve CD shows the relation of pressure and volume. Supposing that compression isothermally is carried out by having the walls of the cylinder suitably constructed, then, as the piston moves inwards, work is being done by the force P against the resistance offered by the gaseous pressure; this work is converted into heat which would produce a rise in temperature of the gas if it were not allowed to flow freely through the walls of the cylinder. Conversely, if isothermal expansion is going on, then work is being done by the gaseous pressure against the resistance offered by P, and the heat necessary to perform this work must come from the store of heat in the gas unless provision is made for the required quantity of heat to flow through the walls into the cylinder.

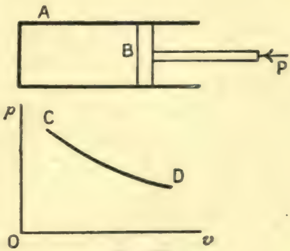


FIG. 230.

If P were removed and if the expansion were performed without any resistance being offered to the free movement of the piston, no work would be done; hence none of the heat energy in the gas would be transformed into work and the temperature of the expanding gas would remain constant, *i.e.* the expansion would again be isothermal. Hence we infer that,

if a gas is expanding isothermally and doing work against an external resistance, heat must be supplied in amount sufficient to be equivalent to the external work done. Thus:

Let W = external work done in foot-lbs.

J = Joule's equivalent (778 foot-lbs.).

Then,

$$\text{Heat which must be supplied} = \frac{W}{J} \; B.T.U.$$

In the same way, if isothermal compression is being conducted, heat must be abstracted to an amount equivalent to the work done by the force applied to the piston in pushing it home, *i.e.*

$$\text{Heat to be abstracted} = \frac{W}{J} \; B.T.U.$$

Suppose now that adiabatic compression is being performed in the same cylinder; in this kind of compression no heat is allowed to escape, hence the work done by P, converted into heat, produces the effect of raising the temperature of the gas. Conversely, if adiabatic expansion is being carried out, as no heat is allowed to enter the cylinder in order to compensate for that converted into work done against the resistance offered by P, the temperature of the gas must fall. The heating of a gas during approximately adiabatic compression may be noticed by working rapidly an ordinary bicycle pump. After several strokes it will be found that the discharge end of the pump has become hot.

Relation of pressure and temperature in a gas.—Taking equation (3) above (p. 222),

$$\frac{p_1 v_1}{\tau_1} = \frac{p_2 v_2}{\tau_2}$$

and put $v_1 = v_2$, thus causing the pressure and temperature to change at constant volume. This will give the law

$$\frac{p_1}{\tau_1} = \frac{p_2}{\tau_2},$$

or, $$p_1 : p_2 = \tau_1 : \tau_2.$$

That is, **the pressure of a perfect gas is directly proportional to the absolute temperature, the volume being kept constant.**

EXPT. 55.—Arrange apparatus as shown in Fig. 231. A is a large bulb connected by a rubber tube to an open top cistern of mercury at B. Air may be introduced to the bulb through a tap at D. The bulb is immersed in water contained in a vessel E; the water may be heated by a steam coil F and its temperature measured by means of a thermometer G. The mercury cistern may be moved along a vertical scale H, the zero of which is at the same level as a mark C on the neck of the bulb. Open the tap at D and lower the cistern until the mercury level is at C and also at zero on the scale; a little mercury may be added or abstracted in order to obtain this result. Close the tap at D; there will now be imprisoned in the bulb a definite volume of air, and its pressure p_1 will be equal to that of the atmosphere. Read the barometer in order to obtain the atmospheric pressure. Take the temperature of the water in the bath and express it as τ_1 on the absolute scale. Warm the water by blowing steam through the coil F; shut off the steam, and allow a minute or two for the temperature conditions to settle down. It will be noticed that the level of the mercury has fallen below C, indicating that the pressure of the air in the bulb has increased; restore the level at C by raising the mercury cistern. Read the temperature τ_2, and also the scale reading on H, the latter giving the height of mercury column between C and B, and hence the superatmospheric pressure on the inclosed air. Obtain the absolute pressure p_2 by adding the observed scale reading to the barometric pressure p_1.

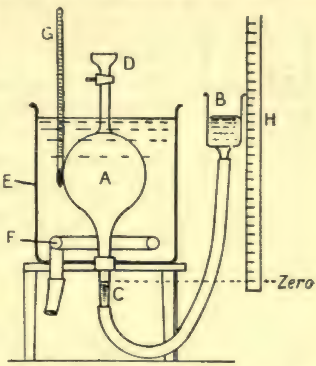


FIG. 231.—Apparatus for showing the relation of pressure and temperature of a gas.

In this manner obtain a number of pressures and corresponding temperatures. Plot these on squared paper. If a straight line results, the graph may be taken as a proof of the

law of proportionality of absolute pressures and absolute temperatures of a gas at constant volume.

Specific heats of a gas.—In Fig. 232 is shown a cylinder fitted with a piston which may be loaded to produce any constant pressure on the gas. Application of heat to the gas will produce two effects.

i. The gas will have its temperature raised.

ii. It will expand, driving the piston upwards, and thus do work against the resistance of the applied pressure.

Now allow the gas to return to its original conditions; then fix the piston, and apply heat so as to raise the temperature of the gas to the same extent as before. This latter operation being conducted at constant volume, and the piston not moving, no work is done against the applied pressure. In the first case, sufficient heat must be given to raise the temperature of the gas, and in addition an amount of heat equivalent to the external mechanical work done against the resistance. In the second case, only an amount of heat sufficient to raise the temperature of the gas need be imparted.

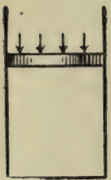


FIG. 232.

It follows therefore that **the specific heat of a gas at constant pressure is greater than the specific heat of the same gas at constant volume.** The specific heat of air at constant pressure is 0·238, and at constant volume is 0·169. Thus, to raise the temperature of 1 lb. weight of air through 1° F. requires 0·238 B.T.U. at constant pressure, and 0·169 B.T.U. at constant volume.

EXERCISES ON CHAP. XIV.

1. Distinguish between a perfect gas and a vapour; also between the gauge pressure and absolute pressure of a gas.

2. Explain briefly the principle of the mercury barometer. Give a sketch and description of a standard mercury barometer.

3. Give sketches and describe the action of an aneroid barometer.

4. Calculate the pressure in lbs. per square foot corresponding to a water gauge reading of 15 inches.

5. Sketch and describe the action of a Bourdon pressure gauge.

6. State Boyle's law and explain how you would verify it experimentally. Under what conditions is Boyle's law followed by a gas?

7. A machine for compressing air takes in 4 cubic feet from the atmosphere at a pressure of 14·7 lbs. per square inch absolute, and compresses it to a pressure of 100 lbs. per square inch as shown by the gauge. Assuming the temperature to remain constant, calculate the final volume. Draw a curve showing the changes of pressure and volume.

8. The mercury column in a barometer has a sectional area of 0·5 square inch. The barometer reads 30 inches and the length of empty tube is 4 inches. Some air is admitted and the barometer falls to 26 inches. Calculate the volume of air admitted at atmospheric pressure.

9. Suppose the barometer containing air in Question 8 to read 24 inches on another occasion when the temperature is the same, find the true reading.

10. State Charles's law. Explain the reason for using an absolute scale of temperature in applying this law.

11. A boiler chimney is 80 feet high and has internal dimensions 2 feet × 2 feet square. Suppose the temperature of the gases inside to be 500° F.; calculate what volume the contents of the chimney would occupy if the temperature were reduced to 60° F. without change in pressure.

12. A cylinder fitted with a piston contains at a certain instant 6 cubic feet of gas at 15 lbs. per square inch absolute and 15° C. Operations are being conducted on the gas involving changes in pressure, volume and temperature, the weight of the gas present being kept constant. At another instant the pressure is found to be 150 lbs. per square inch absolute and the volume 2·5 cubic feet. Calculate the temperature at this instant.

13. Distinguish between isothermal and adiabatic compression of a gas. Explain why adiabatic expansion cools a gas and *vice versa*.

14. Supposing 25,000 foot-lbs. of work to be done in compressing a gas at constant temperature, calculate what heat in B.T.U. must be abstracted during the operation.

15. Some gas contained in a closed vessel at an absolute pressure of 15 lbs. per square inch and at 60° F. is heated at constant volume to a temperature of 200° F. Find the absolute pressure.

16. Explain how you would show experimentally the truth of the principle involved in Question 15.

17. Explain the reason why the specific heat of a gas at constant volume differs from that of constant pressure.

CHAPTER XV.

PROPERTIES OF STEAM.

Water.—Water is a chemical compound of hydrogen and oxygen. This fact may be shown by mixing together two volumes of hydrogen gas with one volume of oxygen gas and exploding them. The gases do not unite until the mixture is ignited, when an explosion occurs; the hydrogen and oxygen unite, forming a compound which cannot be separated into its constituents by simple mechanical means. This compound is water vapour, *i.e.* **steam**, which on cooling condenses into ordinary water. As the experiment is dangerous, the student is recommended not to carry it out himself.

States of water.—Water may exist in the solid state as ice, in the liquid state as ordinary water, as a vapour—ordinary steam, and as a perfect gas if the vapour be raised to a moderately high temperature at constant pressure. Further elevation of the temperature causes the hydrogen and oxygen to dissociate, that is, the molecules of hydrogen and oxygen part company, remaining as a mixture of these two gases until the temperature falls somewhat, when they reunite to form steam.

To cause water to pass successively through the states in the order named, requires heat to be imparted; and, similarly, heat must be abstracted if changes in the reverse order are to be effected.

Sensible heat.—It has been seen already that to raise the temperature of one pound of water through 1° F. requires 1 B.T.U. Heat imparted to a substance which produces a rise of temperature is called **sensible heat.** The fact that sensible heat is entering a substance will, of course, be always rendered

evident by a thermometer. It is customary to reckon the sensible heat of water from 32° F. To raise the temperature of one pound of water from 32° F. to 212° F. requires 180·5 B.T.U., which number therefore gives the sensible heat of water at 212° F. Notice that the sensible heat of water at 212° F. would be $(212-32)=180$ B.T.U. if the specific heat of water were unity at all temperatures, but owing to slight variations in the specific heat the number stated is more accurate. We may assume for ordinary calculations that the specific heat of water does remain constant, in which case the sensible heat, h, required to raise the temperature of w lbs. of water from a temperature t_1° F. to a temperature t_2° F. will be found from

$$h=w(t_2-t_1) \text{ in B.T.U.}$$

If greater accuracy be required, the table giving the values of h (p. 365) may be consulted.

Latent heat.—In testing the fixed points of a thermometer, the student has noticed that the thermometer remains steady throughout the melting of ice and the boiling of water, in spite of the fact that heat is being imparted continually to these substances. The conclusion arrived at is that heat must be imparted to cause water to change its state from solid to liquid or from liquid to gaseous. **Heat imparted to a substance and producing a change of state without change of temperature is called Latent Heat.** If the change of state is from gaseous to liquid, or from liquid to solid, latent heat must be abstracted from the substance.

Expt. 56.—Weigh a copper calorimeter and pour in about ½ gallon of water at a temperature of about 100° F. Weigh again and find the exact weight W of the water by subtraction. Take a piece of ice weighing about ½ lb. Wipe any water from its surface, take the temperature t_1° F. of the water in the calorimeter and plunge in the ice. Keep stirring until the ice is melted and note the temperature t_2° F. at the instant the last piece of ice disappears. Weigh again and subtract from the total weight that of the calorimeter and of the water originally in it, obtaining w, the weight of ice which has been melted.

Assuming that the heat abstracted from the original water has been used altogether

(*a*) in changing the state of the ice from solid to liquid ;

(*b*) in elevating the temperature of the resulting water from freezing point to $t_2°$ F.;

we have, calling the latent heat of 1 lb. of water l, in B.T.U.

$$W(t_1 - t_2) = lw + w(t_2 - 32°) ;$$

$$\therefore \; l = \frac{W(t_1 - t_2) - w(t_2 - 32°)}{w}.$$

Careful experiments show that the latent heat of 1 lb. of water is 144 B.T.U. Compare this with your experimental result. Do they agree fairly closely if the water equivalent of the calorimeter be taken into account? If not, how do you explain the discrepancy?

EXPT. 57.—Arrange the apparatus as shown in Fig. 233. A is a copper vessel about 5″ diameter × 9″ high, serving as a boiler. Steam is taken from it by means of a glass tube B, about $\frac{1}{4}$″ bore, connected to the boiler by means of a cork. B is best made in two pieces connected at C by a piece of rubber tube about $1\frac{1}{2}$″ long. The steam is discharged into a calorimeter D containing a known weight of water, and is condensed there, its latent heat and part of its sensible heat being given up to the water.

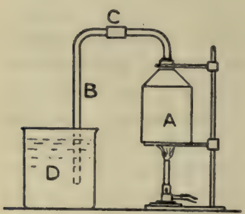


FIG. 233.—Apparatus for determining the latent heat of steam at atmospheric pressure.

To measure the latent heat of steam at a pressure equal to that of the atmosphere by means of this apparatus requires the following quantities to be known :

W = weight of water originally in the calorimeter.

w = weight of steam condensed.

t_1 = original temperature of the water W.

t_2 = final temperature of the mixture consisting of W and w.

The water equivalent of the calorimeter.

First make a preliminary experiment. Bring the water in A to boiling temperature ; and after a minute or two, when steam is being given off freely, take the temperature t_1 of the water in D, and then immerse the open end of the tube B in the water in D, noting the time as you do so. A crackling noise will be

heard proceeding from D, due to bubbles of steam issuing from B and instantly collapsing on coming into contact with the cold water. While the experiment is going on for 10 minutes or so you may observe the following points. Closely examine the glass tube, when you will observe a film of water covering its inner surface and travelling towards D to be discharged finally into the calorimeter. Notice also that if the boiler be strongly heated so that the ebullition is violent, a considerable quantity of water may enter the tube from the boiler. This effect will be magnified if there is too much water in the boiler, thus restricting the volume in the boiler available for steam space.

Now it is evident that any water of condensation entering the calorimeter has already given up its latent heat to some body other than the water in the calorimeter, but as it will still be hot water at a temperature not much below 212° F., it will give up sensible heat to the water in the calorimeter. It would be an advantage if only **dry steam**, *i.e.* steam containing no suspended water, entered the calorimeter, as then no correction for water carried over need be applied. We may approximate to this by using the following precautions:

(*a*) Violent ebullition and too restricted steam space cause **priming**, that is, a large quantity of water passes into the steam pipe from the boiler. Remedy—let the water boil quietly, reducing the flame if necessary, and do not have too much water in the boiler.

(*b*) Condensation takes place in the connecting pipe, which may be reduced by covering the pipe with strips of flannel.

(*c*) Any water now finding its way along the pipe may be separated from the steam by means of a separator. A simple form of separator may be arranged as in Fig. 234, where E is a small bottle fitted with a rubber stopper having two holes for receiving glass tubes. B is the tube coming from the boiler, and reaches about halfway down the bottle. B' is a tube connecting the separator to the calorimeter D; this tube

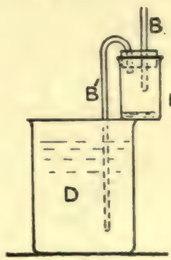


FIG. 234.—Arrangement for separating water from the steam.

terminates immediately on passing through the rubber stopper. Water coming along B will be deposited in the separator and very nearly dry steam will now be discharged into the calorimeter.

Returning to the preliminary experiment, when the temperature of the water in the calorimeter has been raised 20° or 30° F., remove the supply tube, again noting the time, observe the temperature t_2 of the mixture in the calorimeter, and weigh the whole; deduct from this weight that of the calorimeter and of the water originally in it, the result being the weight of stuff w which has come from the boiler as a mixture of water and steam.

Calling the latent heat of 1 lb. of steam L, and neglecting meanwhile the water carried over, we may calculate L on the assumption that, as the weight w has parted with latent heat and with sensible heat in cooling from 212° F. to the final temperature t_2, the sum of these quantities of heat will be equal to that acquired by the water originally in the calorimeter. Hence the equations:

$$wL + w(212° - t_2) = W(t_2 - t_1) \quad \text{.......................... (1)}$$

$$\therefore \; L = \frac{W(t_2 - t_1) - w(212 - t_2)}{w} \quad \text{.......................... (2)}$$

The true result is about 967 B.T.U. Equation (2) will be found to give a result very different from this. Apply corrections to equation (1) as follows: (a) add to W, on the right hand side, the water equivalent of the calorimeter. Calling this e, the right hand side now becomes

$$(W + e)(t_2 - t_1);$$

(b) To correct for water carried over, allow the boiler to discharge steam from the open end of B into the atmosphere, raising the outer end of B for this purpose (Fig. 235). A dish F will receive the drops of water carried over, the steam being blown off clear of the beaker.

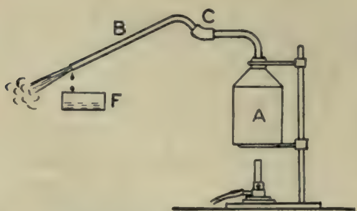


FIG. 235.—Estimation of the quantity of water carried with the steam.

Allow this to go on during the same interval of time as the first experiment lasted, and it may be assumed that the weight of water collected is the same as that which has been carried into the calorimeter as water of condensation during the first experiment. Let this weight be called w_c, then of the whole weight w of stuff coming from the boiler, w_c is water and $(w-w_c)$ is steam. The left-hand side of equation (1) now becomes

$$(w-w_c)L+w(212^\circ-t_2),$$

and the corrected equation will now be

$$(w-w_c)L+w(212^\circ-t_2)=(W+e)(t_2-t_1) \quad \text{.............. (3)}$$

$$\therefore \; L=\frac{(W+e)(t_2-t_1)-w(212^\circ-t_2)}{w-w_c} \quad \text{...................... (4)}$$

Compare the result for L as found from equation (4) with the true result 967 B.T.U. There should now be no serious difference.

EXPT. 58.—Taking the precautions noted above regarding priming, covering the pipe and fitting a separator, repeat the experiment, making corrections for the water equivalent of the calorimeter and for any water carried past the separator, and compare the result with that found in the foregoing experiment.

Saturated and superheated steam.—When water is heated, vapour particles are given off from the surface of the water at all temperatures, but boiling does not occur until a certain temperature is attained, depending on the pressure to which the water is subjected. Steam in contact with the water from which it has been formed is said to be **saturated steam.** A space is said to be saturated with a vapour when there is equality of interchange of particles between the vapour and the liquid from which it has been formed. The steam space in a closed boiler is saturated when the steady conditions are attained of equality between the mass of the particles leaving the water surface per second and the mass of the particles being returned to it per second. Saturated steam at a given pressure can only exist at one temperature. Abstraction of heat from saturated steam does not produce a lowering of the temperature, but causes some of the steam to condense into water. Addition of heat to saturated steam, provided the pressure is maintained constant and

no water is present, produces a rise in the temperature. The steam is no longer saturated steam, but is said to be **superheated.**

Superheated steam at a given pressure can exist at any temperature higher than the boiling temperature corresponding to that pressure. Saturated steam is a vapour, the pressure and temperature of which follow no simple law. Superheated steam behaves more and more like a perfect gas as its temperature is raised.

EXPT. 59.—Fig. 236 shows a small copper boiler A, fitted with a cork having two holes, one of which serves for the insertion of a thermometer E, and from the other is led a copper tube B,

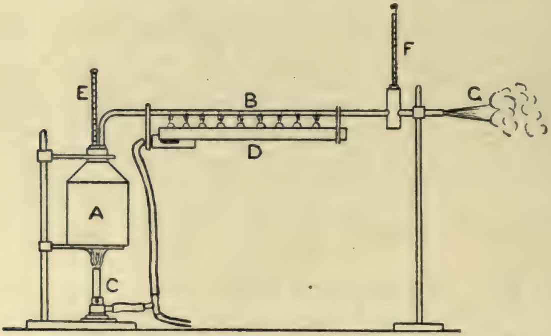


FIG. 236.—Apparatus for showing the temperature of superheated steam.

having a pocket formed near its outer end for the insertion of a thermometer F. D is a row of gas jets by means of which the tube B may be strongly heated. Put some water into the boiler and heat it. When steam has been given off freely for a few minutes read both thermometers. The pressure being practically atmospheric, both thermometers will read 212° F. Now light the row of gas jets and take readings of the thermometers. The pressure is still atmospheric, and the thermometer in the boiler will be found to read 212° F., but that in the pocket will be found to ascend to a temperature much higher than 212° F. The steam in the boiler is saturated steam at atmospheric pressure, that being discharged from the outer end of the tube is superheated steam at atmospheric pressure.

Pressure and temperature of saturated steam.—Experimental data due to Regnault show that the pressure and temperature of saturated steam are not simply proportional to one another. A Table showing corresponding values of these will be found on p. 365. From this Table it may be seen by plotting corresponding values of p and t (Fig. 238), that the pressure rises rapidly as the temperature increases. The relation cannot be deduced from first principles, but may be approximately presented by an empirical formula, *i.e.* one contrived so as to agree closely with the experimental data. Rankine's formula is:

$$\log_{10} p = 6 \cdot 1007 - \frac{B}{\tau} - \frac{C}{\tau^2},$$

where p = pressure in lbs. per square inch,

τ = absolute temperature Centigrade,

$$\log_{10} B = 3 \cdot 1812,$$
$$\log_{10} C = 5 \cdot 0881.$$

This formula may be used, in the absence of a Table, to calculate corresponding values of p and t.

EXAMPLE. What will be the pressure of saturated steam at a temperature of 200° C.?

$$\tau = (200 + 273 \cdot 7) ; \ \log_{10} \tau = 2 \cdot 6755.$$

$$\log_{10}\left(\frac{B}{\tau}\right) = 3 \cdot 1812 - 2 \cdot 6755 = 0 \cdot 5057 ;$$

$$\therefore \frac{B}{\tau} = 3 \cdot 204.$$

$$\log\left(\frac{C}{\tau^2}\right) = 5 \cdot 0881 - 2 \times 2 \cdot 6755 = \bar{1} \cdot 7371 ;$$

$$\therefore \frac{C}{\tau^2} = 0 \cdot 5459.$$

$$\log_{10} p = 6 \cdot 1007 - 3 \cdot 204 - 0 \cdot 5459 = 2 \cdot 3508 ;$$

$$\therefore p = \underline{224 \cdot 3} \text{ lbs. per square inch.}$$

The tabular number will be found to be 225·9 lbs. per square inch, showing an error in the calculated result of about 0·7 per cent. It will be found better to use the Tables where possible. Any pressure intermediate to those given in the Table, p. 366, may be found readily by plotting the temperatures and pressures nearest to the given temperature.

Expt. 60.—To obtain the relation of pressure and temperature of saturated steam, the apparatus illustrated in Fig. 237 will be found useful. To use it a supply of steam from a steam boiler must be available. A is a strongly made vessel constructed of solid drawn brass tube about 4″ diameter, having brass flanges brazed on and brass covers bolted to the flanges. B is a coil made of ⅜″ copper tube. Steam from the boiler enters the coil through a regulating valve C, and is discharged through another valve D. Distilled water is poured into A through a valve H in quantity sufficient to cover the coil. E is a thermometer pocket, containing a thermometer F. G is a steam pressure gauge.

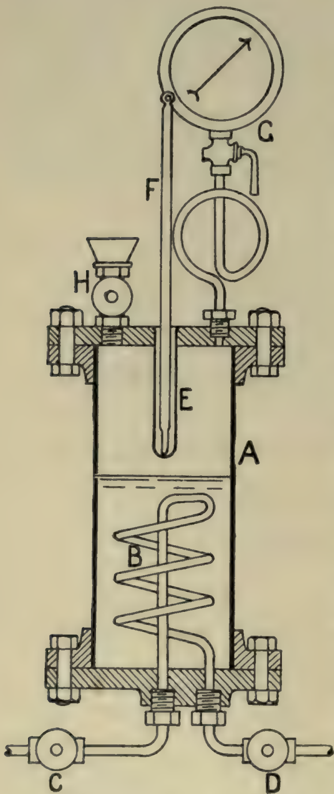


Fig. 237.—Apparatus for determining the relation of p and t for saturated steam.

Open valves C and D slightly, when the water in the vessel will be heated quickly by the heat coming from the steam condensed in the coil. The valve H ought to be left open so as to get rid of air, the pressure of which would cause a higher pressure to be indicated at any given temperature than that existing in a space filled with steam only at the same temperature. When steam is being discharged freely through H, close H, when the pressure of the steam will rise as shown by the gauge G, the temperature for different pressures being

indicated by the thermometer F. It will be found impossible to obtain a pressure of steam in this apparatus higher than that in the steam boiler, hence the advantage over the ordinary experimental boiler heated by gas jets, in which dangerous pressures may be attained through careless use and safety valves which stick. The apparatus may be used for pressures up to 100 lbs. per square inch.

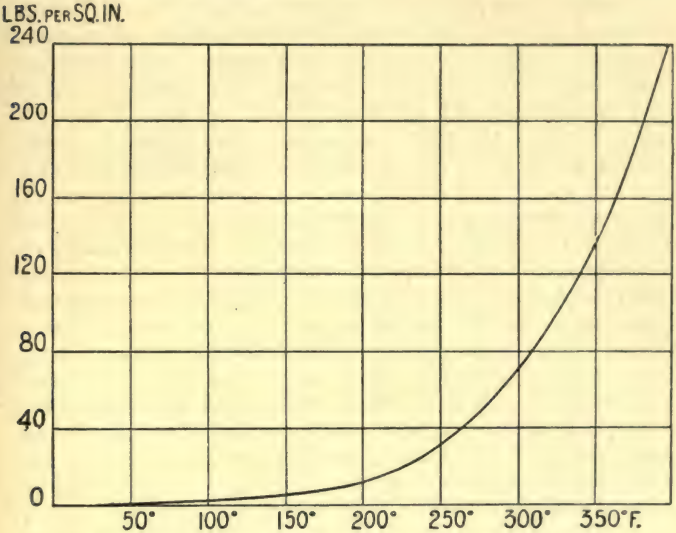


FIG. 238.—Curve showing the relation of p and t for saturated steam.

Read pressures and corresponding temperatures throughout the possible range and tabulate :

Pressure lbs. per square inch.	Temperature from experiment.	Temperature from Table (p. 365).	Error.

Plot columns 1 and 2, and also 1 and 3, and compare the resulting curves.

Expt. 61.—Arrange apparatus as shown in Fig. 203. See that the flask is perfectly clean and introduce some ordinary tap water. Light the Bunsen and observe what happens in the water. After the water has become hot, bubbles ascending to the surface will be noticed; these are composed of air which has been dissolved in the water. Presently, as the temperature becomes higher, bubbles of steam will form near the bottom in proximity to the flame, and will rise towards the surface, but will become collapsed by the colder water before reaching the surface. Ebullition, or boiling, takes place at a slightly higher temperature, called the boiling point; the whole of the water is now at one temperature and bubbles of steam are coming from all parts, reaching the surface and being discharged. Note the vapour temperature, and, by pushing the thermometer down into the water, verify that the temperatures of the vapour and of the water are equal.

Remove the flame and cork and introduce a small quantity of common salt, which will quickly dissolve in the water. Replace the thermometer, arranging that its bulb is in the liquid and heat again. It will now be found that the boiling point is higher than before. Move the thermometer upwards until its bulb is in the vapour space, and again read the temperature; this will be found to be the same as in the first experiment, showing that the temperature of the vapour is apparently the same as that of boiling water, and that the temperature of the salt solution is higher. Actually the vapour temperature is somewhat higher than that indicated by the thermometer; this is owing to water vapour condensing into a film of water on the thermometer bulb; any addition of heat from the vapour to this film simply causes it to evaporate without rise in temperature, which, accordingly, is indicated by the thermometer as that of boiling water.

It follows from the above experiment that the boiling points of solutions should be determined with the thermometer bulb immersed in the liquid. In vapour temperature determinations the bulb should be immersed in the vapour only.

Vapour pressure at pressures lower than that of the atmosphere.—The pressure of water or other vapour at low pressures may be observed approximately as follows:

EXPT. 62.—Arrange two barometer tubes (Fig. 239) as directed on p. 209. Bend the point of a small pipette C and charge it with the liquid to be examined. Put the point of the pipette under the open mouth of B, and by blowing introduce some of the liquid into B. This will rise to the surface of the mercury at D, and will give off vapour which will fill the space BD and will exert pressure on the surface of the mercury at D. Care should be taken to introduce no more liquid than will just leave a thin layer at D after evaporation is complete. Measure the heads of mercury in A and B and take the difference. As that in A shows the atmospheric pressure, it is evident that the difference gives the vapour pressure at the temperature of the room. If possible, the readings should be taken for several different temperatures, either by reading at different times during the day, or by moving the apparatus into a colder or warmer room.

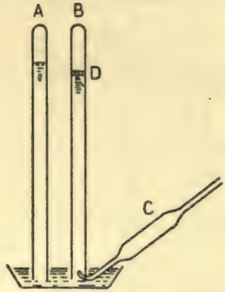


FIG. 239.—Pressure of a vapour at low temperature.

Pressure and volume of saturated steam.—This relation again does not follow any simple law. Various empirical formulae have been devised to show the relation, two of which are here quoted :

$$\text{Rankine's formula, } pV^{\frac{17}{16}} = 479\,;$$

$$\text{Zeuner's formula, } pV^{1\cdot0646} = 479.$$

Where p = pressure in lbs. per square inch,

V = volume in cubic feet per pound weight of steam.

The formulae differ only in the index of the power to which V is raised.

Direct experiments on the volume occupied by one pound of saturated steam at a given pressure are not performed easily, and can never be said to be strictly accurate. Plotting pressure and volume from the data in the Table (p. 365), as shown in Fig. 240, it will be seen that the curve somewhat resembles that representing Boyle's law (Fig. 225).

The student should now plot, on a large sheet of squared paper, p and t, and also p and v, both curves being drawn on the same sheet and preserved for future reference.

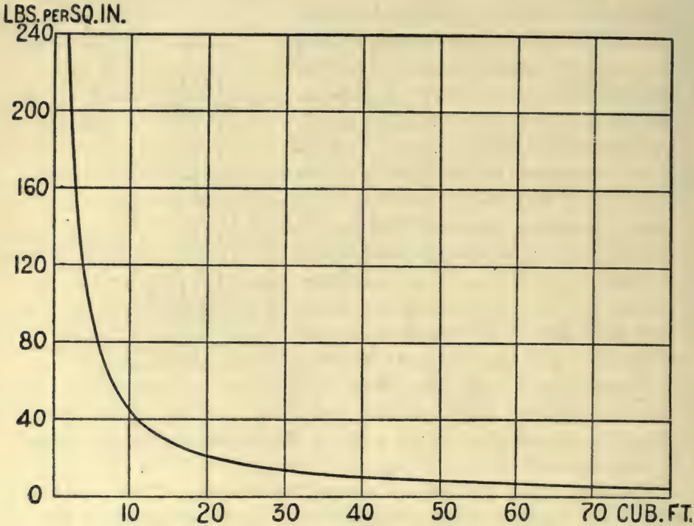


FIG. 240.—Curve showing the relation of p and v for dry saturated steam.

Latent heat of steam.—One pound of steam possesses a latent heat of 967 B.T.U. only when formed at a pressure of one atmosphere. For each degree Fahrenheit of increase in temperature above 212° F. at which boiling occurs, it is found that the latent heat is diminished by 0·695 B.T.U., and is increased by the same amount for each degree Fahrenheit below 212° F.

A formula to represent this would be

$$\begin{aligned} L_t &= 967 - 0{\cdot}695(t^\circ \text{ F.} - 212) \\ &= 967 - 0{\cdot}695\, t^\circ \text{ F.} + 147 \\ &= 1114 - 0{\cdot}695\, t^\circ \text{ F.}, \end{aligned}$$

where L_t = latent heat in B.T.U. of the pound of steam formed at t° F.

The formula may be modified to suit other scales of temperature. Thus: $L_t = 606{\cdot}5 - 0{\cdot}695\, t°$ C. in which the latent heat is stated in lb.-degree-Cent. units of heat.

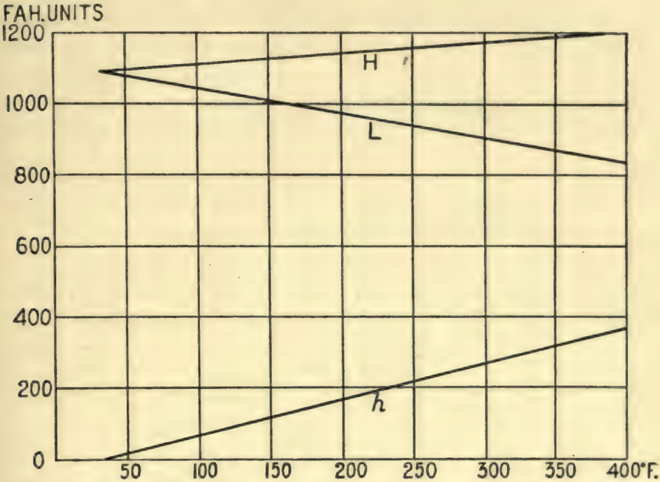


FIG. 241.—Curves showing H, L and h with temperature for dry saturated steam.

Total heat of steam.—Regnault's total heat of steam is defined as **the total heat** (sensible and latent) **which must be imparted to one pound of water at freezing temperature in order to convert it into saturated steam at any given temperature.**

Let H = total heat of steam.
h = sensible heat.
L = latent heat.

Then $H = h + L$.

Putting $L = (1114 - 0{\cdot}695t°\text{ F.})$.
$h = (t°\text{ F.} - 32°)$.

$$H = (t°\text{ F.} - 32°) + (1114 - 0{\cdot}695t°\text{ F.}) = 1082 + 0{\cdot}305t°\text{ F.}$$

This equation may be used to calculate values of H. It may be modified to express the result in lb.-degree-Cent. units, thus giving

$$H = 606{\cdot}5 + 0{\cdot}305t°\text{ C.}$$

Values of H, L and h will be found in the Table, p. 365. Taking these values, plot H, L, h and temperatures ; also plot the same quantities and pressures, putting all three curves on a single large sheet of squared paper. The curves will resemble those shown in Figs. 241 and 242, and should be preserved for future reference.

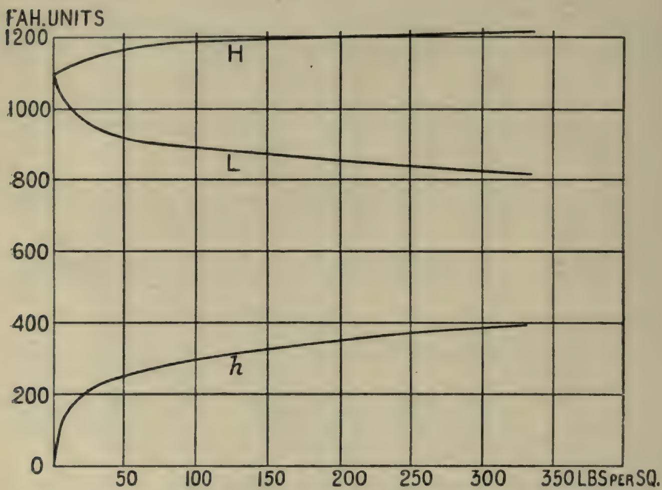


FIG. 242.—Curves showing H, L and h with pressure for dry saturated steam.

It should be noted that H, L and h give straight lines when plotted with temperatures, and that the value of H throughout the range alters very little comparatively. When plotted with pressures it will be noticed that all three change rapidly at low pressures and tend to take up a more uniform rate of change as the pressure increases.*

Water expands on being heated, and, as the temperature at which the generation of steam begins is increased, the volume of

* Recent researches of Callendar have led to exact equations expressing the properties of both saturated and superheated steam. Tables based on these equations have been constructed by Mollier and by Smith and Warren.

steam from a given weight of water diminishes. It follows, therefore, that a point will be reached where the volume of the steam will be equal to the volume of the water; at this point steam and water cannot be distinguished from each other, and the latent heat becomes zero. The temperature at which this occurs lies in the region of from 365° to 370° C., and is known as the **critical temperature.**

Formation of steam at constant pressure.—Fig. 243, *a*, shows a cylinder fitted with a piston which may be loaded to any desired extent. One pound of water is contained in the cylinder, the piston exerting a constant pressure p lbs. per square inch on it. Let the area of the piston be 1 square foot. Since the volume of 1 lb. of water is 0·017 cubic foot, the length of cylinder occupied by the water will be 0·017 foot. Suppose that the water is already at the boiling temperature corresponding to p, and that the latent heat required to form steam at this temperature is supplied to it. The volume will thereby be increased to V cubic feet = volume occupied by one pound of steam at the given pressure p; the piston will rise to accommodate this increased volume, which will evidently occupy a length V feet of the cylinder (Fig. 243, *b*).

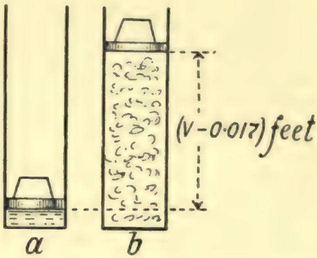


FIG. 243.—Diagram showing the formation of steam at constant pressure.

Two things have occurred in this process, (*a*) the state has been changed from liquid to gaseous, (*b*) external work has been done against the resistance p, and both are due to the latent heat supplied. The external work done may be calculated easily.

Area of the piston = 144 square inches.

Total pressure on piston = $144p$ lbs.

Distance through which piston is moved = $(V - 0{\cdot}017)$ feet.

External work done = $144p\ (V - 0{\cdot}017)$ foot-lbs.

It is usually sufficiently accurate to neglect the volume of one pound of water, and doing this, we may write

$$\text{External work done} = 144p\,V \text{ foot-lbs.}$$
$$= \frac{144p\,V}{J} \text{ heat units.}$$

This expression gives the portion of the latent heat supplied which has been transformed into mechanical work. The remainder of the heat energy supplied, viz.

$$\left(L - \frac{144p\,V}{J}\right) \text{ heat units}$$

remains as internal energy in the steam. The **total internal energy**, or **intrinsic energy** of the steam is defined as **the total heat of formation H diminished by the heat required to perform external work.**

Writing I for the intrinsic energy,

$$I = H - \frac{144p\,V}{J} \text{ heat units.}$$

Isothermal and adiabatic expansion and compression of a vapour.—In Fig. 244 is shown a cylinder A fitted with a piston B and containing one pound of water under pressure, produced by the force P. In the pressure volume diagram, D is the corresponding point. Let the water be at the boiling temperature t_1 corresponding to the pressure, and let the piston move outwards, at the same time permitting heat at a temperature t_1 to flow freely into the cylinder. The water will thus take up its latent heat of evaporation ; its temperature and that of the resultant steam will remain steady at t_1; the volume will increase as the water is converted into steam as shown by DE, a line of constant pressure as P is maintained uniform. It is obvious that the operation is isothermal, and that the final result is a pound of dry saturated steam at E. As there is no more

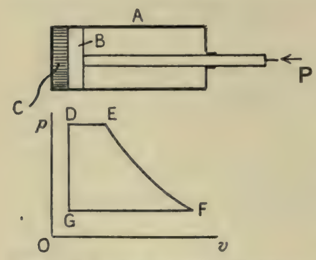


FIG. 244.—Expansion and compression of a vapour.

water available, further movement of the piston will be accompanied by the expansion of the steam, producing diminutions of pressure as shown by *EF*. Suppose heat in sufficient quantity were supplied during expansion to maintain constant the temperature, then the steam at *F* would be superheated. Its temperature would still be t_1, the temperature corresponding to the higher pressure at which it was formed, and this temperature is greater than the boiling temperature corresponding to the lower pressure at *F*. Usually, in practice, an attempt is made not to superheat the steam during expansion, but merely to supply enough heat to maintain the vapour as dry saturated steam. If no heat were supplied during expansion, *i.e.* if the expansion is adiabatic, then the steam, which is dry at *E*, would be wet at *F*, for the reason that work has been done against the resistance of the piston, and the heat necessary to do this work has been abstracted from the steam.

Supposing the steam to be dry and saturated at *F*, its temperature being t_2, reconversion into water may be effected by pushing the piston inwards, at the same time permitting heat to flow freely out of the cylinder. The steam will give up its latent heat at steady temperature t_2 and will condense into water. This operation is isothermal, represented by *FG*, and the final result of it is a pound of water at *G*, having a pressure and temperature lower than the original pressure and temperature represented by the point *D*. Restoration to the original conditions may be obtained by applying pressure and imparting sensible heat to the water.

It will be evident that any isothermal changes effected on a mixture of liquid and vapour will be accompanied by evaporation if the change is one of expansion and by condensation if the operation is one of compression.

EXERCISES ON CHAP. XV.

1. Explain the terms "sensible heat," "latent heat," and "total heat of steam."

2. Using the Table, p. 365, plot the pressure and temperature of saturated steam. State in general terms how the pressure varies with the temperature.

3. Plot the pressure and volume of dry saturated steam, using the Table, p. 365. Beginning at a point near the top of the curve, plot on the same sheet a curve to represent Boyle's law. Mark the curves clearly in the diagram and contrast them.

4. Using the empirical formula

$$\log_{10}p = 6{\cdot}1007 - \frac{B}{\tau} - \frac{C}{\tau^2},$$

calculate the pressure of saturated steam corresponding to a temperature of 300° F. Compare your result with the tabular result (p. 365).

5. Using the empirical formula

$$pV^{\frac{17}{16}} = 479,$$

calculate the volume occupied by one pound weight of dry saturated steam at a pressure of 100 lbs. per sq. inch absolute. Compare your answer with the tabular number, p. 365.

6. Calculate the latent and total heats of dry saturated steam at a temperature of 300° F. Use the empirical formulae

$$L_t = 1114 - 0{\cdot}695t^\circ \text{ F.}$$
$$H = 1082 + 0{\cdot}305t^\circ \text{ F.}$$

Compare your results with those given in the Table, p. 365.

7. What is meant when we speak of a space being saturated with a vapour? Explain the meaning of the term "dry saturated steam." What is meant by superheated steam?

8. Distinguish between the internal and external work done during the formation of dry saturated steam at constant pressure.

9. Taking the quantities required from the Table, p. 365, calculate how much heat must be abstracted from 100 lbs. weight of dry saturated steam at 3 lbs. per square inch absolute in order to condense and cool it to 130° F.

10. The water in a tank containing 40 gallons is heated by blowing dry saturated steam into the water at a pressure of 30 lbs. per square inch absolute. Suppose the initial temperature of the water to be 60° F., calculate what weight of steam must be used to raise the temperature to 150° F. Take any quantities you require from the Table, p. 365.

11. What heat must be given to 1 lb. of water at 80° F. to convert it into steam at 300° F.? Regnault's formula for the *total heat* of a pound of steam from water at 32° F. being $H = 1082 + 0{\cdot}305t$ where t° F. is the temperature of the steam, how many pounds of this steam are equivalent in *total heat* to the calorific power (15,000 units of heat) of a pound of coal?

12. A formula for Regnault's total heat H will be found at p. 241 ; it is the total heat which must be given to 1 lb. of water at 0° C. to raise its temperature as water to t° C., and then to convert it all into steam at t° C. What is the heat which must be given to 1 lb. of water at 40° C. to convert it into steam at 170° C. ?

13. With a small experimental boiler you are finding the pressure of steam when its temperature is, say, 100° C., 110° C., 120° C., etc. Show, with sketches, exactly how you would proceed. In what way does the presence of air with the steam spoil your results ?

14. Show, with sketches, how you would find the latent heat of water experimentally. What precautions must be taken ?

15. Describe, with sketches, how you would determine by experiment the latent heat of steam at atmospheric pressure. What circumstances are likely to interfere with the result ?

16. Describe, giving sketches, how you would find the pressure of saturated steam at a temperature equal to that of the atmosphere in the laboratory.

17. Supposing dry saturated steam at a certain pressure to be expanded (a) isothermally, (b) adiabatically, to a lower pressure, explain its condition at the final pressure and give reasons.

CHAPTER XVI.

APPLICATIONS OF THE PROPERTIES OF GASES AND VAPOURS.

Lift pump.—The **common lift pump** depends for its action on the pressure exerted by the atmosphere. A cylinder at *A* (Fig. 245) contains a piston, or **pump bucket** as it is called, fitted with a valve opening upwards. The cylinder is connected by a pipe *C*, with a foot valve at its bottom, to a cistern of water *E*. On the up-stroke of the bucket, the pressure of the air contained in *C* falls, and the atmospheric pressure on the water in *E* causes some of the water to flow into the pipe *C*. On the down-stroke the valve *D* closes, and the valve *B* opens. No water can pass *D* now, and air will be expelled through *B*. On the next up-stroke *B* will close again and *D* will open, and more water will flow into *C*, and this process repeated again and again will bring water ultimately into the cylinder, when it will pass *B* and be discharged through *F*. The process of starting in this manner is long, and can be hastened by first charging the pump cylinder and pipe *C* with water through a plug placed near the top of the suction pipe.

Feed pump.—Another type of water pump is shown in Fig. 246, and is used for forcing feed water into a marine steam boiler. It consists of a gun-metal body *A* fitted with a brass plunger *B* made hollow for lightness. The plunger is rendered tight against leakage by means of a stuffing-box with brass gland and neck bush. The inlet valve is shown at *C* and the delivery valve at *D*. These valves are of the ordinary conical, or clack, variety and have a limited rise and fall. An **air vessel** is fitted at *E*. In working, during the up-stroke the inlet valve *C* opens,

permitting water to flow into the pump, the delivery valve, D, being closed. During the down-stroke the inlet valve C closes

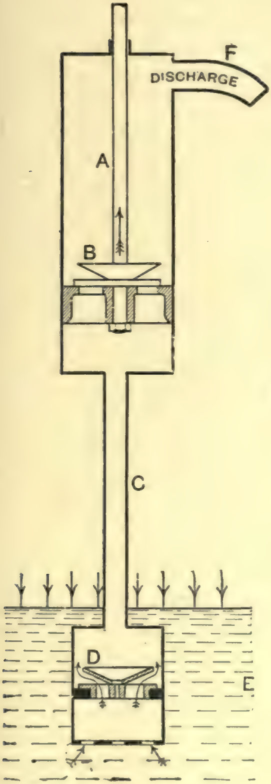


FIG. 245.—Lift pump.

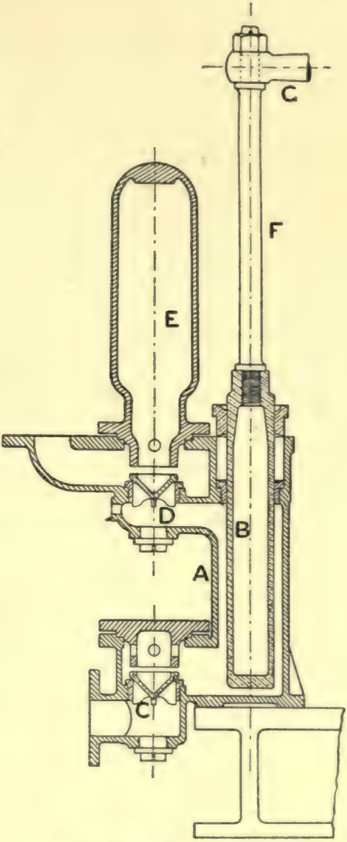


FIG. 246.—Marine feed pump.

and the water is forced by the plunger through the delivery valve D and so into the boiler. The air vessel contains some air, and this, by its elasticity, serves as a cushion for the reduction of

shocks, which would otherwise be produced by the forceful pumping of incompressible water. The plunger is attached by means of a rod F and a crosshead G to the main engines.

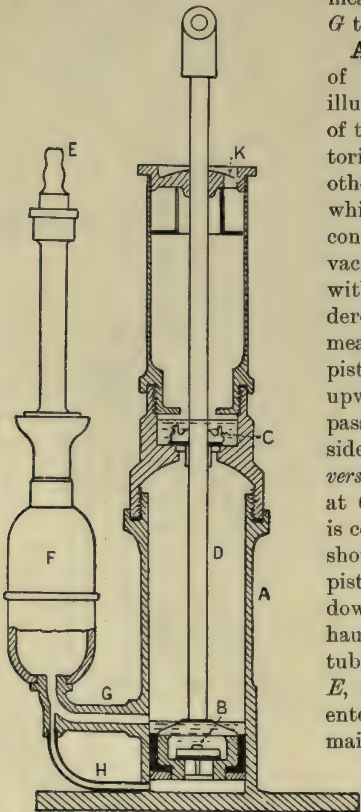


FIG. 247.—Fleuss air-pump.

Air-pumps.—A common type of air-pump—the Fleuss—is illustrated in Fig. 247. Pumps of this kind are used in laboratories for exhausting air, or other gases, from vessels in which experiments are being conducted requiring a partial vacuum. A cylinder A is fitted with a piston B which is rendered tight against leakage by means of a leather ring; the piston contains a valve opening upwards, and will permit gas to pass from the lower to the upper side of the piston, but not *vice versa*. A similar valve is fitted at C. D is the piston rod and is connected to a hand lever (not shown), by means of which the piston may be moved up and down. The vessel to be exhausted is connected by rubber tubing to E; the gases enter at E, pass through F, and may enter the cylinder through a main passage G, or through a by-pass H. The exit orifice to the atmosphere is at K. The action of the pump is as follows: Starting with the piston at the bottom of the stroke, the vessel to be exhausted is in communication with both sides of the piston through G and H, hence the pressures on the top and bottom of the piston are equal and the piston may be moved easily. Immediately the

piston passes the opening of G, it will begin to compress the gas in the space lying between the valve C and the piston. Compression goes on until the pressure of the enclosed gas is equal to that of the atmosphere, when the valve C will lift, and the gas will be discharged through C and K during the remainder of the upward stroke.

As the piston cannot quite arrive at the top of the cylinder, the whole of the gas will not be discharged; a more efficient result would be obtained if the entire gaseous contents of the cylinder were expelled through C. In order to effect this result, some oil is introduced and lies on the top of the piston; as the piston approaches the top of the stroke, this oil will fill ultimately the whole of the upper part of the cylinder, thus driving the entire gaseous contents through C. Some oil will find its way through C, and will remain on the top of this valve during the next downward stroke. During the upward stroke which has just been described, more gas has been flowing into the lower part of A through G and H. The piston now starts to descend; C closes and the space above B becomes more rarefied than that below, hence the valve in the piston opens, and the piston descends with equal pressures on both top and bottom. On the piston passing G, the function of the by-pass H is to permit the escape of any gas, or liquid, which may be trapped between the piston and the bottom of the cylinder; the piston may be thus pushed easily to the cylinder bottom and the action repeated.

The volume of air removed during each upward stroke will be equal to that of the portion of the cylinder lying above G, measured at the particular pressure existing in the vessel under exhaustion at the beginning of the stroke considered. A baffle plate arrangement is fixed to the under side of the top cover of the pump, in order to minimise the nuisance of oil being thrown upwards through K.

Expt. 63.—Connect a flask containing some water at a temperature of 200° F. approximately to an air-pump by means of a rubber cork, glass tube, and rubber tube joints. Make certain that the joints are tight. On lowering the pressure in the flask by working the pump, it will be found that the water begins to boil. The operation should not be kept

up too long or the air-pump will become charged with water of condensation.

In Fig. 248 is shown a simple form of **mercurial air-pump.** A funnel *A* containing mercury is connected at *B*, by means of a short rubber tube having a screw clip, to a long vertical glass tube *BC*. This tube dips below the surface of mercury contained in the vessel at *C*, and is steadied by a cork fitted to the vessel. A spout permits of the escape of excess mercury into a beaker *F*. The tube *BC* has a branch at *D*, and the flask to be exhausted is connected to the branch by a short rubber tube. The tube *BC* should be about 0·1 inch in bore, and should be considerably longer from *D* to *C* than a barometer tube. In action, drops of mercury descend the tube from the funnel, the space separating the drops being filled with air abstracted from *E*. As the action goes on and the air in *E* becomes rarefied, the spaces between the drops become smaller until finally the tube *BC* is full of mercury. The conditions in *E* are now similar to those over the top of the mercury column in a barometer tube. It will be necessary to stop the flow occasionally, while the mercury caught in *F* is returned to the funnel *A*.

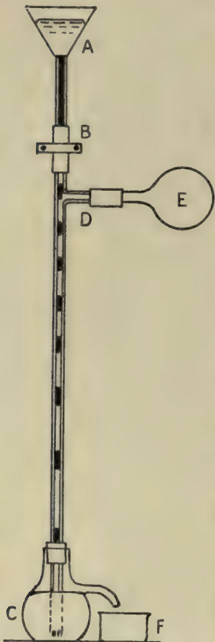


FIG. 248.—Mercurial air-pump.

Action in charging an air reservoir.—In Fig. 249, an air-pump *A*, fitted with a piston *B*, takes in air from the atmosphere through the inlet *C*, compresses it, and delivers it under pressure through the discharge valve *D* and pipe *DE*, into a closed reservoir *F*. Assume that the contents of the reservoir are at atmospheric pressure at the beginning of the charging process, and that the compression follows the simple law $pv =$ a constant. Let the piston be at the right-hand end of the cylinder, and let

the pump be full of air at atmospheric pressure. In the pressure-volume diagram *al* is a horizontal line drawn to represent the constant pressure of the atmosphere, and *b* is the point corresponding to the conditions at the start of the first compression stroke. On pushing the piston inwards, the discharge valve *D* will open practically at once, as there are equal pressures on both its top and bottom, and air will be delivered into the reservoir throughout the stroke. This is represented by the

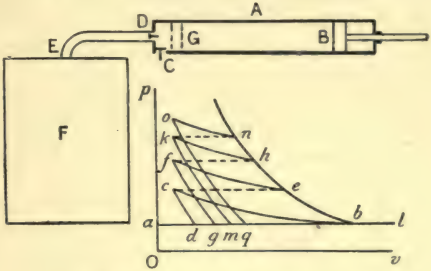


Fig. 249.—Charging an air reservoir.

curve *bc*. The piston comes to rest at *G* and *D* then closes. In this first stroke, the initial volume v_1 will be the total volume of air in the reservoir, pipes, and in the pump chamber up to the initial piston position. The final volume v_2 will be the total volume in the reservoir, pipes and pump chamber up to *G*. The calculation of the pressure p_2 at *c* may be effected by application of the equation

$$p_1v_1 = p_2v_2,$$

where p_1 is the pressure of the atmosphere.

Consider now the return stroke. At its start, both valves *C* and *D* are closed, and there is air in the pump chamber at a pressure p_2 corresponding to *c*. This air must expand to atmospheric pressure before the inlet valve *C* can open; the expansion is shown by the curve *cd* and the remainder of the stroke is represented by *db*, air flowing into the pump chamber through the valve *C*.

The second inward stroke is now effected. During the early portion of this stroke both *C* and *D* will be closed, as the

pressure is rising, but is not yet equal to that in the reservoir. Hence the initial volume being dealt with will be the volume in the pump chamber only. Consequently the pressure rises at a more rapid rate than during the first stroke, and is shown by the line *be*. At *e*, which is at the same height as *c*, the pressure in the pump and reservoir are equal; the valve *D* opens and the remainder of the stroke is completed by compressing the total air in the reservoir, pipes, and pump; this stage is shown by the curve *ef*; the rate of rise in pressure is lower in this stage than in the earlier part of the stroke, owing to the larger volume being compressed. Expansion from *f* back to atmospheric pressure takes place in the same manner as before along the line *fg*, and intake along *gb* follows. Other two similar operations are shown by the curves *bhkmb* and *bnoqb*. The operations are repeated until the reservoir is charged at the desired pressure.

The action in a tyre inflator is similar to that above described, with a slight difference owing to the fact that the reservoir (the tyre) has not constant volume. At first, when the tyre is "down," the volume contained by it is practically zero, and increases as the air is pumped in. The pressure-volume diagram will be altered in this respect only, the curves *bc*, *ef*, *hk*, etc., will not rise so steeply, owing to the expansion of the rubber tyre as the pressure rises. The other portions of the diagrams will remain as shown in Fig. 249.

EXAMPLE 1. A cycle tyre has a volume of 150 cubic inches when inflated and the air pressure in it is 35 lbs. per square inch absolute. How many cubic inches of air at atmospheric pressure does the inflator take in during the entire pumping operation?

$$p_1v_1 = p_2v_2,$$

$$v_2 = \frac{p_1v_1}{p_2}$$

$$= \frac{35 \times 150}{15}$$

$$= \underline{350} \text{ cubic inches.}$$

EXAMPLE 2. The volume of air in the inflator at the commencement of a stroke is 5 cubic inches. The pressure in the tyre is 25 lbs. per square inch absolute. What will be the volume in the

inflator at the moment the valve opens to permit air to flow into the tyre?

Here $p_1 = 15$ lbs. per square inch absolute.

$p_2 = 25$,, ,, ,,

$v_1 = 5$ cubic inches.

$$p_1v_1 = p_2v_2,$$

$$v_2 = \frac{p_1v_1}{p_2}$$

$$= \frac{15 \times 5}{25}$$

$$= \underline{3} \text{ cubic inches.}$$

Caissons.—There are many practical applications of air under pressure, including such tools used in engineering works as pneumatic hammers, riveters, drilling and tapping machines. One important example in engineering practice is the use of air pressure for enabling the submerged foundations of bridges to be constructed. A large closed steel cylinder or **caisson** *A* rests on the bottom of the river (Fig. 250); a diaphragm separates off a space near the bottom in which the men work, excavating the material from under the caisson. The caisson is furnished with cutting edges round its bottom at *C*, *C*, and is fitted with a material hoist in a tube *F*, and another hoist in *D* by means of which the workmen have access to and from *B*. To exclude the water, the level of which is at *H*, *H*, air under sufficient pressure to overcome the head of water between *H* and *C* is supplied to the space *B* by means of pumping machinery. Hence, it is necessary to supply lock-chambers *G* and *E* at the top of each hoist; arrangements of valves and doors are provided for the purpose of gradually reducing the pressure in the lock-chambers to that of the atmosphere and *vice versa*.

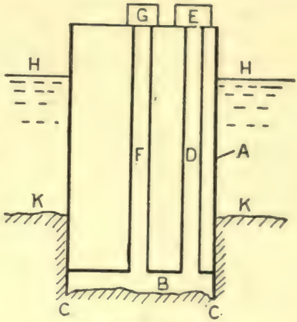


FIG. 250.—Caisson.

The workmen's lock-chamber requires particular attention so as to render it impossible to change the pressure at too rapid a rate. If this were to occur, there would be great danger to the workmen emerging from the caisson. Working under a pressure of say 50 or 60 lbs. per square inch in *B*, sudden liberation of the pressure would probably be fatal. Hence, two lock-chambers are sometimes fitted at *E* with an intermediate chamber in which the men may remain for a few minutes before the pressure is reduced finally to that of the atmosphere.

Pneumatic parcel transmitter.—Small parcels may be transmitted from one station to another by means of carriers

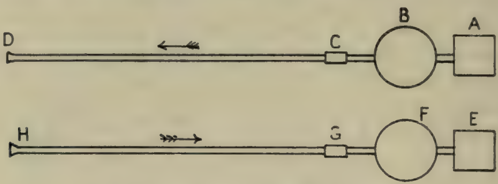


FIG. 251.—Pneumatic parcel transmitter.

impelled through tubes by gaseous pressure. In Fig. 251, *A* is an air pump capable of maintaining a pressure of 10 to 15 pounds per square inch above that of the atmosphere in the receiver *B*. The transmission tube is connected to *B*; a carrier containing the package to be transmitted is placed in the tube at the station *C*, the opening is closed and the air pressure is turned on. The carrier is then blown through the tube and discharged at the station *D*. The type of carrier used by the Post Office consists of a gutta-percha tube *A* (Fig. 252) covered with felt *B* and having felt discs at its front end *C*; these fit the tube and form a buffer when the carrier is being stopped. In transmission from *C* to *D* in Fig. 251, the carrier will have approximately atmospheric pressure on its front end, resisting the motion, and the receiver pressure of 10 to 15 lbs. per square inch above that of the atmosphere acting on its other end; there is thus a net pressure of 10 to 15 lbs. per square inch urging it along the tube.

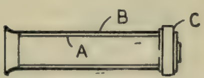


FIG. 252.—Pneumatic carrier.

Service in the opposite direction may be obtained by use of a separate tube. In Fig. 251 E is an exhausting air pump, the function of which is to maintain the gaseous pressure in a reservoir F at 7 to 10 lbs. per square inch below that of the atmosphere. F is connected to the return tube. A carrier inserted at H will evidently have a difference in pressure of 7 to 10 lbs. per square inch urging it from H to G, where it is stopped and removed from the tube.

In the two-tube system explained above, there may be several carriers in the tube at the same time; arrangements are made for securing that the carriers are separated by a definite time interval. In cases where the service is not large, one tube suffices, and may be connected at will either to B or to F in Fig. 251. When connected to B, carriers will be sent outwards from the power end, and when connected to F, carriers will be returned towards the power end of the tube. It is evident that one carrier alone should be in the tube at a time.

The Post Office tubes are of lead, 2¼ to 3 inches in diameter, smoothly jointed together and laid inside cast-iron mains. A speed of 20 to 30 miles per hour can be obtained. The tubes are connected by means of valves to the receivers only when carriers are being transmitted. In small installations, fitted in stores and used for cash purposes, the air receivers are connected continuously to the tubes; this does away with any valve system and the carriers simply require to be inserted in the tubes; the cashier's office must be connected to each station by two tubes, an outwards and an inwards, under pressure and vacuum respectively. In very small installations, the necessary air pressure may be obtained by foot or hand operated pumps.*

Vacuum brake.—In this country passenger trains must have each coach fitted with brakes applied to the wheels, and capable of being operated by either driver or guard independently. There are two systems in use, both of which involve a continuous air pipe passing along the train. In the vacuum brake system, air is abstracted from this pipe by means of an ejector on the engine, and in normal running condition the gaseous pressure in the pipe is considerably below that of the

* "Pneumatic Tube Services," by D. H. Kennedy, *Post Office Electrical Engineers' Journal*, April, 1909.

atmosphere ; the brakes are then off. The action of admission of air to the pipe, which may be accomplished by either driver, or guard, by means of opening valves, is used for operating devices for the application of the brakes.

Each coach is fitted with an appliance shown in outline in Fig. 253. A rod A passes downwards and is connected to the levers which operate the brakes. A is attached to a piston B which may move vertically in a cylinder C and is rendered tight against leakage by means of a rubber ring D, which may roll with very

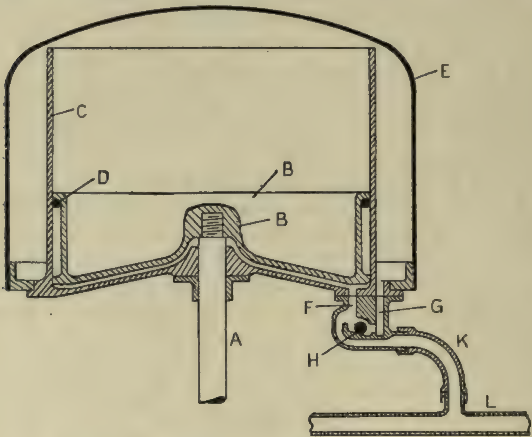


FIG. 253.—Apparatus for operating vacuum brakes.

little friction in a recess in B as the piston moves. The cylinder C is enclosed entirely by another cylinder E, both being connected together at the bottom. A passage F leads from C, and another G from E. These passages communicate with one another, and the opening may be closed by a small ball-valve H, which runs on an incline, and will be held up against the opening should the pressure in F be greater than that in G. The passages are extended by a pipe K which connects the appliance to the continuous train pipe L. The brakes are "off" when the piston B is at the bottom of the cylinder. This is effected by withdrawal of air from the pipe L ; air will thus flow from the

under side of the piston through F and K, and the air in the cylinders above the piston will flow through G, past the ball-valve H and so into L. The action of pumping air from L thus equilibrates the pressures on the bottom and top of the piston, and it will sink consequently to the bottom of its stroke.

To apply the brakes, the driver, or guard, opens an operating valve permitting air to flow into L. The rush of air up K will drive the ball-valve against its seat, closing the communication between F and G; the pressure in F will now be greater than that in G, and the ball-valve will remain closed. The air admitted thus passes to the lower side of the piston, and, as the top is under a partial vacuum, will exert a pressure upwards greater than the downward pressure on the top of the piston. The piston is thus impelled upwards and the brakes are applied. It is obvious that if coaches should break away by a coupling snapping, the pipe L will also be fractured, air will rush in, and all the brakes will be applied. The brakes are taken off again by closing the operating valve and by renewed pumping at the engine.

Atmospheric circulation.—Circulation of the atmosphere—evidenced by winds—is caused by the air at one place being at a higher temperature, and thus having a lower density, than the air at adjacent places. Ascending currents of warm air will thereby be set up and the colder air will rush inwards in the effort to equilibrate the gaseous pressures. Supposing there are two rooms entirely closed, except that there is a door opening from one room into the other, and that one room is warmer than the other. If the door be open, the colder and therefore heavier air will produce a current of air flowing into the warmer room near the floor; air will thus be displaced from the warmer room and will find its way into the colder room by means of a current near the top of the door opening. These currents may be rendered visible by smoke from smouldering brown paper, or by the deflection of the flames of lighted candles placed at different heights in the door opening.

The circulation of the atmosphere on the large scale is produced in a similar way; lower currents of air flow from cold towards hot places, and upper currents are thus induced to flow towards

neighbouring cold places. When very large bodies of air are in motion, the direction of the currents may be modified by the rotation of the earth. Thus, a current of air which would otherwise flow from the north to the equator along a meridian, will reach a point on the equator westwards of the place which it would pass over if the earth were at rest. This is owing to the motion of the earth's surface from the west towards the east, and also to the fact that the whirling velocity of the air accompanying the earth is lower, when at some distance from the equator, than the speed of the earth's surface at the equator. The result is a wind coming from the north-east instead of from due north, and from the south-east in southern hemispheres instead of from due south. These motions of the air towards the hot equatorial regions are evidenced by the trade winds.

Land and sea breezes occur with great regularity in hot countries and are caused by the greater specific heat of the sea compared with that of the land. During a hot day the land will acquire a temperature considerably higher than that of the sea, but will lose it much more rapidly at night. The effect is to cause ascending currents of hot air from the land during the day, while cooler air comes from the sea to take its place, producing a sea breeze. The effect is reversed at night, when the land quickly cools to a temperature lower than that of the sea, and a land breeze blows. Winds, such as the monsoons in the Indian Ocean, may be produced in this way, and extend in one direction or the other for long periods of time. The conditions required are a hot continent and a cooler ocean for sea breezes, produced in summer, and a cool continent and a hotter ocean for land breezes, which predominate in the winter.

Influence of height on the pressure and density of the atmosphere.—Take unit volume of a gas at a pressure p_1; let its weight be w_1 and let the pressure diminish to p_2, keeping the temperature constant. The volume will increase to v_2 and Boyle's law may be applied. Thus:

$$p_2 v_2 = p_1 \times 1.$$

Or,

$$v_2 = \frac{p_1}{p_2}.$$

This volume v_2 will possess the same weight w_1 as at first, and the weight of unit volume, w_2, under the altered conditions may be obtained by dividing w_1 by v_2.

$$\therefore \quad w_2 = \frac{w_1}{v_2}$$

$$= w_1 \frac{p_2}{p_1}.$$

Or,
$$\frac{w_2}{w_1} = \frac{p_2}{p_1}.$$

We may therefore state that the weight per unit volume of a gas at constant temperature is proportional to its pressure.

In Fig. 254 is shown a column of air of one square foot in sectional area standing on a base at A, and reaching upwards to the limit of the atmosphere. The pressure on the base A is caused by the total weight of the column. Suppose that the pressure at A is 2116 lbs. per square foot, and that the temperature is 0° C. throughout the whole column. Under these conditions of pressure and temperature, a cubic foot of air at A would weigh 0·0807 lb., and the volume of one lb. weight of air will be $\frac{1}{0·0807} = 12·391$ cubic feet. Hence, if we ascend through a height of 12·391 feet to B, the pressure there will be diminished by the weight of the column AB, equal to 1 lb. weight, and will be found to be 2115 lbs. per square foot. Effects owing to any change in gravitational effort have been disregarded in this calculation.

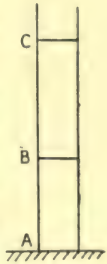


Fig. 254.

At B, the weight of one cubic foot may be calculated by applying the equation:

$$\frac{w_2}{w_1} = \frac{p_2}{p_1},$$

$$w_2 = w_1 \frac{p_2}{p_1}$$

$$= 0·0807 \times \frac{2115}{2116} \text{ lbs. weight per cubic foot.}$$

Or, Volume occupied by 1 lb. weight of air $= \frac{2116}{0·0807 \times 2115}$

$$= 12·395 \text{ cubic feet.}$$

Hence, if we ascend from B to C through a height of 12·395 feet, the pressure at C will be less than that at B by the weight of the column BC, *i.e.* by one pound per square foot; therefore the pressure at C will be 2114 lbs. per square foot.

It will be evident that the additional heights to be ascended as we go higher, in order to produce equal diminutions of pressure, will steadily increase owing to the rarefaction of the atmosphere indicated by the above calculation.

Increase in temperature will increase the heights AB and BC in Fig. 254; this is owing to increase in temperature being accompanied by an increase in volume, and therefore by a diminished weight per cubic foot.

The diminishing pressure of the atmosphere as the height increases has been utilised as a means of roughly estimating heights by the fall in the barometer. Approximately, an elevation of 900 feet corresponds to one inch fall in the mercury barometer column. The aneroid barometer is used generally, and is adapted for the purpose by having two scales on its dial, one showing ordinary barometric heights and the other scale showing elevations in feet. It will be evident that correction for the mean temperature of the air must be applied.

Flotation of balloons.—A balloon consists of a strong light envelope of material as nearly gas tight as possible, and charged with gas having a density less than that of the surrounding atmosphere. A car is attached to the envelope by means of ropes. It will be evident that the same law applies to this case as to that of a body immersed in a liquid (p. 148), viz. the buoyancy, or resultant pressure upwards of the surrounding atmosphere, is equal to the weight of the air displaced by the balloon, and this buoyancy will be equal to the total weight of the balloon and contained gas when the balloon is floating at constant elevation. Thus, if W_1 is the weight of the balloon, W_2 that of the contained gas, and W_3 that of the displaced atmosphere, (Fig. 255), then, for equilibrium of these forces,

$$W_1 + W_2 = W_3.$$

Ascent to a higher level may be accomplished by diminishing W_1, usually performed by throwing away some ballast, in the form of sand or paper. The buoyancy will then be in excess of

$(W_1 + W_2)$, and the balloon will rise until, by the rarefaction of the atmosphere, a level is reached at which the weight of the displaced air is again equal to $(W_1 + W_2)$. There is a limit to the height which may be thus attained ; at higher levels, the pressure of the atmosphere, acting inwards on the envelope, has diminished, while the pressure of the gas inside the envelope has not changed to any great extent. The difference between these pressures constitutes a bursting pressure which may ultimately tear the fabric of the envelope.

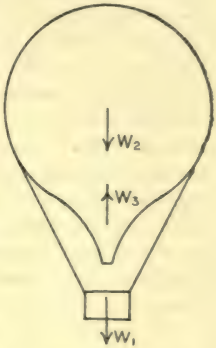


FIG. 255.—Equilibrium of a balloon.

Descent to a lower level may be accomplished by diminishing the volume of the balloon and hence decreasing the volume of air displaced by it ; this is effected by means of a valve at the top of the balloon, operated by a rope from the car. Opening the valve permits some of the gas to escape and thus diminishes the volume of displacement. $(W_1 + W_2)$ will now be greater than W_3, and the balloon will sink until, by the increasing density of the atmosphere, the buoyancy again becomes equal to the total weight.

Hydrogen may be used for charging the envelope ; one cubic foot of hydrogen at 0° C. and 1 atmosphere pressure weighs 0·00559 lb. One cubic foot of air under the same conditions weighs 0·0807 lb.

EXAMPLE. Find the total weight which may be raised at a temperature of 0° C. by a balloon having a capacity of 10,000 cubic feet, and filled with hydrogen.

$$\text{Weight of displaced air} = W_3 = 0{\cdot}0807 \times 10{,}000 = 807 \text{ lbs.}$$

$$\text{Weight of hydrogen} = W_2 = 0{\cdot}00559 \times 10{,}000 = 55{\cdot}9 \text{ lbs.}$$

$$\text{Load which may be just raised} = W_1 = W_3 - W_2 = 807 - 55{\cdot}9 = \underline{751{\cdot}1} \text{ lbs.}$$

Coal gas is obtained conveniently and is therefore often used; its density is greater than pure hydrogen, and hence the lifting power is diminished.

It will be noted that the duration of a balloon voyage is limited by the necessity for parting with both ballast and gas in seeking favourable currents of wind at different elevations. The loss of gas causes the envelope to lose its shape, and this difficulty is met in dirigible balloons by having **ballonets**, or small balloons inside the large one, and containing air which is maintained under pressure by a pump. The loss of gas is compensated automatically by the pump forcing more air into the ballonet, and the shape of the envelope is preserved, and the tension of the fabric of which it is made is maintained constant.

Mist, cloud, dew.—Our perceptions regarding the dryness of the atmosphere depend on its hygrometric state, a quantity which may be expressed as the ratio of the mass of water vapour present per cubic centimetre of air, to the mass of water vapour which would be required to saturate one cubic centimetre of air if the temperature remained unaltered. If the air be not saturated with water vapour at a given temperature, it may become so if the temperature falls; less water vapour is required in order to produce saturation at low temperatures. Hence the presence of a cold body in an atmosphere which has not quite reached saturation point may cool the air in its immediate vicinity sufficiently to produce saturation or over-saturation. Some of the water vapour will then condense on the surface of the cold body, producing the phenomenon of **dew**.

Should a large body of air be cooled slowly, saturation point will be reached throughout the mass simultaneously. The atmosphere is charged more or less with particles of dust always, and condensation will take place on each dust particle, giving rise to a mist. The fogs of large towns are due to condensation of this kind on soot particles floating in the atmosphere.

Clouds are caused by similar condensation in the upper strata of the atmosphere; heated air containing water vapour, but above the saturation point, becomes colder as it rises in the atmosphere, and condenses to a mist or cloud on the saturation point being reached.

If a piece of wet muslin be exposed to the atmosphere,

evaporation of the water will take place, accompanied by a fall in temperature. It is on this principle that a well-known type of butter-cooler acts ; the plate containing the butter is covered with a basin and is placed on the top of a soup plate containing water. A piece of wet muslin is spread over the basin and has its ends tucked into the water. Evaporation will proceed and the butter is kept cool. The rate of evaporation, and hence the cooling effect, will depend evidently on the state of the atmosphere as regards saturation. It follows that the temperature produced as compared with the temperature of the atmosphere will afford a means of deducing the hygroscopic state, and this method is adopted in the wet and dry bulb hygrometer. Two thermometers are suspended a few inches apart; one bulb is kept moistened by a piece of muslin wrapped round it and connected by a piece of clean lamp wick to a small basin of water. The other thermometer is kept dry. Readings of both thermometers are taken until steady temperatures are attained, and the hygrometric state may then be obtained from tables constructed from data arrived at by other methods of experimenting.

Ventilation.—The outside atmosphere contains carbonic acid gas to the amount of 3 to 4 parts in 10,000 parts of air. Indoors the proportion is usually higher, especially so if many people be present in a room. An adult person at rest produces about 0·6 cubic feet of carbonic acid gas per hour, and unless this is diluted largely by a plentiful supply of air admitted to the room, the conditions will become objectionable. Should the proportion in a room amount to over 10 parts of carbonic acid gas in 10,000 of air, it will prove a source of discomfort. In ventilation problems a limit of 6 to 7 parts in 10,000 is aimed at, although cases are on record of theatres and other public buildings having proportions up to 48 parts of carbonic acid gas in 10,000 of air. To dilute the gas exhaled by an adult requires from 1,800 to 3,600 cubic feet of air per hour, depending on the particular conditions existing in the building.

In ordinary houses open doors and windows in summer generally afford the solution of the problem of admission of sufficient air. In winter the current of air drawn up the chimney of an open fire-place is very helpful ; it will, however,

be observed that this action will be more energetic the hotter the chimney ; hence if more people be in the room, assisting by their presence to raise the temperature, the provision of additional air by chimney ventilation will have to be accompanied by a hotter fire.

Special methods of ventilation may consist of the provision of openings in the ceiling leading to ducts through which the heated vitiated air escapes. Such openings should be accompanied also by ducts and openings for distributing the incoming fresh air so as to ensure that all parts of the room are supplied without the production of excessive draughts in any part.

In mechanical methods of ventilation, power-driven fans are used for impelling the air. These may be of the exhausting type, placed in the exit opening of the room, or a fan may be used in order to blow fresh air into the ducts leading to the room. The latter method, if properly carried out, has much to commend it. The fan may be placed in a chamber away from the room to be ventilated ; incoming air may be warmed by passing it over pipes through which steam or hot water circulates, it may be cleansed by filtering, and brought to a proper hygrometric state by passing over water or by water being sprayed into it. The success of such installations, however, is often spoiled completely by the arrangement of inlet ducts producing strong draughts. As has been noted above, these ducts should be so arranged as to distribute the air throughout the room, and the speed of flow should be reduced very low before admission.

Thermostats.—Any device intended to maintain constant the temperature of a body, or of a space, is called a thermostat. If the desired temperature is above that of the surroundings, heat must be supplied, and the thermostat may operate by supplying heat in excess of that required and removing this excess at constant temperature. A space may be maintained at a temperature lower than that of the surroundings by the abstraction of heat at constant temperature. A temperature of 32° Fah. may be maintained by surrounding the space with melting ice, the temperature of which will remain constant provided that too much of the water is not allowed to remain. Examples of this method are illustrated on pp. 182 and 201.

An ordinary glue-kettle (Fig. 256) is an example of a thermostat in which the boiling of one liquid controls the temperature of another substance. The pot A contains water and is closed by the inner pot B containing glue. The water in A is brought to boiling point, and the steam coming from it is condensed on the upper walls of A and on the outer skin of B, giving up latent heat at constant temperature which passes through the walls of B into the glue. The steamer used for cooking food operates on precisely the same principle as the glue-kettle.

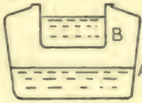


FIG. 256.

The steam jacket sometimes supplied to steam engine cylinders (p. 342) illustrates the same method applied on a large scale.

Other types of thermostats aim at controlling the supply of heat by regulating the supply of fuel; elevation of temperature in the space which is to be maintained at a nearly constant temperature is used in order to operate a device for reducing the supply of fuel and *vice versa.* It will be noted that any appliance of this kind can operate only after some rise or fall of temperature has taken place, and hence the temperature in the space will not keep quite steady, but will oscillate about a mean temperature. The changes may be kept very small by the use of a suitable device. The method is analogous to the results achieved by the centrifugal governor of a steam engine (p. 313) which so controls the supply of steam as to keep the speed oscillating slightly above and below the mean speed.

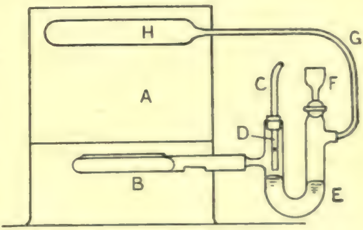


FIG. 257.—Thermostat for controlling a gas supply.

An arrangement for automatically controlling the temperature of a small oven A is shown in Fig. 257. The oven is heated by gas jets B supplied with gas from a rubber tube C connected to a metal tube D. D passes through

a tightly fitting cork in one limb of a U tube E, which contains some mercury. The U tube has its other limb closed by a cock F, and is connected by a fine bore metal tube G, to a vessel H placed in the oven. H contains air, or a liquid may be used. Rise in temperature of the oven above the desired temperature will cause the air in H to expand, and will thus cause the surface of the mercury to approach the mouth of the tube D, thus restricting the supply of gas to the burner, or cutting it off entirely should the mercury cover the mouth of D. The consequent cessation of flame at B may be prevented by having a small hole in the side of D, sufficiently large just to pass enough gas to keep the burner alight. Adjustment of the action may be controlled by raising or lowering D, by pushing it through the cork, or by use of the tap at F, which enables the quantity of air in H and the spaces connected to it to be adjusted.

Distillation.—Distillation processes consist in first converting a liquid into vapour by heating it, and then reconverting it

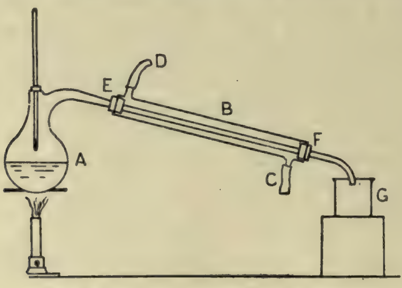


FIG. 258.—Distillation apparatus.

into the liquid form by abstracting heat from it. The essential appliances are a retort or still A (Fig. 258), containing the liquid to be distilled, and means of heating it, together with a condenser B, which may consist of an outer tube, through which the discharge tube EF from the still passes ; cold circulating water enters the space between the tubes at C and is discharged at D ; the discharged condensed products are caught in a vessel G. For scientific purposes, the retort may be fitted with a thermometer.

Distillation affords a ready means of separating a liquid from undesirable constituents. Thus, it is very objectionable to feed sea water into the steam boilers on board a ship. A supply of fresh water is rendered available for this purpose by means of a distilling plant, in which sea water is heated in a still by means of a coil of copper pipe, through which steam from the boiler passes and gives up its heat to the surrounding sea water. The vapour thus driven off from the sea water may be led through a pipe into the main condenser, in which the steam from the engines is condensed, and will there be condensed and fed into the boiler as fresh water. The salts contained in the sea water remain in the still and require to be cleaned out occasionally.

Turpentine is manufactured by distilling a viscous product obtained from the bark of pine trees. Common turpentine is thus obtained from the Scotch fir, and Venice turpentine from the larch. The raw material is a solution of common rosin in oil of turpentine. In the operation of the still, the oil of turpentine comes off as a vapour, leaving the rosin in the still, and is condensed into ordinary commercial spirits of turpentine.

Various useful products are obtained from crude petroleum by distillation processes. Crude petroleum contains a number of constituents having different temperatures of vaporization. The still is heated gradually ; at low temperatures the lighter constituents, such as naphtha and illuminating oils, are first given off and condensed. As the temperature is increased, constituents such as are useful for the manufacture of lubricating oils come off and are condensed. Paraffin wax and vaseline are the products of the later stages of distillation at high temperature ; in these stages superheated steam is employed in order to prevent overheating and damage to the material. The residue left in the retort is called astatki in Russia.

Expt. 64.—Arrange apparatus as shown in Fig. 258. Prepare a solution of common salt in distilled water, noting the quantities used. Weigh the retort and introduce the solution. Start the distillation process, using a graduated flask to catch the condensed water. Stop the process when about 10 per cent. of the water in the retort has been driven off ; weigh the condensed water. Weigh the retort and its contents together, deduct the

weight of the retort and also the known weight of water left in it. Compare the result with the weight of salt used in making the solution. Has any salt passed out of the retort during the distillation process?

Melting point.—The temperature at which a substance passes from the liquid into the solid state may be determined by simple methods when the temperature is low.

Expt. 65.—Determine the melting point of paraffin wax by dipping a thermometer bulb into some melted wax. Keep the bulb in the wax for a few seconds, then withdraw it and spin the thermometer between the hands until the film becomes opaque; note the temperature. Dip the bulb into warm water and gradually increase the temperature until the film disappears. Again note the temperature. The mean of these will be the desired melting point.

Expt. 66.—Determine the melting point of sulphur by enclosing a small portion in a piece of capillary tube and tying it to a thermometer bulb. Fit a cork, bored to receive the thermometer, to a test tube; insert the thermometer, and heat gradually the test tube until melting occurs. The temperature shown by the thermometer will be the melting point nearly.

The freezing point of a solution is always lower than that of the solvent; for example, the freezing point of a salt solution, such as sea water, is lower than that of pure water. The freezing point of an aqueous solution of salt may be lowered to -22° C., and of crystalline calcium chloride and ice to -55° C. Such substances are known as freezing mixtures, and are useful in the laboratory for experimental purposes. They are of little importance for the commercial production of low temperatures.

The freezing point of some substances is influenced by the pressure to which they are subjected. Water and iron, both of which expand in solidification, have their solidifying points lowered by increase in pressure. Paraffin wax contracts in solidifying, and an increase in pressure raises the solidifying point.

Refrigerating machines.—The earliest commercially successful refrigerating machines, used for transporting mutton, operated by taking advantage of the heating and cooling that occurs when a gas is compressed and expanded. Fig. 259 shows

in diagram form the action of the Bell-Coleman refrigerating machine. A is the room which is to be kept at a low temperature. A pump B draws air from this room and compresses it; during this operation the air rises in temperature and is delivered hot, and at a pressure of 50 or 60 lbs. per square inch, into a pipe coil C, which is kept cool by means of water circulating round it. The air, thus cooled, is fed from C into a motor cylinder D. Here it does work on the piston, thus assisting the pump B, and is expanded to a lower pressure, falling in temperature as it does so. Finally it is discharged from D at low temperature into the refrigerator chamber A, where it again takes up some heat from the walls of the chamber and from the articles being chilled. A steam engine or other motive engine is used for driving the pump.

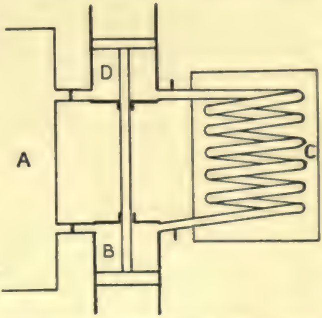


FIG. 259.—Arrangement of the Bell-Coleman refrigerating machine.

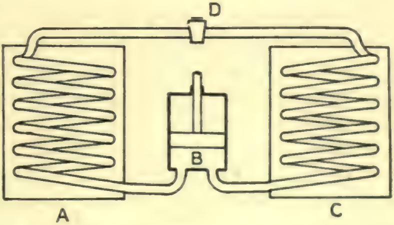


FIG. 260.—Arrangement of a refrigerator using a vapour.

In modern refrigerating machines, vapour is used as the working fluid, and alternately is condensed into the liquid form and evaporated again into vapour. The action depends on the giving out and taking up of latent heat, and is illustrated in diagram form in Fig. 260. The vessel A contains a pipe coil which is surrounded by the substance to be chilled—usually

brine. The working fluid is allowed to evaporate in this coil, and in doing so obtains its latent heat of evaporation at the expense of the heat in the brine, which accordingly is chilled further. A pump B draws the resulting vapour from the coil in A and compresses it, the temperature rising as the pressure increases; the pump then discharges the compressed vapour into another pipe coil contained in a vessel C, which is kept cool by means of circulating water. The pressure being high, and the temperature being lowered in this pipe coil, the vapour condenses again, giving up latent heat to the circulating water. The resulting liquid is fed through a regulating valve D, into the pipe coil in A, where, the pressure being much lower than that in C, evaporation takes place again. The chilled brine is taken from the vessel A through pipes, and is circulated through the room which has to be maintained at low temperature, becomes heated somewhat during its passage by abstraction of heat from the walls and substances in the room, and is returned again to A to be chilled further. Pumps are used in order to keep the brine circulating.

Three substances have been used in the operation of refrigerating machines, viz. anhydrous ammonia, NH_3, carbonic anhydride (carbonic acid), CO_2, and sulphurous anhydride, SO_2. It is obvious that water cannot be employed, as it would freeze at the low temperatures required. The substances mentioned possess physical properties which render them suitable as refrigerating agents. Curves are given exhibiting these properties in Figs. 261, 262, and 263;* these correspond to those in Chap. XV. for saturated steam.

It will be noticed in Fig. 261 that the pressure of SO_2 vapour becomes very low at temperatures below the freezing point of water; at $-22°$ F. the pressure is 5·54 lbs. per square inch absolute, and the volume occupied by one pound weight is 13·177 cubic feet. It follows that at refrigerating temperatures, there is a possibility of air leaking into the apparatus and thereby interfering with the working; also the dimensions of the machine must be large, as the volume of the vapour is large. SO_2

*Figs. 261-3 have been plotted from Tables given in a paper on Refrigerating Machines by Dr. J. H. Grindley; *Proc. Inst. Mech. Eng.*, November, 1912.

machines are convenient, however, for dairy installations in which the working temperatures are not very low. NH_3 vapour has a pressure of 16·93 lbs. per square inch absolute at $-22°$ F., and gives a more compact machine. CO_2 vapour has a pressure of 213 lbs. per square inch absolute at $-22°$ F., and gives a very compact machine. The very high working pressures in the CO_2

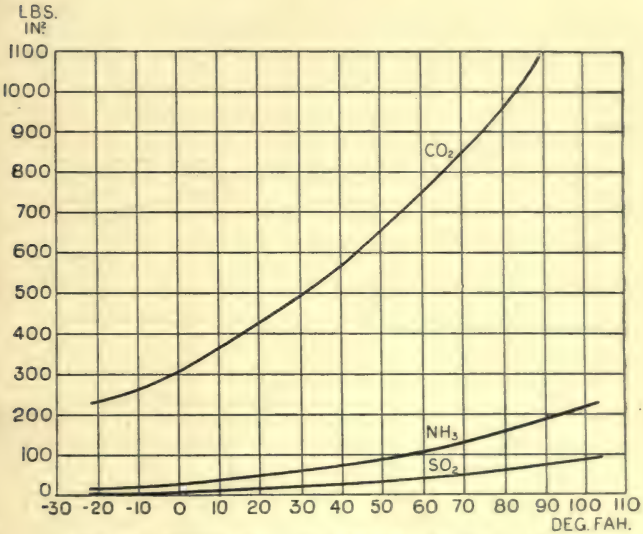


Fig. 261.—Pressure-temperature curves for CO_2, NH_3 and SO_2.

machine offer no vital objection in practice. CO_2 machines are used very largely on board vessels carrying meat, etc.; NH_3 machines are much used for land refrigerators.

A refrigerating machine may be looked upon as a heat engine reversed. In an ideal heat engine heat is taken in at a high constant temperature, and the unused heat is discharged at a constant lower temperature. In the ideal refrigerating machine heat is taken in by the working fluid at a constant low temperature τ_2, effected in the refrigerating coil, and this heat, together with that equivalent to the work done by the

compressing pump, is discharged in the cooling coil at constant temperature τ_1. The coefficient of performance, corresponding to the efficiency of a perfect heat engine (p. 343), is defined by

$$\text{Coefficient of performance} = \frac{\text{Heat taken in from the cold body}}{\text{Heat equivalent of work done}}$$

$$= \frac{\tau_2}{\tau_1 - \tau_2},$$

where τ_1 and τ_2 are the absolute temperatures of the cooler C and of the refrigerator A respectively (Fig. 260). The fraction

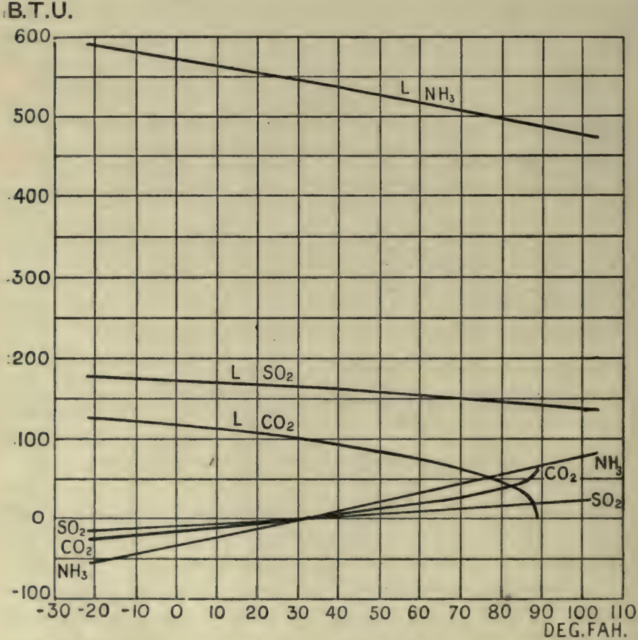


FIG. 262.—Latent and sensible heats of CO_2, NH_3 and SO_2. The curves marked L show the latent heat.

is always greater than unity, and will be greater the smaller the difference between τ_1 and τ_2; this leads to the important

practical conclusion that the temperature range in the whole machine should be small in order to obtain efficient working.

EXPT. 67.—Stand a beaker containing some ether in a small quantity of water on the bench. By means of a glass tube and foot-bellows, assist the evaporation of the ether by blowing air through it. The rapid removal of latent heat thus produced will cause the water to freeze quickly.

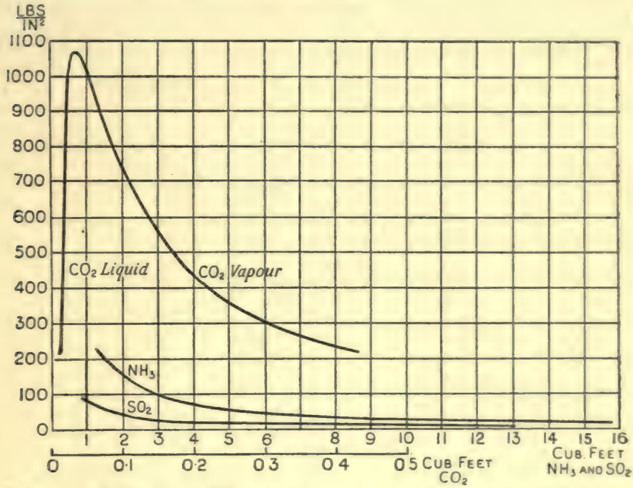


FIG. 263.—Pressure-volume curves for CO_2, NH_3 and SO_2.

Reversed heat engine as a warming machine.—A reversed heat engine working with air may be used as a means of heating the air in a building; the method was suggested by Lord Kelvin. Let an air pump take in air at ordinary atmospheric temperature and pressure, and let the air be expanded doing work. The pressure and temperature will fall; allow sufficient time to elapse so that the air may come again to the temperature of the atmosphere by heat flowing in from the outside. Compress the air again to atmospheric pressure and discharge it into the room to be heated; it is evident that the temperature will rise during compression, and that the air will be discharged at a temperature higher than that of the atmosphere.

Liquefaction of gases.—Any gas may be liquefied by cooling and pressure to extents which depend on the kind of gas. Thus in Andrews's experiments on carbonic acid gas, CO_2, it was found that the gas could be liquefied by application of pressure provided that its temperature does not exceed 30·92° C.; at this temperature, a pressure of approximately 73 atmospheres is required. The corresponding temperature in the Fahrenheit scale is 88·43° and the pressure is 1074 lbs. per square inch.

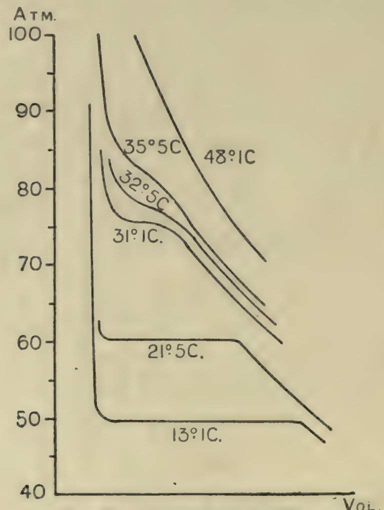


FIG. 264.—Isothermals for CO_2.

Fig. 264 shows a few pressure-volume curves for constant temperatures for CO_2. In such curves the presence of a horizontal line implies the addition or removal of latent heat, *i.e.* evaporation, or liquefaction. It will be noted that liquefaction may be obtained at 13·1° C. with a pressure of roughly 50 atmospheres. No liquefaction will occur if the temperature exceeds 30·92° C., a temperature which is called the **critical temperature** for this gas. The critical temperature of ammonia is 130° C. At temperatures higher than the critical value, the substance behaves more and more nearly like a perfect gas.

The problem of obtaining sufficient pressure in liquefying a gas may be solved practically by the employment of a pump; the problem of lowering the temperature sufficiently may be solved by taking advantage of the latent heat taken up when a liquid is allowed to evaporate. Carbonic acid gas may be liquefied by simple application of pressure by means of a pump at ordinary atmospheric temperatures. Fig. 265 shows in diagram form Pictet's method of liquefying oxygen. In this method advantage is taken of the ease with which sulphur

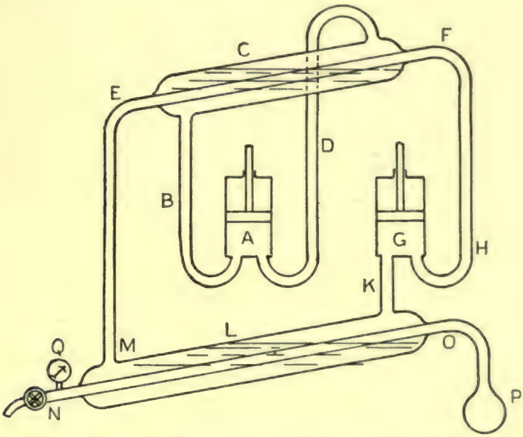


Fig. 265.—Pictet's method for liquefying oxygen.

dioxide (SO_2) may be liquefied. A pump *A* draws SO_2 vapour from a tube *C*, forming a jacket round another pipe *EF*, compresses the vapour and delivers it back to *C* as liquid SO_2. Evaporation of this liquid takes place in *C*, and, by the taking up of latent heat, chills the pipe *EF* and its contents.

Another pump *G* draws CO_2 vapour through *K* from a tube *L*, which forms a jacket round another tube *NO*; the vapour is compressed in *G* and is delivered through *H* into *FE*; here the chilling which it undergoes causes it to condense into the liquid form, and it is returned as liquid CO_2 into *L* through the pipe *EM*. Evaporation of the CO_2 in *L* takes place during the

suction stroke of the pump G, and the consequent taking up of latent heat will chill the contents of the pipe ON. The complete arrangement as described constitutes a two-stage cooling device for chilling the pipe ON.

P is a strong steel vessel connected to ON and containing potassium chlorate, which gives off oxygen when heated. The pipe ON is closed by a valve at N, and has a pressure gauge fitted at Q. On heating P, oxygen is given off, and, as the pipe is closed, a great pressure of gas is produced. This, combined with the chilling of ON, as described above, is sufficient to liquefy the oxygen, and this occurred at a pressure of 500 atmospheres.

Using an improved form of this apparatus, Dewar has liquefied both oxygen and air in bulk. The resulting liquids may be stored in a vacuum vessel devised by Dewar, consisting of a double-walled glass vessel, the space between the walls being exhausted of air. The traces of mercury vapour left between the walls condenses into a mirror surface on the glass and assists the heat insulation of the vessel. A similar plan is employed in the vacuum flasks which are in common use for the transportation of hot or cold liquid.

Joule and Lord Kelvin proved by experiment that, when certain gases are expanded from one pressure to another by passage through a porous plug of cotton-wool, a small fall in temperature occurs. Thus, in the case of air, a fall in temperature of about 0·25 degree Centigrade takes place for each atmosphere difference in pressure on the two sides of the plug. Advantage is taken of this fact in Dr. Linde's apparatus for liquefying air ; air which has been chilled slightly by expansion through a small orifice is used to cool more air approaching the orifice, and the same air is expanded again and again through the orifice and undergoes successive cooling during each passage. One form of the arrangement is shown in diagram form in Fig. 266. A pump A compresses air and delivers it through a pipe B into a cooler C, where its temperature, raised during compression, is lowered. The air passes from the cooler into a pipe DE, having a throttling valve at E, by means of which the air experiences a drop in pressure and expands into a box F. The air, thus cooled somewhat, passes upwards through a pipe G

which encloses the pipe DE and abstracts some of the heat from the air passing through DE on its way to the orifice. The air is then drawn through the pipe H and is compressed again by the pump. The action is thus continuous, and, if maintained long enough, will result in liquid air accumulating in the box F. A valve L enables the liquid to be withdrawn. The pipes G and DE are enclosed in a case K, the space being packed with heat-insulating substance, and constitute an interchanger.

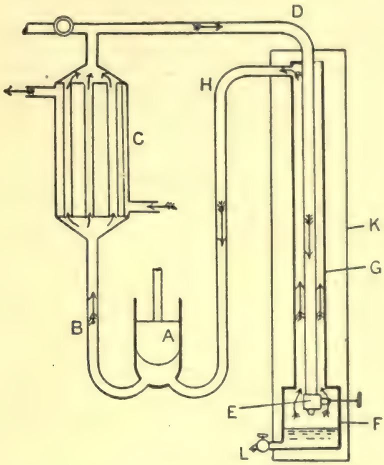


FIG. 266.—Linde apparatus for liquefying gas.

Usually a two-stage air compressor is used, consisting of two pumps, one of which delivers air partly compressed into the other. The air is cooled on its passage from pump to pump, and the second pump delivers the air into a cooler surrounded by a mixture of ice and salt. The interchanger may consist of three tubes arranged concentrically and coiled into a spiral. Liquid air may be obtained in a few minutes by the use of apparatus thus modified; the production is accelerated if carbonic acid snow is used for preliminary cooling.

EXERCISES ON CHAPTER XVI.

1. What is the maximum height that a common lift pump may be placed above the level of the supply water?

2. A pump, the diameter of the plunger being 2″, forces water against a pressure of 700 lbs. per square inch. If its stroke is 6 inches, and it makes 40 effective strokes per minute, how much work is done per minute? Suppose the efficiency to be 60 per cent., what horse-power is absorbed in driving the pump?

3. Sketch in section, and describe the action of, the ordinary lifting pump. In such a pump the pump rod is $\frac{3}{4}$ inch in diameter, and the pump barrel is 5 inches in diameter, while the spout at which the water is delivered is 20 ft. above the surface of the pump bucket when the latter is at its lowest point; what would be the maximum tension on the pump rod in the upstroke of the pump, neglecting the weight of the pump rod and the pump bucket (the weight of a cubic foot of water is 62·5 lbs.)?

4. If, in an ordinary suction pump for raising water from a well, 200 gallons of water are raised per hour from a depth of 20 feet, and if the efficiency of the pump is 60 per cent., what horse-power is being given to the pump?

5. A bicycle tyre has a capacity of 150 cubic inches when fully inflated, the pressure being then 25 lbs. per square inch by gauge. Supposing the tyre to be quite flat at first, calculate what volume of atmospheric air must be used in order to inflate it.

6. An air receiver has a capacity of 16 cubic feet and contains air at an absolute pressure of 15 lbs. per square inch. How many cubic feet of air at absolute pressure must be pumped into it in order to attain a pressure of 90 lbs. per square inch absolute? Assume that the temperature does not alter. Sketch an approximate pressure-volume diagram for the first three strokes of the pump.

7. A vessel to be exhausted of air contains 100 cubic inches at a pressure of 30 inches of mercury absolute. The air pump is similar to that shown in Fig. 247, and removes 4 cubic inches of air during the first upward stroke. What will be the pressure in the vessel at the end of the third upward stroke?

8. Sketch and describe the action of a piston air pump used for exhausting a vessel.

9. Describe with sketches the action of a mercurial air pump.

10. Explain, with sketches, the method whereby workmen may excavate foundations at the bottom of a river.

11. Give a brief description of the method used for transmitting packages by the use of pneumatic tubes.

12. Describe with sketches the action of the vacuum brake used on railway trains.

13. Give a brief description of atmospheric circulation, explaining especially land and sea breezes, trade winds, and monsoons.

14. Explain how heights above sea level may be obtained roughly by use of a barometer.

15. A balloon has a capacity of 55,000 cubic feet and is filled with hydrogen gas at atmospheric pressure. Find what total weight may be lifted at a temperature of 60° F. At freezing point and one atmosphere pressure, air and hydrogen weigh 0·0807 and 0·00559 lb. per cubic foot respectively.

16. Explain the formation of dew, fog, and clouds. What is meant by the dew point? How is it measured?

17. Explain briefly how we may deduce the quantity of fresh air per adult person which must be introduced per hour into a room.

18. State the conditions which must be complied with in maintaining at uniform temperature a vessel in which an experiment is being conducted. Deal with both cases (*a*) temperature lower, (*b*) temperature higher than the surroundings.

19. Sketch and describe the action of a mechanical thermostat for controlling the supply of gaseous fuel to a burner.

20. Give a brief description, with sketches, of any distillation process. Explain how the different constituents of petroleum are separated.

21. How would you find the melting point of a substance such as sulphur?

22. Describe the action of the Bell-Coleman refrigerating machine. Give an outline sketch.

23. Explain, with sketches, the action of a refrigerator using the vapour of some liquid, such as carbonic acid.

24. How may the freezing of water be carried out on a small scale in a laboratory?

25. A reversed heat engine may be used as a warming appliance. Explain this.

26. Describe a method of liquefying oxygen. Give sketches. What is meant by the critical temperature and pressure?

27. Explain, with sketches, the Linde process for the liquefaction of air.

CHAPTER XVII.

FUELS.

Combustion.—Substances which are mixed with one another may, in general, be separated easily by some mechanical process. For example, a mixture of sugar and sand can be separated by placing the mixture in water, stirring up and then allowing to settle. The sand will sink to the bottom, and the water in which the sugar will have dissolved may be drained off. Gentle evaporation will drive off the water and the sugar may be recovered.

Substances in **chemical combination** with one another cannot be so separated, such bodies being known as **chemical compounds.** Water is the commonest example of a chemical compound, being composed of definite proportions of hydrogen and oxygen chemically united with one another. Air is a mixture consisting chiefly of oxygen aud nitrogen. Many substances possess constituents which are able to unite in chemical combination with the oxygen of the atmosphere, the process being accompanied by the evolution of heat and light. This operation is called **combustion.** Substances which are suitable for the supply of heat to be used in commercial operations are called **fuels.** The principal combustible constituents of fuels are **carbon** and **hydrogen** together with compounds of these bodies called **hydrocarbons.**

Some important definitions.—An **element** in chemistry is a substance which has never been separated into two or more substances. Examples of elements are hydrogen, oxygen, carbon, copper, and iron. All substances are considered to be composed of very small particles, called **atoms**, which are

regarded as indivisible. A **molecule** consists of a group of atoms, and forms the smallest possible portion of a substance capable of independent existence. For example, a molecule of water is composed of two atoms of hydrogen and one atom of oxygen. **In all gases, the same number of molecules exist per cubic foot under similar conditions of pressure and temperature.**

By chemists, the elements are denoted generally by writing the initial letter only as a capital. Thus H stands for hydrogen, O for oxygen, C for carbon, one atom of each being denoted by the symbol. H_2 means 2 atoms of hydrogen, other numbers of atoms being indicated in the same way. If the weight of an atom of hydrogen be taken as 1, the atomic weights of some other elements are as follows:

Name of element.	Chemical symbol.	Atomic weight.
Hydrogen	H	1
Carbon	C	12
Nitrogen	N	14
Oxygen	O	16
Phosphorus	P	31
Sulphur	S	32

The constitution of chemical compounds is indicated by their **chemical formulae.** Thus the chemical formula of water is H_2O. This indicates not only that 2 atoms of H and 1 atom of O are present in the molecule, but that the weights are in the proportion of 2 parts of H to 16 parts of O. The molecules of hydrogen, oxygen, and other simple gases, are each made up of two atoms. Thus, the molecule of hydrogen is H_2 and of oxygen O_2. The molecule of water, H_2O, will occupy the same volume as that of a molecule of hydrogen, H_2. No element can be made to combine with any other elements in any proportion by weight other than its atomic weight or even multiples of its atomic weight.

Chemical equations afford the means of representing the combination of various elements or compounds with the

formation of new compounds. Such an equation representing the formation of water would be

$$H_2 + O = H_2O.$$

We may read this equation as:

(*a*) Two atoms (one molecule) of hydrogen when combined with one atom of oxygen give one molecule of water;

(*b*) Two parts by weight of hydrogen when combined with sixteen parts by weight of oxygen give 18 parts by weight of water. (Notice that as nothing can be destroyed in any chemical operation, the weights on the two sides of the equation must be equal.) The gram, or the pound avoirdupois, may be used as the unit of weight.

In reading (*a*), volumes may be substituted for molecules, because all gases at the same pressure and temperature have the same number of molecules per cubic foot. Thus, if one cubic foot of oxygen be used, there will be a certain number of oxygen molecules, and, to obtain double this number of hydrogen molecules at the same pressure and temperature, two cubic feet of hydrogen must be used, so that the equation may be read:

(*c*) Two cubic feet of hydrogen when combined with one cubic foot of oxygen at the same temperature and pressure give two cubic feet of gaseous water.

The student should be cautioned here to write always **water**, not H_2O, when he means the liquid. H_2O may exist as ice, water, or steam.

Other chemical equations useful in the study of fuels are

$$C + O = CO.$$

We may take this equation to mean that 12 lbs. of C when combined with 16 lbs. of O give 28 lbs. of CO. CO, **carbon monoxide**, or **carbonic oxide**, is a highly poisonous gas produced by the partial combustion of carbon with a limited supply of oxygen. To burn 1 lb. of carbon until it is converted completely into carbon monoxide requires, as indicated by the above proportions, $1\frac{1}{3}$ lbs. of oxygen.

Carbon also combines with double the quantity of oxygen, the reaction being expressed by the equation

$$C + O_2 = CO_2.$$

Expressing this equation as weights, the student will find that to burn 1 lb. of C to CO_2 requires $2\frac{2}{3}$ lbs. of O. CO_2, known as **carbon dioxide**, or **carbonic acid gas**, is the gas produced by the complete combustion of carbon, which may be secured by a plentiful supply of oxygen. It will not support combustion, and produces death in animals by suffocation.

CO is combustible, producing CO_2 as represented by the equation

$$CO + O = CO_2.$$

The combustion of sulphur is represented by the equation

$$S + O_2 = SO_2,$$

the compound SO_2 is known as **sulphur dioxide.**

The student should express in weights the reactions represented in the two above equations for himself.

CH_4, **methane** or **marsh gas**, and C_2H_4, **ethylene** or **olefiant gas**, are the most important hydrocarbons with which we have to deal.

Rates of combustion.—Hydrogen, when mixed with its proper proportion of oxygen, may be exploded with violence. Hydrocarbon gases are also explosive when mixed with oxygen. Carbon burns fairly slowly unless it is powdered finely and mixed as a dust with oxygen, in which case an explosion may be produced. Carbon monoxide, CO, is also explosive when mixed with oxygen.

Atmospheric air.—The atmosphere forms the principal supply of oxygen necessary to support combustion. The proportions of oxygen and nitrogen in the atmosphere are roughly 4 parts of nitrogen to 1 part of oxygen by volume, or 56 parts of nitrogen to 16 parts of oxygen by weight. More accurately, the proportions are

79 of N to 21 of O by volume;

77 of N to 23 of O by weight.

Taking the rough numbers, it will be seen that if we require one cubic foot of oxygen in any combustion we must supply 5 cubic feet of air. If 16 lbs. of oxygen are required, we must supply $(16+56)=72$ lbs. of air, or $\frac{72}{16}=4\frac{1}{2}$ lbs. of air must be supplied if 1 lb. oxygen is required, or 4·35 lbs. using the more exact data.

Nitrogen does not aid or hinder combustion in any way other than it gets heated in the process, and so carries off a considerable portion of the heat developed, thereby preventing a higher temperature from being attained.

Air required for combustion.—The quantity of air required for the combustion of a substance may be estimated from its chemical composition. The result may be called the **theoretical quantity** of air. Actually, from 50 to 100 per cent. more must be given to ensure complete combustion.

EXAMPLE. Calculate what weight of air must be supplied for the complete combustion of 1 lb. of carbon.

$$C + O_2 = CO_2.$$

12 lbs. carbon require 32 lbs. oxygen.

1 lb. carbon requires $2\frac{2}{3}$ lbs. oxygen.

For 1 lb. oxygen, $4\frac{1}{2}$ lbs. of air are required.

$$\therefore \text{ Air required} = 2\tfrac{2}{3} \times 4\tfrac{1}{2}$$
$$= \underline{12} \text{ lbs.}$$

In practice, from 18 to 24 lbs. of air would be required.

Heating values.—The heat available from the combustion of 1 lb. of a fuel is called its **heating value.** The heating value of hydrogen may be taken as 62,000 B.T.U. or 34,500 lb.-deg.-Cent. units; and of carbon 14,500 B.T.U. or 8040 lb.-deg.-Cent. units when completely burned to CO_2. Carbon burned to CO gives 4450 B T.U. or 2470 lb.-deg.-Cent. units. CO burned to CO_2 gives 10,080 B.T.U. or 5600 lb.-deg.-Cent. units. Sulphur has a heating value of 4060 B.T.U. or 2250 lb.-deg.-Cent. units.

Fuels.—Fuels are either solid, liquid, or gaseous. The principal fuels in use are **coal** (solid), **petroleum** and **paraffin oil** (liquid), **lighting gas** and **power gas** (gaseous).

The value of a fuel for steam raising purposes is estimated from its heating value, percentage of volatile constituents and of ash, ease with which it can be handled, stored and fired, and its cost.

Solid fuel.—Coal consists of mineralised vegetable matter. The vegetation of past ages being buried in the earth undergoes compression and slow mineralisation; the first product is **lignite,**

a coal which ranges in colour from yellow to dark brown, and is still rich in volatile constituents. Further mineralisation produces **bituminous coal**, a fuel which contains a large proportion of volatile matter. **Anthracite** is a coal produced by the elimination of the volatile constituents, and is the most perfectly mineralised coal we have. **Semi-bituminous** and **semi-anthracite** are coals intermediate in composition.

Principal constituents of coal.—These are carbon, hydrogen and oxygen. The best qualities of anthracite contain more than 90 per cent. of carbon ; bituminous coal contains from 50 to 60 per cent. of carbon and up to 30 per cent. of volatile matter. Semi-bituminous coals have proportions of carbon and volatile matter between these. The percentage of oxygen ranges from about 1·5 in the best anthracite to 30 or 40 in the poorest lignite. Ash varies from 3 per cent. in the highest classes of coal to 25 or 30 per cent. in the poorest.

The volatile matter in coal is composed chiefly of hydrocarbons. These make the coal easy to burn, and facilitate the production of flame. To burn hydrocarbons completely a high temperature and a plentiful supply of air are required simultaneously. Failure to secure these conditions with bituminous coal will cause black smoke to be formed. Much ash causes the furnaces to become dirty, and clinker may be formed by the fusing of the ash, with the consequent choking of the air spaces in the furnace. Sulphur in coal is deleterious to the furnace plates.

Anthracite is usually jet black in colour and is very clean. Sudden application of heat causes it to break up into small pieces. It is best worked with light fires and forced draught, and the furnace should be of ample capacity, as the heat is very intense. There is no smoke produced during the combustion.

Semi-anthracite and **semi-bituminous** coals are much used for steam raising purposes. The best Welsh coal contains just sufficient volatile matter to secure easy combustion without the production of black smoke. Other qualities require more care in the stoking to secure this result.

Bituminous coals are useful for the manufacture of gas, for which purpose the large quantity of volatile matter present makes them valuable.

Lignite may contain from 60 to 70 per cent. of carbon. The poorer qualities, containing as little as 30 per cent. of carbon, are not suitable for use in boiler furnaces.

Other solid fuels are **coke**, produced from coal by destructive distillation so as to drive off the volatile constituents—coke is the residue, consisting of carbon and ash ; **wood** ; **charcoal**, produced from wood by driving off moisture and volatile matter, leaving practically pure carbon ; **peat**, which is the remains of comparatively recent vegetation found in bogs.

A few heating values of solid fuels are here given.

HEATING VALUES OF SOLID FUELS.

Fuel.	Heating Value, B.T.U. per lb. of fuel.
Best Welsh coal, - -	16,000
Average Newcastle coal, -	14,900
Average Derbyshire coal, -	13,900
Average Lancashire coal, -	13,900
Average Scotch coal, - -	14,200
Lignite, - - - -	11,500 to 14,000
Fairly dry peat, - -	10,000
Coke, - - - -	13,000
Wood, - - - -	about ½ that of coal

Liquid fuel.—Mineral oils suitable for fuels are obtained as (*a*) **crude petroleum**, (*b*) **paraffin oil.** These oils consist of mixtures of various hydrocarbons.

Crude petroleum is discharged from natural reservoirs in the earth's crust through wells. The bulk of the supply comes from the United States and Russia. Crude petroleum is refined by a process of distillation described on p. 269. Gasoline, burning oils, oils suitable for gas making, lubricating oils, paraffin wax, etc., are thus produced. Crude petroleum and the residue from the distillation process may be used instead of coal in boiler furnaces for steam raising purposes. The light gasoline oils and the heavier burning oils are suitable for use in the cylinders of internal combustion engines.

Oils of low specific gravity (0·6 to 0·75) such as **gasoline,**

petroleum spirit, benzoline require careful handling, as at ordinary atmospheric temperatures they continually give off inflammable vapour. A light brought near any of them, but not in actual contact, will cause the oil to take fire. Special precautions must therefore be taken in their storage, and in no circumstance should a naked light be introduced to the storeroom. Oils of this class are used largely for motor-cars and for this purpose are very suitable, being brought very easily into the state of vapour. The vapour, when mixed with a proper proportion of air, forms an explosive mixture to be used in the engine cylinder.

The heavier burning oils, specific gravity 0·78 to 0·828, are safer to handle. Oils such as Royal Daylight (American) and Russolene (Russian) do not give off inflammable vapour at ordinary atmospheric temperature. These oils may be vaporised by first finely dividing them by spraying, and then heating the spray. The resulting vapour, when mixed with air, gives the required explosive mixture.

Paraffin oil is manufactured by distilling bituminous shales and boghead coal.

Flash point.—The temperature at which an oil begins to give off inflammable vapour is called its **flash point**. In this country, the legal test is performed in the Abel apparatus. The apparatus consists of a cup to contain a certain quantity of the oil under test when filled up to a standard mark. The cup is surrounded by a bath of water which is adjusted at 130° F. before the cup is placed in position. The cup is closed by a cap, furnished with a small slide which may be drawn open by hand, the same action lowering a tiny gas jet into the interior of the cup. A pendulum 24″ long is supplied with the apparatus. Three swings of the pendulum give the proper time for opening the slide and one swing for closing it. Thermometers are inserted in the water bath and into the space in the cup just above the oil surface. As the temperature of the oil rises and attains 66° F., the slide is drawn as described, and this is repeated at every subsequent degree until a flame is observed to spread inside the cup when the slide is drawn. The temperature at which this occurs is called the flash point of the oil by Abel's close test. (*Close* because a closed cup is used.) No oil may be

sold for illuminating purposes in this country which has a flash point by this test lower than 73° F. This is equivalent to a flash point of about 100° F. when an open cup is used.

Properties of some well-known oils.—Royal Daylight oil has a flash point of 81° F., Russolene, 88° F., and White Rose (American), 105° F.

The composition of refined petroleum may be taken as averaging 86 per cent. of carbon and 14 per cent. of hydrogen. Russolene oil has a specific gravity of 0·825 and its heating value is about 20,300 B.T.U. per lb. Royal Daylight oil has a specific gravity of 0·797 and a heating value of 20,100 B.T.U. per lb.

Petrol of specific gravity 0·678 has a heating value of 19,800 B.T.U. per lb. Crude petroleum, American, has a specific gravity 0·886, Russian, 0·938. The heating value varies from 19,000 to 19,500 B.T.U. per lb.

Petroleum refuse has a specific gravity from 0·906 to 0·928. Its heating value averages 19,000 B.T.U. per lb.

Gaseous fuel.—The fuels of this class in common use are (*a*) ordinary lighting gas, (*b*) producer gases manufactured specially for power purposes, (*c*) gases produced as by-products from other processes, (*d*) natural gas.

Ordinary lighting gas is produced by heating bituminous coal in closed retorts, when the volatile constituents are driven off, and, after purification, are available for lighting and heating. The contents of the retorts at the completion of the process consist of coke. Lighting gas is enriched often by the addition of oil vapour. The constituents of lighting gas vary considerably in different localities. Ordinary proportions by volume are :

LIGHTING GAS.

Constituent.	Percentage volume.	Constituent.	Percentage volume.
H_2	51·81	CO	8·95
CH_4	35·25	O_2	0·08
C_2H_4	3·53	N_2	0·38

The heating value ranges from 550 to 750 B.T.U. per cubic foot.

The quantity of gas produced from a ton of coal varies from 7000 to 15,000 cubic feet, depending on the kind of coal used. 10,000 cubic feet may be taken as an average yield.

Producer gases.—**Dowson gas** is produced by blowing a mixture of air and superheated steam through incandescent anthracite or coke. This fuel is used to prevent choking of the producer by tarry matter and clinker. The resulting gas is composed chiefly of hydrogen, carbonic oxide, and nitrogen. The average proportions by volume are:

DOWSON GAS.

Constituent.	Percentage volume.	Constituent.	Percentage volume.
H_2	18·73	CO_2	6·57
CH_4	0·31	O_2	0·03
C_2H_4	0·31	N_2	48·98
CO	25·07		

The average heating value is 160 B.T.U. per cubic foot. Modern gas engines use about 75 cubic feet of Dowson gas per indicated horse-power per hour, which works out to a consumption of about 1 lb. of coal per horse-power hour.

Mond gas is produced from bituminous slack by blowing air saturated with steam at a temperature of 70° C. through the coal, which is kept burning at a dull red heat. The combustion is effected without the production of clinker or tarry matter, and the whole process is designed to avoid loss of heat. The average proportions by volume are:

MOND GAS.

Constituent.	Percentage volume.	Constituent.	Percentage volume.
H_2	28	CO_2	15
CH_4	2	O_2	—
C_2H_4	traces	N_2	43
CO	12		

The average heating value is 160 B.T.U. per cubic foot. About 70 cubic feet per indicated horse-power per hour are used in gas engines. About 150,000 cubic feet of gas are obtained from 1 ton of bituminous slack.

Suction gas plants have come into considerable use for supplying gas to small engines of about 10 to 50 H.P. Anthracite or coke is burned slowly in a small producer, using about 1·1 lb. per hour per brake horse-power. The air required to maintain the combustion is drawn through the producer by the action of the engine piston during the suction stroke. The air is heated and charged with water vapour before entering the producer, where chemical action results in a gas being given off containing roughly 29 lbs. of CO, $1\frac{1}{2}$ lbs. H, 57 lbs. N, and 12 lbs. of CO_2 per 100 lbs. The gas is cooled and is drawn into the cylinder, together with the further proportion of air required to form an explosive mixture.

Oil gas is manufactured by treating heavy mineral oils. **Water gas** is produced by blowing superheated steam through incandescent anthracite or coke.

By-product gases.—**Blast-furnace gas** is given off from blast-furnaces during the smelting of iron; this gas may be used in boiler furnaces or in the cylinders of gas engines. The gas given off from coke ovens during the manufacture of coke from bituminous coal may be used also for heating purposes.

Natural gas occurs largely in the United States. Wells are bored until gas is reached, when it is discharged at a high pressure and is available for direct use for lighting and power purposes. The principal constituent is marsh gas. Pittsburg natural gas has an average heating value of 1000 B.T.U. per cubic foot.

Calculation of heating value.—The heating value of a fuel may be calculated from its composition as found by a chemical analysis. To carry out such an analysis requires the services of a highly skilled chemist, and the results obtained from the subsequent calculation are doubtful. The method is to estimate the heat available from each constituent, using the heating value of the element in the calculation. If oxygen and hydrogen are both present, it is assumed that some of the

hydrogen is combined with the oxygen as water, and, as no heat will be available from this part of the hydrogen, which is already completely in combination, it is deducted from the total hydrogen present, thus :

Let O = quantity of oxygen present,
H = „ hydrogen „

Then, since 8 parts by weight of oxygen are required to form water with 1 part by weight of hydrogen,

$$\left(H - \frac{O}{8}\right) = \text{hydrogen available for heating.}$$

Example i. A sample of petroleum contains 86 per cent. of carbon and 14 per cent. of hydrogen by weight. Calculate its heating value.

In 1 lb. of the oil there will be $\frac{86}{100}$ lb. of carbon and $\frac{14}{100}$ lb. of hydrogen. The heating values of these elements are respectively 14,500 and 62,000 B.T.U. per lb.

Heat available from the carbon in 1 lb. of the fuel

$$= \frac{86}{100} \times 14{,}500 = 12{,}470 \text{ B.T.U.}$$

Heat available from the hydrogen in 1 lb. of the fuel

$$= \frac{14}{100} \times 62{,}000 = 8{,}680 \text{ B.T.U.}$$

Total heating value of 1 lb. of the fuel = 21,150 B.T.U.

It will be noticed in this calculation that it is assumed that the water vapour resulting from the combustion of the hydrogen has been condensed and cooled to ordinary atmospheric temperature, giving up latent and sensible heat. In very few practical operations will this occur, and the heating value as found above consequently represents a greater quantity of heat than will be available usually in practice. Since 1 lb. H gives 9 lbs. water, the fuel in the above example will produce on combustion $\frac{14}{100} \times 9 = 1{\cdot}26$ lbs. water. Deducting say 1100 B.T.U. of latent and sensible heat for each lb. of water gives a total deduction of $1100 \times 1{\cdot}26 = 1385$ B.T.U.

The heat available in the fuel will therefore be

$$21{,}150 - 1385 = \underline{19{,}770} \text{ B.T.U. per lb. fuel.}$$

EXAMPLE ii. A sample of Welsh coal has the following percentage composition:

C - - - - -	88·26
H - - - - -	4·66
O - - - - -	0·6
S - - - - -	1·77
Ash, etc. - - -	4·71
	100·00

Calculate its heating value.

$$\left.\begin{array}{r}\text{Available hydrogen}\\ \text{in 1 lb. coal}\end{array}\right\} = H - \frac{O}{8}$$

$$= \left\{\frac{4\cdot66}{100} - \left(\frac{6}{1000} \times \frac{1}{8}\right)\right\} \text{ lb.} = 0\cdot0458 \text{ lb.}$$

Constituent.	Heating value of constituent B.T.U. per lb.	Weight of constituent available in 1 lb. of fuel. lb.	Heating value B.T.U.
C	14,500	0·8826	12,800
H_2	62,000	0·0458	2,840
S	4,000	0·0177	71

Total heating value =15,711.

It is customary to neglect the small quantity of sulphur in coal in estimating the heating value.

Equivalent evaporation.—The heating value of a fuel is stated by engineers often as the weight of water in lbs. at a temperature of 212° F. which could be converted into steam at the same temperature by the application of the heat contained in one pound of the fuel. This quantity is called the **equivalent evaporation of the fuel.**

Each pound of water so evaporated would take up 967 B.T.U., consequently, for a given fuel:

$$\text{Equivalent evaporation} = \frac{\text{heating value in B.T.U. per lb.}}{967}.$$

EXAMPLE. Calculate the equivalent evaporation of a fuel having a heating value of 15,461 B.T.U. per lb.

$$\text{Equivalent evaporation} = \frac{15{,}461}{967}$$

$$= 16 \text{ nearly.}$$

To obtain the heating value of a given sample of fuel by chemical analysis requires a skilled chemist, as has been stated, and therefore it is more customary for engineers to estimate the heating value by use of a calorimeter in which the fuel is burned in an atmosphere of oxygen, the resulting heat being imparted to a measured quantity of water.

Testing for heating value of coal.—It is by no means easy to devise a calorimeter for determining the heating value of a given sample of coal which shall be simple and at the same time certain in its action. The Darling calorimeter probably is the one which most nearly possesses these qualities. In this instrument (Fig. 267), a weighed quantity of the powdered coal is contained in a crucible *C* held in clips secured to the top of a short brass tube *A*. A plate *R*, is also secured to the tube, and serves for the attachment of three legs *L*, on which the instrument rests, and also for the support of a bell glass *B* which is made water and gas tight at *R* by means of rubber rings and a brass ring secured by thumb screws. A very short cylinder *H*, formed of two brass plates and a ring at the edge to keep them apart, is fixed to the bottom of *A*. The upper plate is perforated with a number of small holes. The top hole of the bell glass *B* is closed by means of a rubber stopper through which passes a central brass tube connected by a flexible tube at *O* to an oxygen cylinder. Two brass wires *W*, *W* also pass through the stopper and are connected at *P* and *Q* to leads, so that an electric current may be passed through a platinum, or iron, wire which dips into the powdered coal. The whole instrument is immersed in a measured quantity

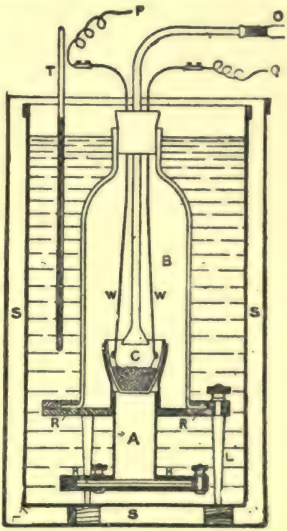


FIG. 267.—Darling calorimeter for testing the heating value of coal or other solid fuel.

of water contained in a vessel *S*. On passing the current, the wire will become heated and ignite the coal, the action being assisted by the oxygen. Combustion of the coal is completed under a continuous supply of oxygen, the products of combustion passing downwards through *A*, then through the perforations in *H* into the surrounding water. Passing upwards, the escaping bubbles are detained long enough by the plate *R* to give up most of their heat to the water before finally escaping to the surface of the water. The temperature of the water is taken before and after the combustion by a thermometer *T*. The following directions should be followed.

Expt. 68.—Grind up an average sample of coal in an iron mortar. Weigh out one gram in the crucible. Measure out 1400 c.c. of water into the vessel *S*, first bringing the temperature of the water to about 2·5° C. *below* the temperature of the room. Place the crucible in position and fasten the bell jar down. Insert the rubber stopper in the neck so that the ignition wire is embedded in the fuel. Adjust the oxygen tube so that its mouth is about ½″ above the coal. Turn on a gentle stream of oxygen and immerse the apparatus in water, taking care to adjust the pressure of the oxygen supply so that the gas may be able to overcome the head of water and so keep water out of the bell glass. Note carefully the temperature of the water and complete the battery circuit. As soon as the fuel is ignited, disconnect the battery and allow the combustion to proceed; direct the stream of oxygen so that every particle of fuel is burned; the rubber stopper permits this to be done easily. When combustion is complete, thoroughly mix the water by raising the combustion arrangement up and down until no further rise in temperature is observed. Note this temperature, then remove the combustion arrangement from the water vessel and shut off the oxygen supply.

The heating value may now be calculated.

Let Q = heating value in gram-calories per gram of coal,

W = weight of water used, grams,

W_e = water equivalent of instrument, grams,

W_f = weight of fuel burned,

t = rise in temperature of water, degrees C.

Then

$$Q = \frac{(W + W_e)t}{W_f}, \text{ gram-calories per gram of coal.}$$

$$= \frac{(W + W_e)t}{W_f} \times 1{\cdot}8, \text{ B.T.U. per lb. of coal.}$$

In the case of a platinum wire being used to start the combustion, none of it will be consumed, and the heating effect of the current used may be neglected, provided care is taken to have the time during which the current is on as short as possible. If an iron wire is used, a deduction should be made for the quantity of heat from the wire which has been burned. This amounts to 1575 gram-calories per gram of wire actually burned. Sulphur may be used to start the combustion prior to placing the rubber stopper in position; 0·05 gram of sulphur will be a sufficient quantity, and in this case a deduction of 110 gram-calories should be made from the final result for the heat evolved from this quantity of sulphur. The oxygen stream should be adjusted before placing the stopper in position, and the sulphur ignited by passing a red hot wire into the bell glass so as to touch the sulphur. Place the stopper at once into position and immerse as quickly as possible, afterwards proceeding as before.

Heating values of gaseous and liquid fuels may be determined also by use of the Darling Calorimeter in a slightly altered form to permit of the safe and complete combustion of the gas or oil. The makers are Messrs. Gallenkamp.

Boys's gas calorimeter.—There is no simple form of apparatus available for the beginner to employ in determining the heating value of gaseous fuel. The following apparatus is in use at the Metropolitan Gas Works, and has been devised by Prof. C. V. Boys, F.R.S. The makers are Messrs. Griffin & Sons, Ltd. The method consists in causing a steady current of circulating water to take up the heat evolved by the burning gas, and is of particular interest to engineers.

The calorimeter is shown in section in Fig. 268. The gas is burned at two small union jets *B*, giving off products of combustion, which rise into the bell *H* and then descend outside the chimney *E*. Here the hot gases come into contact with coils containing the circulating water; the coils are made of Clarkson's

motor car radiator tube, so as to be capable of rapidly abstracting the heat from the gases. The circulating water enters the outer coil at the union O, and, after passing through the outer coil N and the inner coil M, enters the space K above the bell H,

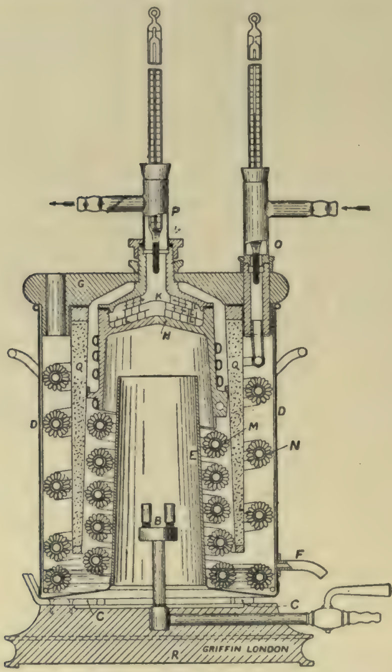


Fig. 268.— Section of Boys's calorimeter for testing the heating value of gas.

where it circulates between two dished mixing plates, finally leaving at the union P. Thermometers at O and P enable the inlet and outlet temperatures of the circulating water to be measured.

Part of the products of combustion consists of steam. This steam will condense on coming into contact with the water-cooled surfaces, and the resulting water will fall to the bottom

of the instrument, where a water bath is formed in which the two lower turns of the coil are immersed. The bath serves to keep the chimney E cool, but not cool enough to cause condensation to occur on its inner surface. An overflow F is provided for drawing water from the bath. The coils M and N are separated by a heat insulator Q, containing cork dust.

The instrument is used in conjunction with an accurate gas meter, a gas governor, and vessels for measuring the quantity of circulating water. Observations are taken of the quantity of gas, and of the quantity of circulating water with its initial and final temperature. From these data the heating value is calculated, certain corrections being applied.

EXERCISES ON CHAPTER XVII.

1. Define the terms "chemical compound," "combustion," "fuel."

2. Calculate what weight of oxygen is required to burn 3·5 lbs. of hydrogen; also the quantity required to burn completely 85·6 lbs. of carbon.

3. Describe the properties of carbon monoxide. Calculate what volume of oxygen is required to burn 100 cubic feet of carbon monoxide.

4. State the composition of the atmosphere. In Question 2, calculate approximately in each case what weight of air would be required.

5. Calculate the heat available in one ton of carbon.

6. Write a brief description of the common varieties of coal.

7. A coal contains 80 per cent. by weight of carbon, 5 of hydrogen and 10 of oxygen. Calculate its heating value.

8. In Question 7, calculate approximately the weight of air required for complete combustion of 1 lb. of the coal. What weight of air would be required in practice?

9. What is petroleum? In what forms is it used as fuel?

10. Give a brief account of any kind of gas manufactured for power purposes.

11. Define "equivalent evaporation." In a certain boiler trial, coal was used having an equivalent evaporation of 15 lbs. of water. State the heating value of the coal in B.T.U. per lb.

12. In comparing the following methods of generating heat, pay attention only to cost, leaving convenience and other matters out of account.

1 lb. of average coal gives out 8500 Centigrade pound heat units.

1 cubic foot of average London gas gives out 380 Centigrade pound heat units.

A Board of Trade unit of electrical energy is $1\frac{1}{3}$ horse-power hours. How much heat is generated by one ton of coal? If the gas costs 3 shillings per thousand cubic feet; if the Board of Trade unit cost sixpence; what is the cost in these two cases of the amount of heat given out in burning one ton of coal.

13. A pound of oil contains 0·85 lb. of carbon and 0·15 lb. of hydrogen. What weight of oxygen is sufficient to produce CO_2 and H_2O by combustion? (Take the atomic weights of C, 12; of O, 16; of H, 1.) If 1 lb. of oxygen is contained in 4·35 lbs. of air, how many lbs. of air are needed for complete combustion?

CHAPTER XVIII.

ACTION OF THE STEAM ENGINE.

Action of simple engines.—The steam engine and boiler may be looked upon as contrivances for converting into **mechanical work** the energy contained in the coal or other fuel in the form of potential **heat.** Steam is generated from water by the application of heat. When an open vessel is used, the steam is given off at the same pressure as that of the atmosphere, but a much higher pressure may be secured by generating steam in a closed vessel. Steam may be used for the production of work by allowing it to push a piston to and fro in a cylinder; or, by causing it to discharge against vanes formed round the circumference of a wheel, thus producing rotation. In this chapter, sufficient information will be given to enable the student to understand the action and arrangement of the parts of a small engine of the first-mentioned type.

Steam engines having pistons working in cylinders are generally employed to give a motion of rotation to a shaft. This result is effected by means of a mechanism called the crank and connecting rod. In Fig. 269, A is a **cylinder** shown in section. B is a **piston** capable of sliding in the cylinder and fitted so as to prevent leakage of steam past its edge. A **piston rod** C is attached to the piston and passes through a hole in one end of the cylinder, formed so as to be steam-tight. The outer end E of the piston rod is jointed by means of a pin to one end of a **connecting rod** D; this joint is called the **crosshead.** The other end of the connecting rod is attached to a pin F secured to a **crank** G, which is mounted on a **crank shaft** H. As the shaft H rotates, the pin F will describe the circumference of a circle, and the pin E will move to and fro in a straight line.

Steam is admitted to the cylinder first through the opening or **port** K and will exert pressure on the left-hand side of the piston, the other side being put into communication, through the port L, with the atmosphere, or with a vessel called a **condenser**, in which the pressure is kept low. The steam, by its pressure will cause the piston to travel to the right-hand end

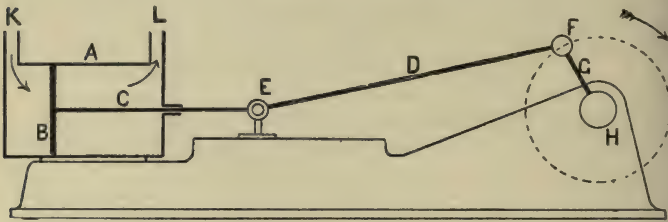


FIG. 269.—Outline diagram of engine mechanism.

of the cylinder, and thus, by means of the intervening mechanism, the crank shaft is made to execute half a revolution. Steam is then directed into the right-hand side of the cylinder through L, the left-hand portion being put into communication with the atmosphere or with the condenser through K, and the piston will be driven back to the left-hand end of the cylinder, the crank shaft meanwhile completing the revolution. To enable the crank shaft to rotate smoothly without jerky action, a heavy wheel called a **fly-wheel** may be attached to it.

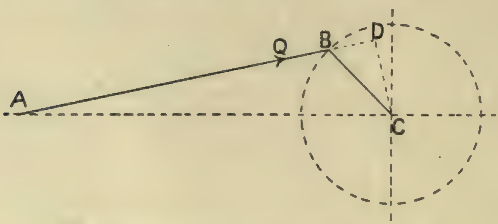


FIG. 270.—Measurement of the turning moment.

Turning moment.—The forces acting on the piston of an engine are transmitted through the piston rod and connecting

rod, and so exert push, or pull, on the crank pin; these forces acting on the crank pin have a **tendency to rotate the crank shaft,** which tendency is described as the **turning moment.** In Fig. 270, AB is the connecting rod and BC the crank. Q is a force of 15,000 lbs. acting along the connecting rod. To measure the turning moment of Q, drop a perpendicular CD from C, the axis of rotation, on to the line of the connecting rod, producing it, if necessary. Suppose CD to be 22 inches, then

$$\begin{aligned}\text{Turning moment} = T &= 15{,}000 \times 22\\ &= 330{,}000 \text{ lb.-inches}\\ &= \underline{27{,}500} \text{ lb.-feet.}\end{aligned}$$

It will be evident that the turning moment will be zero when the connecting rod and the crank both lie in the line AC; this will happen twice in each revolution; in other positions the turning moment will vary considerably. Fig. 271 shows a turning moment diagram for a single-cylinder engine, constructed by calculating the moment and setting it off to scale along the crank produced, as at BF. This has been done for several positions throughout the revolution, and a curve drawn through the various points such as F. The want of uniformity in the turning moment is now very apparent. The crank is said to be on the dead point when its direction lies along AC (Fig. 270). If B falls between A and C, the crank is at the inner dead point, and if B falls outside, the crank is at the outer dead point.

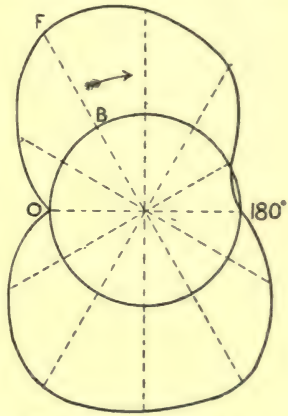


FIG. 271.—Turning moment diagram for a single crank engine.

The flywheel operates by storing a large quantity of energy in the kinetic form while the engine is getting up speed. It is then able to meet any deficiency which might be caused by a sudden demand for more energy, or by variation in the

turning moment, by giving up some energy from its store. This it is able to do by changing its speed to a comparatively small extent. The kinetic energy of a flywheel may be calculated by imagining that its whole mass is concentrated at the mean radius of the rim. Thus:

Let m = the total mass in pounds.
r = the mean radius of the rim, in feet.
v = the mean speed of the rim, in feet per second.
N = the revolutions per minute.

Then,

Mean circumference of the wheel $= 2\pi r$ feet.

Distance travelled by a point on the mean circumference $= 2\pi r N$ feet per minute.

$$\therefore\ v = \frac{2\pi r N}{60} \text{ feet per second.}$$

Hence,

$$\text{Kinetic energy of the wheel} = \frac{mv^2}{2g} \text{ foot-lbs.}$$

Supposing that the wheel lowers its speed from v_1 to v_2 feet per second, we have

$$\text{Change in kinetic energy} = \frac{mv_1^2}{2g} - \frac{mv_2^2}{2g},$$

$$= \frac{m}{2g}(v_1^2 - v_2^2) \text{ foot-lbs.}$$

If the wheel is called upon to give up W foot-lbs. of energy, we have

$$W = \frac{m}{2g}(v_1^2 - v_2^2),$$

or,

$$v_1^2 - v_2^2 = \frac{2g}{m} W.$$

From this result the change in speed of the wheel may be calculated.

EXAMPLE. A wheel of mass 2000 lbs. at a mean radius of 3 feet has a speed of 180 revolutions per minute. Suppose 4000 ft.-lbs. to be abstracted from it and calculate its new speed.

$$v_1 = \frac{180}{60} \times 2\pi r = 3 \times 2 \times \frac{22}{7} \times 3$$

$$= \frac{396}{7} = 56{\cdot}57 \text{ ft. per sec.,}$$

and

$$v_1^2 = 3200.$$

Now
$$v_1^2 - v_2^2 = \frac{2g}{m} W$$
$$3200 - v_2^2 = \frac{64 \cdot 4}{2000} \times 4000$$
$$= 128 \cdot 8.$$
$$v_2^2 = 3200 - 128 \cdot 8,$$
$$v_2 = \sqrt{3071}$$
$$= 55 \cdot 4 \text{ ft. per sec.}$$

Let new speed $= N_2$ revolutions per minute.
$$N_2 = \frac{55 \cdot 4 \times 60}{2\pi r}$$
$$= \frac{55 \cdot 4 \times 60 \times 7}{2 \times 22 \times 3}$$
$$= \underline{176 \cdot 3} \text{ revolutions per min.}$$

The wheel therefore loses 3·7 revolutions per minute while giving up 4000 ft.-lbs. of energy.

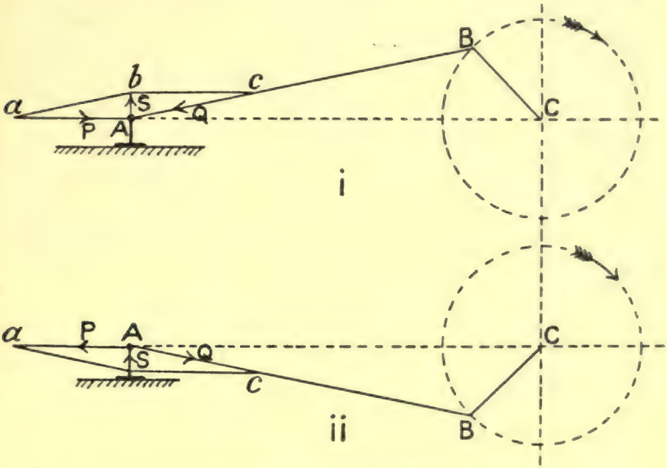


Fig. 272.—Diagram of the forces acting at the crosshead for a crank having clockwise rotation.

Forces acting at crosshead.—In general, there will be three forces acting at the crosshead pin, viz. the force P acting along the piston rod (Fig. 272, i), the force Q along the connecting

rod, and a reaction S coming from the guides, these three forces being in equilibrium. If we neglect friction, S will act always at right angles to the line of the stroke. Assuming friction absent for our present purpose, the values of Q and S can be

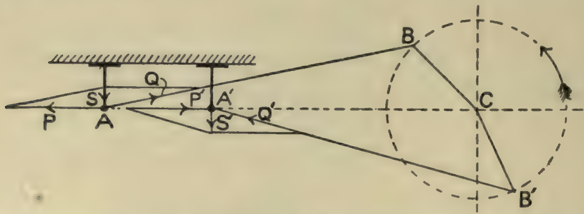


FIG. 273.—Diagram of the forces acting at the crosshead for a crank having anti-clockwise rotation.

found, if we know P, by an application of the parallelogram of forces. Thus, set off Aa to represent P to a convenient scale of force, draw ab parallel to AB cutting the line of S in b; draw bc parallel to AC cutting the connecting rod in c. Then, to the scale of force, cA gives the force Q in the connecting rod and Ab gives S the reaction of the guide.

Fig. 272, ii, shows the crank in another position in the return journey. P and Q now act in opposite directions, the piston and

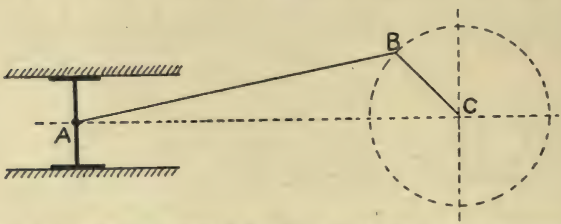


FIG. 274.—The slipper must be guided on both sides in engines intended to reverse.

connecting rods being under pull. Notice that S, the reaction of the guide, still **acts upwards.** Suppose, however, we reverse the direction of rotation and draw diagrams as in Fig. 273, it will be found that the reaction of the guide is always a **downward force.** This shows the necessity for designing the crosshead so

that both upward and downward reactions are provided for in engines in which the direction of motion may be reversed (Fig. 274); and it is customary to do so in all engines as there is a liability for the direction of the reaction of the guide to be reversed when the crosshead is nearing the ends of its stroke.

It must be understood that the mechanism of the engine does not diminish in any way the energy which the steam gives to the piston. Neglecting frictional waste, the work done on the piston by the steam is equal to the work done on the crank shaft.

Construction of the cylinder.—The cylinder is constructed with various passages, or ports, formed in it so that the steam

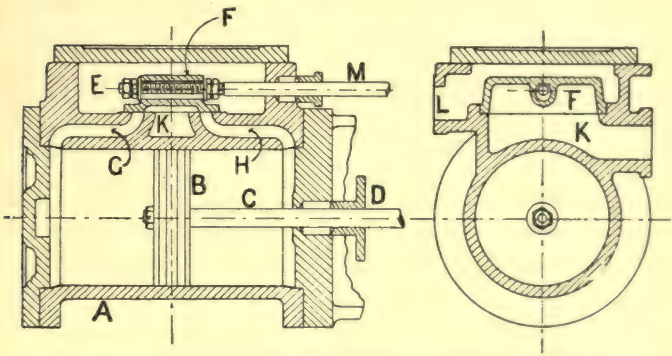


FIG. 275.—Sectional side and end elevations of a steam engine cylinder.

may flow as required. The distribution of the steam first to one side, and then to the other side, of the piston is effected by means of **valves**, of which there are many varieties. These valves are opened and closed at the proper instants by means of a mechanism driven from the crank shaft, so that the engine is self-acting.

A common form of cylinder is shown in section in Fig. 275. *A* is the cylinder, made of cast-iron, and fitted with a cast-iron piston *B*. The piston has spring rings fitted into grooves on its rim; these rings press outwards against the cylinder walls and so prevent the steam from leaking past the piston. *C* is

the piston rod secured firmly to the piston, and passing through a stuffing box and gland at D in the cylinder end. The stuffing box contains packing for rendering tight the piston rod against steam leakage. A box E of rectangular shape, called the **steam chest**, is cast on to the side of the cylinder, and is provided with a movable cover. Two steam ports G, H, lead from the steam chest, one to each end of the cylinder. A third port, K, called

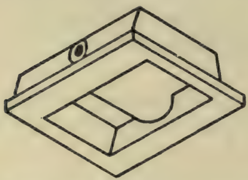


FIG. 276.—Perspective view of a slide valve.

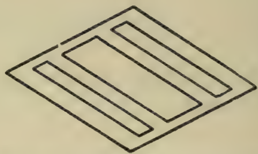


FIG. 277.—Perspective view of the cylinder face and ports.

the **exhaust port**, leads from E into the atmosphere, or condenser. These ports open into the steam chest at a flat face (Fig. 277) over which a valve F (Fig. 276) is arranged to slide to and fro, being driven by a rod M (Fig. 275) actuated by mechanism mounted on the crank shaft. Steam is brought from the boiler into the steam chest through an opening L, and is distributed by means of the **slide valve** F. This valve is made like a rectangular box turned upside down (Fig. 276).

Action of the valve.—To understand the action of the valve reference should be made to Fig. 278, in which a perspective view of the valve and ports is shown in section. As the valve is positioned in this figure, the steam may flow from the steam chest through G into the left-hand part of the cylinder. The other side of the piston is in communication with the exhaust port K through the port H and the cavity in the valve. The piston B will thus be caused to travel towards the right. The return stroke of the piston is effected by first moving the valve into the position shown in Fig. 279. The steam will now be directed through the port H into the right-hand portion of the cylinder, the other side at the same time being put into communication with the exhaust through the port G and the cavity of the valve. Motion of the piston towards the left will

now occur. The valve is designed generally so as to admit steam to the cylinder only during the early part of the stroke, and then to cut off the supply, the remaining part of the stroke

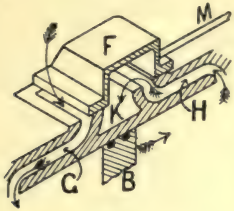


FIG. 278.

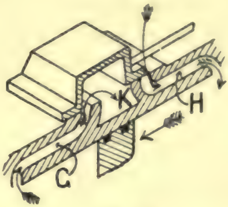


FIG. 279.

Diagrams showing the way in which the slide valve distributes the steam.

being accomplished by the **expansive action** of the steam, giving a continually diminishing pressure on the piston. This arrangement is adopted as being more economical.

Method of driving the valve.—The valve is driven by means of a device called an **eccentric,** which consists (Fig. 280) of a circular disc A, having a hole bored through it to receive the crank shaft B. The hole is bored a short distance away from

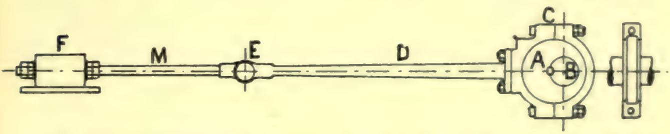


FIG. 280.—Arrangement of eccentric and rods for driving the slide valve.

the centre of the disc, and a key is fitted so as to secure the disc firmly to the shaft. The disc is called the **eccentric sheave**; its edge is made to receive a **strap** C, which is a working fit on the sheave in order that the sheave may rotate with the crank shaft without producing motion of rotation in the strap. The strap is made in halves, secured together by means of bolts, so that it may be got into position round the sheave. Attached to the strap by means of studs is an eccentric rod D, the other end of which is attached by means of a pin, at E, to the end of the valve rod M. As the crank shaft rotates, the valve F will be

driven to and fro a distance equal to twice the distance from the centre of the eccentric sheave to the centre of the crank shaft. The motion of the valve so obtained is precisely the same as would be obtained from a crank having a radius equal to the distance of the centre of the eccentric sheave to the centre of the crank shaft.

Events determined by the valve.—For each side of the piston the valve determines the instant at which four events will occur. These are:

(1) **Point of admission.**—This is the instant at which steam begins to flow into the cylinder (0, Fig. 281), and occurs when the crank is still a few degrees from reaching the dead point. The valve is thus open a little (1, Fig. 281) before the piston is ready to begin its stroke, and thus a supply of steam has had time to enter the cylinder and fill the clearance space between the piston and the steam port; this is said to be giving **lead** to the valve.

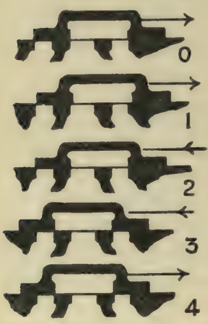


Fig. 281.—Valve positions for the events of the left-hand side of the piston.

(2) **Point of cut-off.**—This point may occur when the piston has completed any convenient fraction of its stroke. At the required instant, the valve closes the steam port (2, Fig. 281), and the remainder of the stroke is completed by the expansive action of the steam.

(3) **Point of release.**—This is the point at which exhaust from the cylinder begins. To permit a rapid discharge of the steam when done with, and so to reduce the back pressure on the piston, it is customary to open to exhaust (3, Fig. 281) some little distance before the piston reaches the end of its stroke.

(4) **Point of cushioning.**—At this point the exhaust port closes (4, Fig. 281) and no further steam is allowed to flow from the cylinder. Usually about 0·8 of the return stroke has been completed when this occurs, and the steam entrapped in the cylinder is compressed by the returning piston and so has its pressure raised. The object of this arrangement is to provide a cushion of steam to bring the piston to rest at the end of the stroke and so to prevent the bearings knocking; cushioning also

fills the clearance spaces with steam at pressure higher than that of the exhaust steam, and so reduces the quantity of boiler steam which is required for filling these spaces each stroke.

The student will find it helpful to study these events on a good working valve model; most laboratories possess such models. Failing this, cardboard models can be constructed easily by the student himself, or the excellent set of models designed by Messrs. Jones[1] may be used.

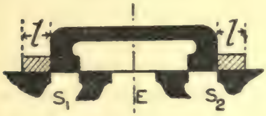


FIG. 282.—Measurement of lap.

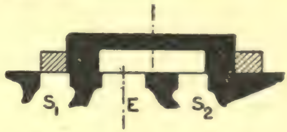


FIG. 283.—Lap in a slide valve for expansive working.

A slide valve is adapted to give expansive working, by making its face longer than the cylinder parts, as shown in Fig. 282 by the parts shaded. This is called **lap,** indicated by the length l in the figure. It will be evident that admission of steam into the left-hand steam port S_1 will occur when the valve has been drawn to the right by an amount equal to the lap (Fig. 283). Hence, at the dead point, the valve must be at a distance to the right of its middle position by an amount equal to the lap plus the lead, *i.e.* the amount by which the valve is open when the crank is at the dead point.

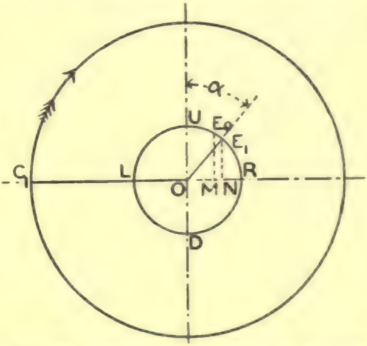


FIG. 284.—Setting of the eccentric for a slide valve having lap and lead.

In Fig. 284, OC_1 is the crank at the inner dead point. The

[1] *Working Models for Engineering Students*, T. Jones and T. G. Jones; The Technical Publishing Co., Manchester.

circle $LURD$ marks the path of the eccentric centre. Make OM equal to the lap and MN equal to the lead, and draw ME_0 and NE_1 perpendicular to LR and cutting the circle in E_0 and E_1. Then E_0 will be the position of the eccentric centre at admission, and E_1 will be its position when the crank is at the dead point. The angle between the crank and the eccentric will be C_1OE_1, and they will rotate clockwise keeping this angle constant, as both are fixed to the shaft. UOE_1 is called the angle of advance of the eccentric.

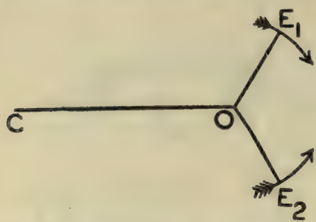


FIG. 285.—Setting of two eccentrics employed in link motions.

Link motions.—Link motions have for their principal object the reversal of the direction of rotation of the engine. Two eccentrics are employed and are set as shown in Fig. 285. An arrangement of links and rods is introduced whereby either eccentric may be used to drive the slide valve. Should E_1 be in gear with the valve, clockwise rotation will occur, and anticlockwise rotation will be produced when E_2 gears with the valve.

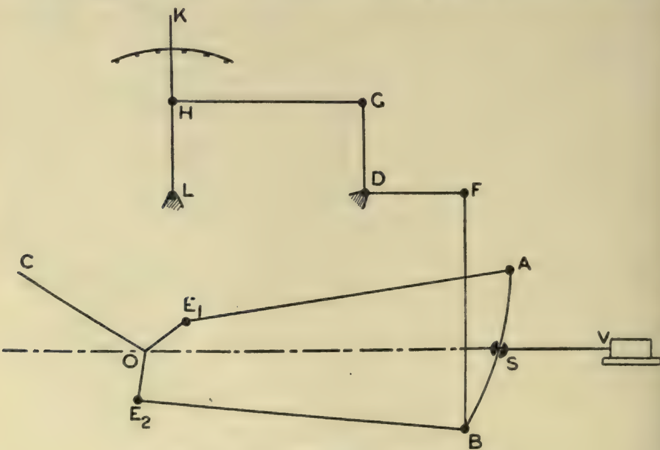


FIG. 286.—Diagram of Stephenson's link motion.

In **Stephenson's link motion**, an outline diagram of which is shown in Fig. 286, a curved link has its ends A and B connected to the eccentric rods E_1A, E_2B. SV is the valve rod, engaging with the link at S by means of a block which may slide in the link. GDF is a bent lever pivoted at D, and connected to the link AB by a rod FB. The bent lever is operated by means of a hand lever KL and rod HG, and when this is done, the link AB will be raised or lowered until either B or A coincides with S. In the former case, E_2 will drive the valve direct, the other eccentric meanwhile causing the link AB to vibrate without communicating any of its motion to the valve. The radius of the link AB is equal to the length of the eccentric rod, or nearly so. Details of construction of this link motion will be found on p. 329.

The governor.—The engine is kept at a steady average speed of rotation by means of a device called a **governor**. The governor effects this by regulating the supply of steam to the cylinder. The arrangement is shown in Fig. 287, where A is the governor and B is the steam pipe supplying steam to the cylinder. A **throttle valve** C is placed in the steam pipe; in the example shown, this valve consists of a disc which may be rotated partially on an axis D, by means of a lever E. If the disc is situated transversely to the pipe, steam will be cut off almost entirely, and there will be a more or less free passage past the valve depending on the angle at which it is set. The function of the governor is to control this angle. Two heavy balls FF are mounted at the end of arms GG, which are attached

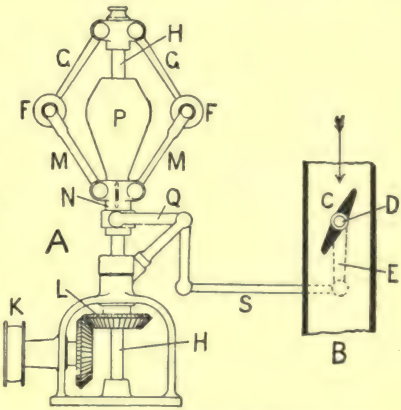


FIG. 287.—Diagram showing how the governor controls the steam supply.

by pins to a spindle *H*; the latter is rotated by a belt from the crank shaft running on the pulley *K*, the motion being transmitted to *H* by means of bevel wheels *L*. Other arms *MM* connect the balls to a sleeve *N* which may slide on the spindle *H*. A heavy weight *P* bears downwards on the sleeve. A bent lever *Q* has one arm connected to a collar embracing the sleeve, and the other arm is connected by a rod *S* to the throttle valve lever *E*.

When the engine is running and driving the governor spindle, centrifugal force acts on the balls, causing them to move outwards until a steady position is reached which depends on the speed of rotation. Any increase in the speed will cause further outward movement of the balls, producing an upward movement of the sleeve *N*, and this, being transmitted through the levers and rod to the throttle valve, will close it partially, and so reduce the supply of steam to the engine; the engine speed will thus fall. Should the speed be lowered below the proper amount, the balls move inward, thus lowering the sleeve *N* and so opening the throttle valve more and thus permitting more steam to pass to the engine.

Arrangement of small steam plant.—The student will find it instructive to examine carefully the drawings of a small steam engine and boilers shown in Figs. 288 and 289 and reproduced here by the courtesy of Messrs. E. S. Hindley & Sons. The engine is of the **horizontal type,** *i.e.* the centre line of the cylinder is horizontal. The steam cylinder is shown in section at *A* (Fig. 289); the cylinder is bolted to the end of the sole-plate *B*. The crank shaft *C* is supported by main bearings *DD*, and has mounted on it a fly-wheel *E*, an eccentric *F* for driving the slide valve, and a belt pulley *G* for driving the governor. The governor is shown at *H* in Fig. 288, and operates a throttle valve contained in the casing *K*. A **stop valve**, by means of which the steam supply may be turned on or off to the engine, is contained in the same casing, and is operated by a hand wheel *L*.

The **boiler** is shown in section in Fig. 288. It consists of an outer cylindrical shell *M*, made of plates riveted together. Another smaller cylindrical box *N* is contained within the shell and is riveted to it at the bottom edge. This forms the

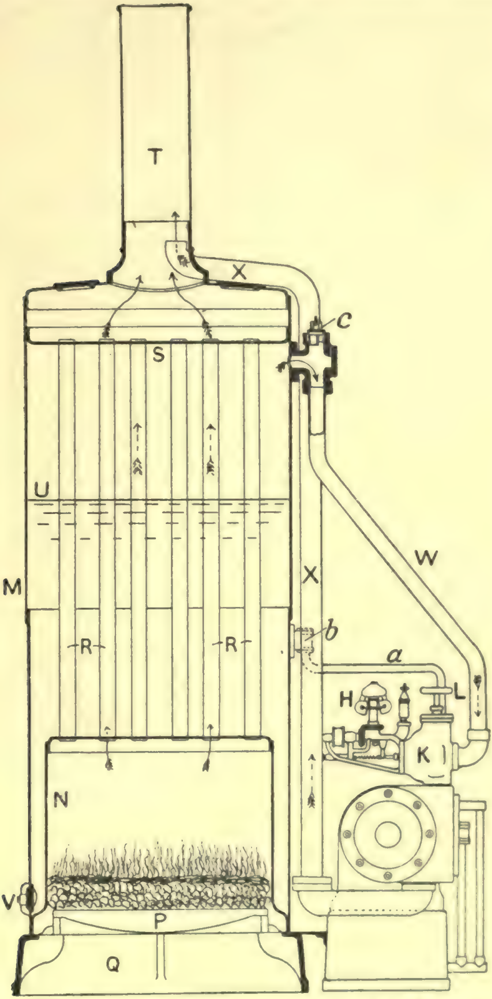


FIG. 288.—Elevation of a small horizontal steam engine, with vertical boiler; the latter is shown in section.

fire box, and is furnished with a grate formed of a number of **fire bars** P laid side by side with small spaces between to admit

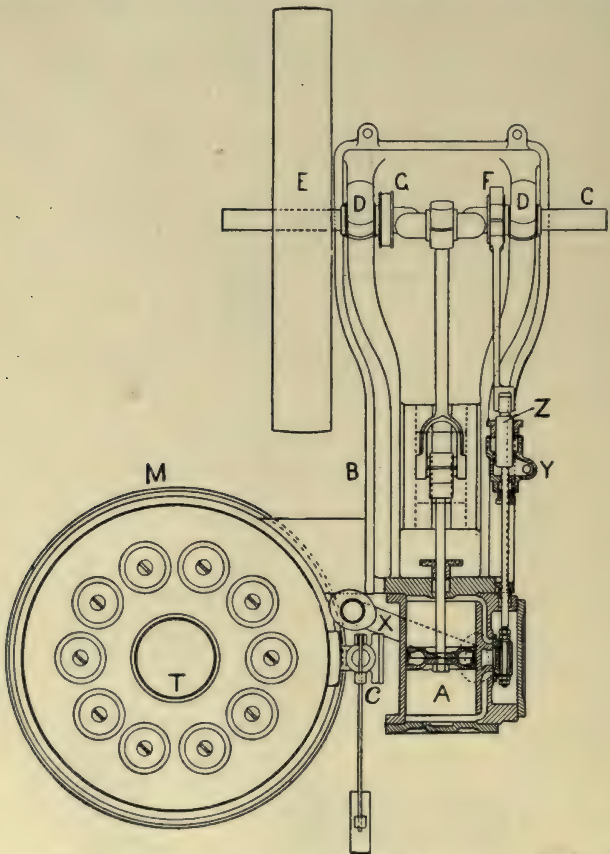


FIG. 289.—Plan of the steam plant shown in Fig. 288. The cylinder and feed pump are shown in section.

air from the **ash pit** Q. A number of tubes R are fitted tightly into the top of the fire box and into a tube plate S

near the top of the shell. The hot gases from the fire pass upwards through the tube, as shown by the arrows, giving up their heat to the surrounding water, and finally escape by the chimney *T*. The water level in the boiler stands at *U*, the space between this level and the tube plate *S* being filled with steam. Several hand holes, rendered tight by covers, enable sludge, etc., to be cleaned out of the bottom of the boiler; one of these is shown at *V*.

Steam is led from the boiler through the pipe *W* to the throttle valve and so to the steam cylinder. The exhaust steam from the engine cylinder escapes through a pipe *X* which is led into the chimney, and so discharges into the atmosphere. The resulting upward blast ensures a draught which will cause the air necessary for the combustion of the coal to pass freely into the fire.

The boiler is kept supplied with water to make good that which is evaporated into steam by means of a **feed pump** *Y* (Fig. 289). This pump has a plunger *Z*, which forms part of the valve rod, and is fitted with suction and discharge valves and pipes which enable it, at each revolution of the engine, to discharge the necessary quantity of feed water into the boiler through the pipe *a* and a non-return valve *b* (Fig. 288). The **non-return valve** prevents any water flowing out of the boiler through the feed supply pipe, should the pump not be delivering water.

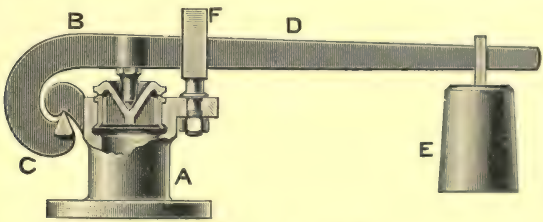


Fig. 290.—Lever safety valve.

A **safety valve** is fitted to the boiler at *c*; its function is to enable steam to escape from the boiler should the pressure exceed a prearranged amount and so endanger the boiler. The type of valve illustrated is called a **lever safety valve**; details of such a valve are shown more clearly in Fig. 290, where it will

be seen that the valve is held down by means of a weight attached to the end of a lever which bears downwards on the valve. The resulting downward pressure on the valve is sufficient to counteract the upward steam pressure under normal working conditions, but should the steam pressure exceed the proper amount, the valve will lift and permit the escape of the steam.

EXERCISES ON CHAPTER XVIII.

1. Explain, with diagrams, how the motion of the piston in the cylinder is converted into motion of rotation at the shaft.

2. Give sketches showing the construction of a cylinder for a steam engine, omitting the valve chest, but showing the cylinder covers and the piston rod stuffing-box.

3. Sketch and describe a common slide valve. Explain how it allows steam to enter and discharge from the cylinder.

4. Give sketches of the arrangement for driving a slide valve by means of an eccentric.

5. Explain how the governor controls the speed of the engine. Illustrate your answer by reference to an outline diagram.

6. Why is it necessary to guide the outer end of the piston rod?

7. Explain clearly the object of fitting each of the following valves to a steam boiler: (*a*) stop valve, (*b*) safety valve, (*c*) non-return feed valve.

8. Explain the meaning of the term "turning moment." An engine crank is 1 foot, and the connecting rod $4\frac{1}{2}$ feet, in length. Draw the crank and connecting rod in the position when the angle between them is 90°. In this position the force exerted by the piston rod is 4500 lbs. Find the turning moment.

9. In Question 8 find the total pressure on the guide for the given position. What area of sliding surface must the slipper have if the pressure on it is not to exceed 70 lbs. per square inch?

10. Explain carefully the reason for often providing two slippers, one on each side of the crosshead, in engines which frequently reverse.

11. A steam engine cylinder is 20″ diameter. The steam pressure on one side of the piston at a certain instant is 65 lbs. per square inch and on the other side is 17 lbs. per square inch. Calculate the resultant force acting on the piston.

12. The travel of a slide valve is $2\frac{1}{2}''$, the outside lap is $\frac{1}{2}''$ and the lead is $\frac{1}{8}''$. Find the angle of advance.

13. Explain by reference to an outline diagram the construction and action of Stephenson's link motion.

14. A fly-wheel has a mean radius of 5 feet. Its mass is six tons and it runs at 120 revolutions per minute. Calculate its kinetic energy.

15. Suppose the fly-wheel in Question 14 to give up 55,000 ft.-lbs. of energy, what will be its speed then?

16. Explain why both the fly-wheel and governor are needed to regulate or govern the speed of an engine.

17. The ratio of expansion in a steam engine is defined as the ratio of the volume of steam present at the end of the stroke to the volume present in the cylinder at the point of cut off. Find the ratio of expansion in a cylinder of 24 inches stroke; the steam is cut off when the piston has travelled 7 inches from the beginning of the stroke.

CHAPTER XIX.

OTHER FORMS OF STEAM ENGINES AND BOILERS. HORSE POWER.

The Lancashire boiler.—For general factory purposes, the Lancashire boiler is the most popular. The construction and arrangement in this boiler will be understood by reference to Fig. 291, in which is shown a complete boiler with its brick-work seating and flues, and to Fig. 292, showing the finished shell. The drawings are reproduced here by the courtesy of Messrs. Spurr, Inman & Co., Ltd. Referring to Fig. 291 it will be observed that the boiler consists of a large cylindrical shell, generally from 25 to 30 feet long, and from 6½ to 9 feet in diameter. Two large tubes of diameter about 0·4 that of the shell pass from end to end. A furnace *C*, about 6 feet long, is placed at the front end of each tube (Fig. 292). The hot gases from the burning fuel pass along the furnace tubes, emerging at the back end, where they pass downwards and unite in a bottom flue *E* (Fig. 291). Passing along *E*, the gases divide at the front end of the boiler into side flues *FF*, and again travel to the rear of the boiler, where they make their exit to the chimney through the flue *G*. The flues are built of common brick, lined with firebrick in order to withstand the heat. **Dampers** consisting of doors sliding vertically in frames are placed at *JJ*; these serve to regulate the draught. Doors by means of which access can be obtained to the flues are placed at *K*, *L*, *L*. *H* is a shallow pit, called the **blow-off pit**, covered with iron plates forming part of the floor of the boiler room. The positions and names of the various fittings attached to the boiler are indicated in Fig. 292.

Lancashire boilers are worked at steam pressures ranging from 100 to 150 lbs. per square inch.

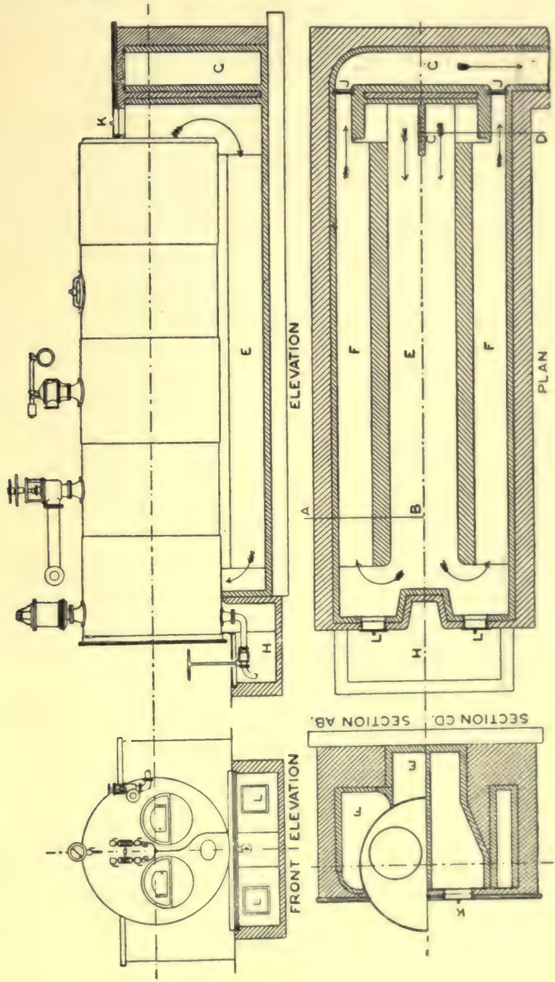


FIG. 291.—Lancashire boiler, showing the brickwork seating and flues.

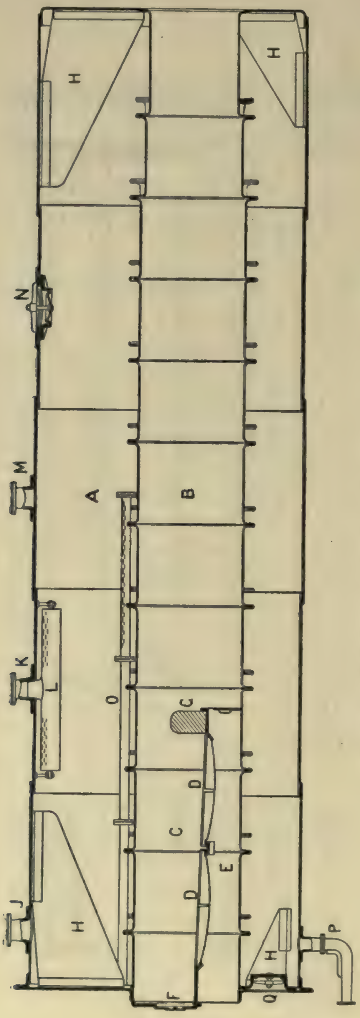


FIG. 292.—Longitudinal section of the shell of a Lancashire boiler.

- **A.** Shell.
- **B.** Furnace tube.
- **C.** Furnace.
- **D.** Fire bars.
- **E.** Ash pit.
- **F.** Furnace door.
- **G.** Fire bridge.
- **H.** Gusset stays, for staying the flat ends to the shell.
- **J.** Mounting block for dead weight safety valve.
- **K.** Mounting block for the main steam stop valve.
- **L.** Anti-priming pipe, for preventing water from passing into the main steam pipe.
- **M.** Mounting block for low water and high steam safety valve.
- **N.** Manhole door.
- **O.** Internal feed pipe, through which the feed-water enters the boiler.
- **P.** Blow-off pipe, for discharging water and sediment from the boiler.
- **Q.** Mud hole door.

Locomotive boiler.—The construction of a locomotive boiler will be understood by reference to Figs. 293 and 294, illustrating

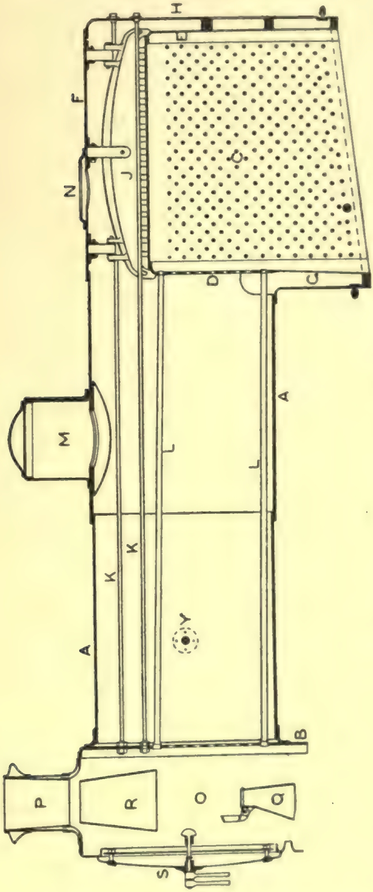


Fig. 293.—Longitudinal section of a locomotive boiler.

A. Barrel.
B. Front end plate.
C. Internal firebox.
D. Tube plate.
E. Firebox back plate.
F. Wrapper plate.
G. Throat plate.
H. Shell back plate.
J. Roof bar stays.
K. Longitudinal stays.
L. Tubes.
M. Steam dome.
N. For safety valve.
O. Smoke box.
P. Chimney.
Q. Blast pipe.
R. Nozzle.
S. Smoke box door.

an express locomotive boiler constructed by the Great Eastern Railway Co. to the designs of Mr. James Holden.

The boiler consists of a cylindrical barrel *A* (Fig. 293), having an internal firebox *C* at one end, and a smoke box *O* at the other. The firebox is connected to the smoke box by 274 tubes $1\frac{3}{4}''$ external diameter. The furnace gases pass from the firebox through these tubes into the smoke box, and are discharged through the chimney *P*. The draught is obtained by discharging the exhaust steam from the cylinders through a blast pipe *Q* and a nozzle *R*, so situated in the smoke box as to induce a strong draught of air through the furnace and tubes. *M* is the steam dome, from the interior of which is taken the steam supply for the cylinders. A safety valve is mounted at *N*. The feed water is introduced through a valve at *Y*. A large door *S* gives access to the smoke box and tubes for examination and cleaning. This boiler works at a steam pressure of 180 lbs. per square inch.

Liquid fuel in locomotives.—The Great Eastern Railway locomotives are fitted with apparatus for the consumption of oil fuel. Mr. James Holden has taken a prominent part in the development of this method of raising steam. In Fig. 294, *A* is one of two injectors for spraying the mixture of coal tar and green oil, oil gas tar, creosote or other oil used as fuel into the furnace. These injectors are inserted in orifices in the front of the boiler, one on each side, and are so disposed that they do not interfere with the use of coal in the furnace. The construction of the Holden injector is shown in Fig. 295. The object is thoroughly to break up the oil fuel into a spray of fine particles. There are three internal cones in the injector, having finely adjusted spaces between. The oil is fed into the outer space, and on emerging from the end of the cone at *A* is met by a mingled jet of steam and air coming from the other spaces. Air is supplied through the centre, and steam between the inner and middle cones. On emerging from the mouth of the injector at *B*, the mingled jet of air, steam, and partially atomised oil, is met by jets of steam coming from a ring *C*. These jets effectually complete the atomising, with the result of securing complete combustion of the fuel. The steam required to run the injector is taken from the boiler. The jet action of the steam flowing through the nozzles enables it to draw in the air supply through the central orifice ; indeed the vacuum, which may be

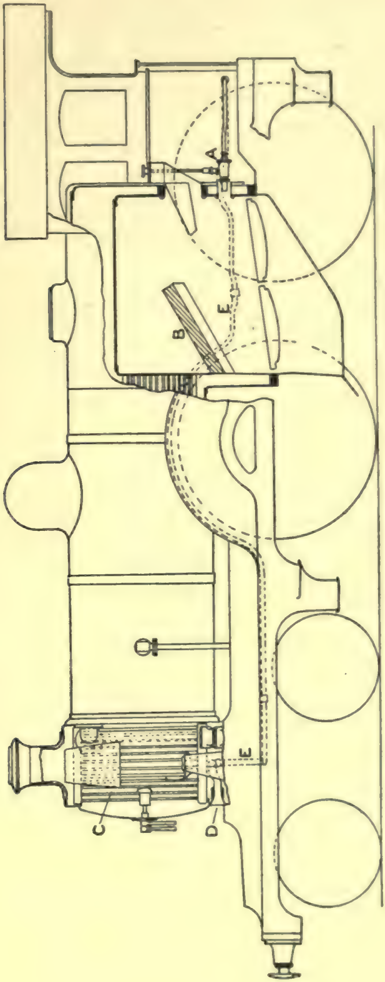


Fig. 294.—Oil fuel arrangement on a Great Eastern Railway express passenger locomotive.

maintained by the injector, has been applied for the working of the vacuum brakes. Regulation of the oil supply is accomplished by means of valves operated by hand wheels under the control of the driver.

Coal is used for raising steam in the boiler. The furnace is fitted with a specially large fire-brick bridge *B* (Fig. 294), towards which the injector jets are directed. This bridge intercepts and vaporises any oil which has escaped being atomised by the injectors. A thin layer of incandescent fuel is kept on the fire-bars during running. The air required for the injectors

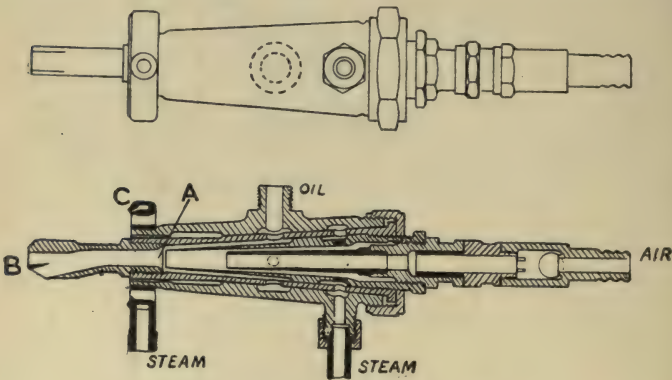


Fig. 295.—Holden's liquid fuel injector.

may be supplied hot by drawing it through a steam heating arrangement consisting of a number of pipes *C* placed in the smoke box. The air enters through the orifice *D*, and after passing through the heater is led through the pipe *E* to the injectors. The oil supply is contained in tanks placed in the tender and connected by pipes to the injector.

The arrangement renders it possible to use coal firing alone, or oil firing alone, or both coal and oil may be burned simultaneously. The alteration from one system to the other can be effected while the engine is actually running. The adaptability of the Holden system has led to its adoption all over the world.

Locomotives.—Locomotives consist in general of an engine having a pair of cylinders, the pistons of which are connected by ordinary crank and connecting rod mechanisms to a crank shaft having a pair of cranks placed at right angles; this arrangement gives fairly uniform turning moment on the crank shaft, and permits of the engine being started from any position of rest. The crank shaft has a pair of driving wheels, one at each end, which are properly constructed so as to run on the rails. The engines are mounted on a frame, on which also is mounted a boiler, situated over the engines. The whole is supported on wheels, springs being placed between the axles and the frame in order to reduce shocks. Frequently there are four, six, eight or even ten driving wheels, these being mounted on axles in pairs, one axle being driven direct by the engines and the others being connected to the first by means of coupling rods and cranks placed at right angles. The locomotive is said to be four, six, etc., coupled, depending on the number of driving wheels.

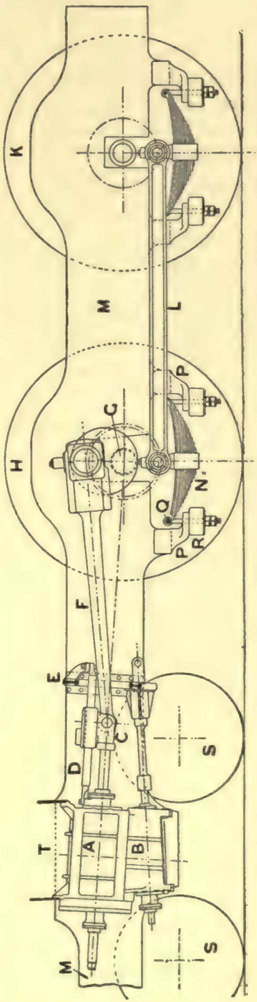


Fig. 296.—Driving mechanism of a Great Eastern Railway express passenger locomotive.

Inside cylinder locomotives have the cylinders placed between the frames, thus necessitating the use of cranked driving axles. **Outside cylinder** locomotives have the cylinders placed outside the

frames, working on crank pins secured to the driving wheels, the driving axles being straight in this arrangement. The tractive effort of the locomotive is secured by reason of the friction between the driving wheels and the rails. There must therefore be a sufficient proportion of the weight of the locomotive carried by the driving wheels in order to secure the required frictional adhesion.

The valves are usually of the simple slide valve type. Generally in this country Stephenson's link motion is the gear used for enabling the engine to run in either direction.

By the courtesy of Mr. James Holden, some of the details of a Great Eastern Railway express passenger locomotive are given here. The locomotive is four coupled, of the inside cylinder type. The boiler has already been described (p. 323).

The engine mechanism.—The arrangement of the principal parts of the mechanism will be understood by reference to Fig. 296, which shows the connections to one of the cylinders, the valve gear being omitted. The cylinder A has the valve chest B placed underneath and is bolted securely to the side frames M. The piston rod is connected to a crosshead C, which slides on a single guide bar D, the latter being bolted to the cylinder at one end and to the motion plate E at the other end. The motion plate consists of a casting running between the two side frames M and is bolted to them at each of its ends; it serves to carry one end of the guide bars and also guide brackets in which the slide valve spindles move. The connecting rod is shown at F and the driving axle at G. The driving wheels H and K on the side of the locomotive shown are coupled by the coupling rod L. A similar coupling rod connects the pair of wheels on the other side of the locomotive.

The driving axles run in axle boxes, which rest on coach springs N; these springs are slung from brackets P, bolted to the side frames, by means of suspending rods Q. The rods pass through cases R, containing several rubber washers, which are put under compression by the weight of the machine and assist in damping vibrations. The weight of the front part of the locomotive is taken by a bogie or small truck having four wheels, two of which are indicated at S, S.

The valve gear.—The valve gear is of the Stephenson link

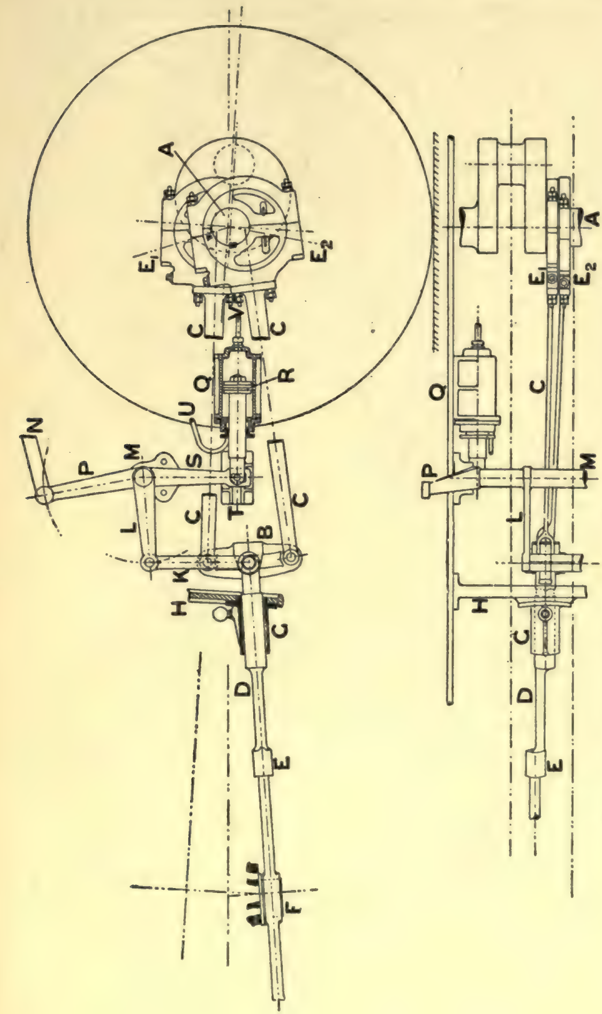


FIG. 297.—Valve gear of a Great Eastern Railway express passenger locomotive.

motion type, the arrangement for one cylinder being illustrated in Fig. 297. The eccentrics E_1 and E_2 are secured to the driving axle A and are connected to the link B by means of eccentric rods C, C, which are shown broken in the illustration. The valve rod D is in two parts, connected at E by a cotter joint, and is connected to the valve at F. The valve rod works in a guide G secured to the motion plate H, and carries at its outer end a block which slides in the slotted link B. The link is suspended by lifting rods K from a lever L on the reversing shaft M. The link of the motion belonging to the other cylinder is connected in a similar manner to a lever fixed to the same shaft. Both links are put over simultaneously on operating the reversing shaft by means of the rod N, which is connected to the reversing lever at the foot plate, and to the lever P on the reversing shaft.

An air cylinder is fixed to the side frame at Q, its function being to support the weight of the links and half the weight of the eccentric rods, and also to supply a means of reversing the engine by power, thus reducing the manual effort which must otherwise be applied to the reversing lever. The piston R is attached to a piston rod of large diameter, which is connected at its outer end to the lever S fixed to the reversing shaft. The piston rod is guided by a block sliding in a bracket T secured to the side frame. Pipes for supplying air under pressure are connected to each end of the cylinder at U and V, and lead to a valve under the control of the driver, and not shown in Fig. 297. In the running position, both ends of the air cylinder are in communication with the main air reservoir, the effect being to produce a resultant force on the piston urging it towards the left. The ratio of the diameter of the piston rod to that of the cylinder is adjusted so that the resultant force may be sufficient to support the weight of the motion hanging from the links K. In reversing, the valve is operated so as to put one side of the cylinder in communication with the exhaust, the other end being supplied still with air under pressure, thus causing the piston to move from one end of the cylinder to the other and so to raise or lower the links.

The indicator.—In ordinary engines with pistons reciprocating in cylinders, the energy delivered to the engine is

measured from the diagram of work done on the piston. These diagrams are drawn by means of an instrument called an **indicator.** The essential parts of an indicator consist of a small cylinder (Fig. 298), which can be connected with the cylinder of the engine by means of a cock. A piston moving in this small cylinder is controlled by a spring. The elastic properties of this spring cause the piston to take up a definite position in its cylinder depending on the pressure exerted on it. The movement of the piston is communicated by linkwork to a pencil, the

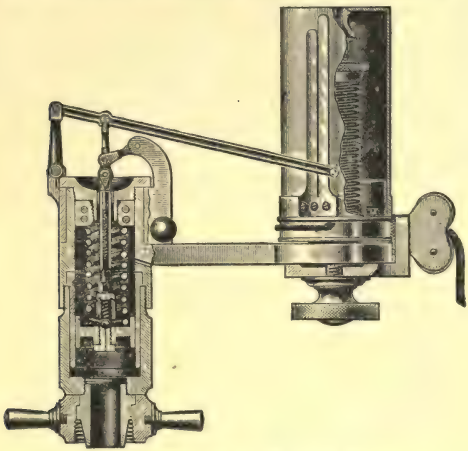


Fig. 298.—Crosby Steam Engine Indicator.

linkwork being arranged so that a straight line motion is given to the pencil. The pencil takes up a position corresponding with the pressure in the cylinder. A paper, stretched usually over a drum, is caused to move transversely under the pencil, being driven to and fro by being connected to some part of the engine so as to give a faithful copy of the motion of the piston of the engine to a reduced scale. When the pencil is pressed on the paper, the indicator being connected to one end of the engine cylinder, it will trace a curve showing the pressure on the piston at any part of the double stroke of the engine. From this curve the average pressure on the piston may be

found. A *datum line*, showing atmospheric pressure, is traced on the paper by putting the indicator cylinder in communication with the atmosphere. A small side hole in the communication cock enables this to be done. Fig. 298 shows the Crosby Steam Engine Indicator.

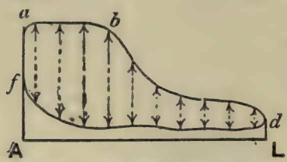


FIG. 299.—Indicator diagram for one end of a cylinder.

The diagram (Fig. 299) represents what the indicator might draw if connected to a steam engine cylinder. AL is the datum atmospheric line. ab shows the admission of steam at high pressure to the engine cylinder. At b the steam is cut off and expands during the remainder of the stroke, falling in pressure as it does so. df shows the back pressure on the piston during the return stroke.

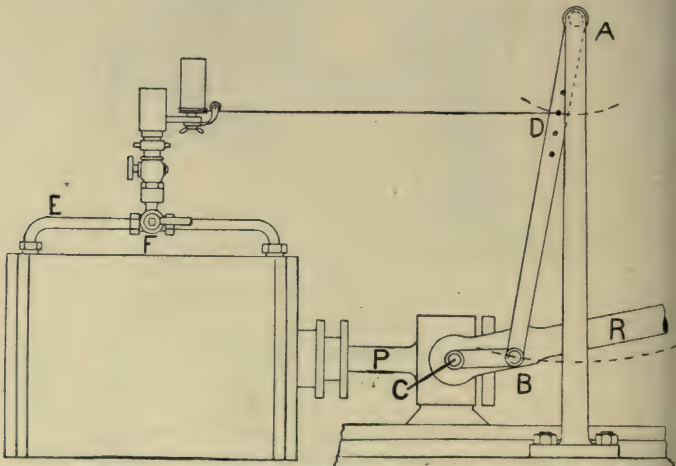


FIG. 300.—Arrangement for connecting up and driving an indicator on a steam engine.

Connecting up the indicator.—The indicator is connected usually to the cylinder by means of a bent pipe E (Fig. 300), which is attached to both ends of the engine cylinder and has

a branch near the middle of the pipe for mounting the indicator. A three-way cock *F*, at the branch connection, enables either end of the cylinder to be put into communication with the indicator, so that diagrams may be taken from either end of the cylinder.

Usually the stroke of the engine is too great to permit of the paper drum being driven by direct connection to the crosshead. A **reducing gear** has generally to be employed. A simple form of such gear is shown in Fig. 300. A rod *AB* has one end *A* pivoted to a fixed axis and its other end *B* connected to the centre of the crosshead pin, by means of a short link *BC*. The cord driving the indicator drum is attached to the link *AB* by fastening it through one or other of the small holes at *D*. In Fig. 311, *P* is the piston rod and *R* the connecting rod.

The function of the reducing gear is to give to the drum a faithful copy of the motion of the piston on a reduced scale. Too many joints in such gears are objectionable on account of their liability to work loose.

The indicator in use.—In using an indicator, attention should be given to the following points:

1. See that the indicator is mounted properly, and that the joints of the connection to the cylinder pipes do not leak. Arrange the position of the drum and guide pulleys so that the cord runs freely when connected to the reducing gear.

2. Adjust the length of the cord so that the drum does not knock against the stops at either end of its stroke.

3. Choose a spring of proper strength to suit the maximum pressure to be expected.

4. See that the pencil is sharp; but in the case of a metallic pencil, the paper may be cut if the pencil is too sharp.

5. Oil the piston before inserting it. If in use during a long test, remove the piston occasionally for cleansing and oiling.

6. When finished, remove the indicator immediately and clean it thoroughly before putting it away. Do not leave the spring in the cylinder; it should be cleansed and oiled to prevent corrosion.

Taking diagrams.—The following procedure may be followed:

1. Fold over about $\frac{1}{4}''$ of one short edge of the paper. Insert this edge in one of the drum clips. Bend the paper round the drum and insert the opposite edge in the other clip. Pull the

paper down the drum and see that it is pressed tightly against it. Fold the short edges over the clips.

2. Connect the drum cord to the reducing gear.

3. Bring the pencil to the paper and see that the pencil is adjusted so that undue pressure does not occur, otherwise the friction of the pencil will be excessive and will introduce errors in the pressures shown.

4. Connect the indicator cylinder to the atmosphere by means of the cock and draw the atmospheric line.

5. Open the cock communication to one end of the cylinder, and allow the indicator piston to rise and fall several times so as to warm the indicator cylinder. Turn the indicator cock two or three times open to the atmosphere so as to blow any water out.

6. Bring the pencil to the paper to draw the diagram. Repeat the operations for the other side of the engine cylinder.

7. Disconnect the drum cord and remove the paper. Be careful that dirty fingers, steam, oil and water do not touch the diagram on the paper.

8. Write at once the following information on the card:

   (*a*) Date and time card was taken.
   (*b*) Engine from which card was taken.
   (*c*) Scale of spring.
   (*d*) Revolutions per minute at time card was taken.
   (*e*) Diameter and stroke of the engine piston.

Indicated horse-power.—Fig. 301 shows a pair of indicator diagrams taken from the two sides of the piston of a steam engine. From these the mean pressures exerted on the opposite sides of the piston may be determined. Having ascertained the mean pressures, the work done on the piston during each forward and backward stroke can be calculated; and, knowing the number of such double strokes per minute, the work done on the piston per minute can be estimated. If this result, stated in foot-pounds, be divided by 33,000, the calculation will give the horse-power developed on the piston of the engine. This is called the **Indicated Horse-Power** (written generally I.H.P.), as it depends on a knowledge of the mean pressure obtained by use of the indicator.

Calculation of I.H.P.—The mean pressures have been deduced from the diagrams in Fig. 301 by first drawing lines touching

the extremities of the diagrams and perpendicular to the atmospheric line AL. The distance between these is then divided into ten equal parts and an ordinate drawn from the middle of each part. By this process the same points of division serve for both diagrams. The heights of these ordinates measured

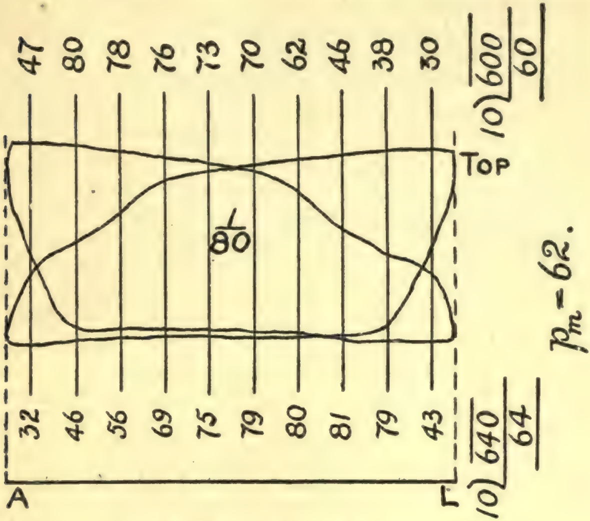


FIG. 301.—Example of an indicator card.

between the curves of each diagram are written down, using the scale of the spring, those belonging to the left-hand diagram being placed above the curves, the others being below. The sums of these columns divided by 10 give the mean pressure for each side of the piston.

The average value of the mean pressures so obtained may be calculated approximately by taking their sum and dividing by 2. Using this as an average pressure common to both sides of the piston and writing it p_m lbs. per square inch, we have:

Work done on piston per stroke $= (p_m \times A \times L)$ ft.-lbs.

where $A =$ area of piston in square inches,

$L =$ length of stroke in feet.

Let $\qquad N$ = revolutions per minute,

then $\qquad 2N$ = number of strokes per minute ;

$\therefore$ work done on piston per minute $= (p_m AL2N)$ ft.-lbs.

$$\text{I.H.P.} = \frac{2p_m ALN}{33{,}000} \quad \ldots\ldots (1)$$

Brake horse-power.—A considerable portion of the horse-power developed on the piston is not available for driving outside machinery. Generally from 5 to 25 per cent. of the I.H.P. is used in overcoming frictional resistances in the mechanism of the engine itself. To obtain an estimate of the horse-power which the engine can deliver, the energy delivered by the engine may be absorbed by means of a brake mounted on the fly-wheel. A common form of brake is illustrated in Fig. 302, and consists of a double rope passed round the wheel and held in position by means of loosely fitting wood blocks secured to the rope. Four such blocks are shown in the illustration, a separate view being given of one to indicate the manner in which they embrace the rim of the wheel. One end of the rope is attached to a spring balance suspended from overhead, the other end carries a load W lbs. The direction of rotation of the wheel being as shown by the arrow, it will be noticed that the pull, P lbs., of the spring balance is helping to turn the wheel, and W is opposing its rotation. The friction between the ropes and the rim communicates these forces to the wheel.

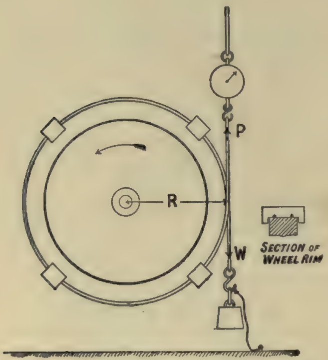


FIG. 302.—A common form of rope brake.

Let R = radius, in feet, to the centre of the rope, then

Net resistance against which the wheel is working $= W - P$ lbs.

As this resistance is applied at a radius R, the distance through which it is overcome in one revolution will be

$$2\pi R \text{ feet.}$$

$$\therefore \text{ Work per revolution} = (W - P)2\pi R \text{ ft.-lbs.}$$

For N revolutions per minute,

$$\text{Work per minute} = (W - P)2\pi R \,.\, N \text{ ft.-lbs.},$$

$$\text{Horse-power} = \frac{(W - P)2\pi RN}{33{,}000}.$$

The result of this calculation, called the **Brake Horse-Power** of the engine (written B.H.P.), gives the horse-power which the engine can deliver for driving machines, etc.

Mechanical efficiency.—The efficiency of any machine is defined as the ratio of the energy which may be obtained from the machine to the energy supplied to the machine. Applying this definition to the mechanism of an engine, the energy supplied to the piston per minute will be obtained by multiplying the I.H.P. by 33,000; also the energy which may be obtained from the engine in one minute will be the product B.H.P. $\times$ 33,000. The efficiency of the mechanism, or **mechanical efficiency** as it is called, will be

$$\text{Mechanical efficiency} = \frac{\text{B.H.P.} \times 33{,}000}{\text{I.H.P.} \times 33{,}000}$$

$$= \frac{\text{B.H.P.}}{\text{I.H.P.}}.$$

The result is multiplied by 100 generally so as to have it as a percentage. The mechanical efficiency generally lies between 75 and 95 per cent.

EXERCISES ON CHAPTER XIX.

1. Give sketches and describe the arrangement of the brickwork flues of a Lancashire boiler. Indicate the flow of the furnace gases by arrows.

2. Sketch and describe the shell of a Lancashire boiler.

3. Give sketches and description of the shell of a locomotive boiler.

4. Give sketches showing how liquid fuel may be used in a boiler furnace.

5. Sketch and describe a liquid fuel injector.

6. Give an outline sketch showing the cylinder and driving mechanism of a locomotive.

7. Sketch in outline and describe the valve gear of a locomotive.

8. Give an outline sketch of the parallel motion, piston, and piston rod of any indicator.

9. Sketch in section and describe the cylinder and piston of any indicator.

10. Give sketches and description of any kind of reducing gear for connecting up an indicator to the engine crosshead.

11. In a steam engine cylinder, the mean pressure was found to be 26·7 lbs. per square inch for one side of the piston and 28·2 lbs. per square inch for the other side. The piston is 24″ diameter × 36″ stroke. Calculate the I.H.P. when running at 95 revolutions per minute.

12. Give a sketch and description of any form of brake which may be used for obtaining the B.H.P. of an engine.

13. In testing an engine for B.H.P. a brake similar to that shown in Fig. 302 was used. The speed of the engine was 230 revolutions per minute; the pull of the spring balance 15 lbs., and the dead load 84 lbs. The brake wheel was 4′ 9″ diameter to the rope centre. Calculate the B.H.P.

14. In Question 13 the mechanical efficiency was found to be 82·5 per cent. What was the I.H.P.? How much energy in foot-lbs. per minute is used in overcoming frictional resistances of the engine?

15. Describe, with sketches, how you would take an indicator diagram of a steam engine. Sketch a possible diagram and explain how you would calculate the indicated horse-power. What information is necessary?

16. The mean effective pressure on the piston, both in the forward and back strokes, is 62 lbs. per square inch; cylinder 18″ diameter; crank 18″ long. What is the work done in one revolution?

CHAPTER XX.

EFFICIENCY OF THE STEAM ENGINE AND BOILER.

Waste of energy.—Some of the sources of loss in the steam engine and boiler, and the means taken to produce the best results, should be examined now. The student will find it useful to consider the following calculations.

EXAMPLE i. In a certain economical steam plant, the engine gives one horse-power for an hour for each $1\frac{1}{2}$ pounds of coal fed into the boiler furnace. The heating value of the coal is 15,000 B.T.U. per lb. What percentage of the heat energy of the coal is transformed into mechanical work?

$$\begin{aligned}\text{Energy derived from 1 H.P.} &= 33{,}000 \text{ ft.-lbs. per min.}\\ &= (33{,}000 \times 60) \text{ ft.-lbs. per hr.}\\ \text{Energy supplied to produce this} &= 15{,}000 \times 1\tfrac{1}{2}\\ &= 22{,}500 \text{ B.T.U.}\end{aligned}$$

Taking $J = 778$,

$$\begin{aligned}\text{Energy supplied} &= 22{,}500 \times 778\\ &= 17{,}500{,}000 \text{ ft.-lbs.}\\ \text{Required percentage} &= \frac{33{,}000 \times 60}{17{,}500{,}000} \times 100\\ &= \underline{11 \cdot 3}.\end{aligned}$$

The remaining 88·7 per cent. of the heat contained by the coal has disappeared in various ways as waste.

EXAMPLE ii. In the boiler of the above steam plant, the feed water enters at a temperature of 240° F. The steam pressure is 175 lbs. per sq. in. absolute, and 11 lbs. of water are converted into steam for each lb. of coal burned. Calculate what percentage of the heat energy of the coal is taken up by the steam leaving the boiler, assuming such steam to be dry.

From the Table, p. 365, the total heat of steam at 175 lbs. per sq. in. absolute is

$$H = 1194{\cdot}9 \text{ B.T.U. (reckoned from } 32^\circ \text{ F.).}$$

As the water enters the boiler with a sensible heat given by

$$\begin{aligned} h &= 240 - 32 \\ &= 208 \text{ B.T.U.,} \end{aligned}$$

it follows that

$$\begin{aligned} \text{Heat supplied to 1 lb. of steam in the boiler} &= 1194{\cdot}9 - 208 \\ &= 986{\cdot}9 \text{ B.T.U.} \\ \text{Heat entering steam for each lb. of coal} &= 986{\cdot}9 \times 11 \\ &= 10{,}860 \text{ B.T.U.} \\ \text{Required percentage} &= \frac{10{,}860}{15{,}000} \times 100 \\ &= \underline{72{\cdot}4.} \end{aligned}$$

Example iii. In the above steam plant, the engine produces a horse-power for an hour for every 18 lbs. weight of steam supplied. The steam is exhausted and condensed into water at a temperature of 130° F. Calculate what percentage of available energy is being converted into work, assuming the steam in the steam chest to be dry.

The steam entering the engine has a total heat of 1194·9 B.T.U., and the heat in each lb. of water leaving the condenser amounts to (130 − 32) = 98 B.T.U. The available energy per pound weight of steam is therefore given by

$$\begin{aligned} \text{Available energy} &= 1194{\cdot}9 - 98 \\ &= 1097 \text{ B.T.U. per lb.} \\ \text{Available energy per hour} &= 1097 \times 18 \\ &= 19{,}750 \text{ B.T.U.} \\ \text{Work produced per hour} &= 33{,}000 \times 60 \\ &= 1{,}980{,}000 \text{ ft.-lbs.} \\ &= \frac{1{,}980{,}000}{778} \\ &= 2545 \text{ B.T.U.} \\ \text{Required percentage} &= \frac{2545}{19{,}750} \times 100 \\ &= \underline{12{\cdot}9.} \end{aligned}$$

The remaining 87·1 per cent. of the heat supplied to the engine is wasted.

It may be observed from these examples that the boiler is by far the most efficient part of the plant. In the boiler the action

consists in the mere transference of heat energy from the furnace to heat energy in the steam, and such an operation is conducted with but little loss. In the engine the energy has to be converted from the form of heat into that of mechanical work, and operations of this character are accompanied always with great waste.

Causes of waste in the engine.—In the attempt to minimise so far as possible the waste of energy in the engine, engineers have been led to study the important effects of the **action of the cylinder walls** on the steam and water present in the cylinder, and also the **leakage** past the valves and piston. The student must guard against the error of supposing that the mechanism intervening between the piston and crank has anything to do with wastage other than the unavoidable waste due to frictional resistances. The principle of the conservation of energy teaches that whatever amount of mechanical work is done on the piston will appear as work done on the crank pin, less that required to overcome the frictional resistances of the mechanism. The wasted power due to this cause may be calculated by taking the difference between the I.H.P. and B.H.P., and may be from 5 to 25 per cent. of the power developed.

Action of the cylinder walls.—Unless the steam supplied is superheated, the contents of the steam chest always consist of steam with a comparatively small percentage of water. On admission, this mixture enters the cylinder and meets surfaces which have been cooled during the previous exhaust stroke, having been in contact with cool exhaust steam. Consequently, much of the entering steam is condensed, its latent heat being imparted to the walls of the cylinder. During expansion, the mixture falls in pressure and also in temperature. A point will be reached beyond which, as expansion goes on, the temperature of the mixture will be lower than that of the walls. Heat will then pass from the walls into the mixture and will be expended in re-evaporating some of the water formed from the steam condensed during the admission period. On the exhaust valve opening, the contents of the cylinder still will consist largely of water, but the pressure, and consequently the temperature, of the mixed steam and water will be much lower. The action of the hot walls will therefore be very vigorous, and the water will

be evaporated rapidly, thus increasing the back pressure by the generation of steam and also producing the undesirable effect of cooling the walls so that condensation will be energetic during the next admission period.

Means of limiting the action of the walls.—The action of the cylinder walls would be practically negligible if we could prevent the entry of water into the cylinder, and also its formation afterwards by condensation of some of the steam. Dry steam only should enter the cylinder, and, in order to secure this result, **separators** are often placed in the steam supply pipe close to the steam chest. This device gets rid of most of the water carried along the steam pipe. A more efficient way is to employ **superheated steam.** Such steam contains no moisture, and may be cooled considerably before the saturation temperature is reached and condensation begins.

Should any water form in the cylinder, means must be provided for its prompt **drainage** immediately exhaust begins. It is obvious that any water present is better got rid of as water which will carry away sensible heat only, than by being boiled off into steam at the expense of the heat contained by the cylinder walls.

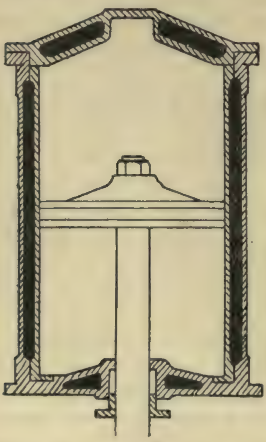


FIG. 303.—Section of a cylinder, steam-jacketed at the parts shown black.

Higher speed of revolution diminishes the action of the walls, the reason being the shorter time given for the action to take place.

Steam jackets are employed often, the method consisting of surrounding the whole of the cylinder with steam at boiler pressure (Fig. 303). The idea originally was to keep the walls of the cylinder as hot as the steam entering the cylinder. The jacket is only partially successful in effecting this, as the action takes place at the inner skin of the cylinder wall while the jacket steam is in contact with the outer skin. The action of

the jacket is beneficial, however, in reducing the range of temperature to which the walls are subjected during the stroke, thus tending to reduce initial condensation during admission and also to hasten the re-evaporation. The steam in the jacket giving up its heat to the walls will undergo condensation, and means must be provided for draining the resulting water away through a steam trap. In fact, a great deal of the benefit derived from the use of a steam jacket consists in removing the region of condensation from the cylinder, where drainage is imperfect, to another place, viz. the jacket, where drainage can be very perfect. Steam at boiler pressure alone should be used in the jacket.

Leakage.—Leakage of steam past the slide valve into the exhaust port probably takes place by the condensation of a film of water on the port face of the cylinder. The valve slides over the film, thus uncovering it to the exhaust, where, as its temperature is higher than that corresponding to the lower pressure, it instantly boils off and is so wasted. The problem is complicated by the fact that a slide valve may be quite tight when cold and yet leak considerably when heated, the effect being due to unequal expansion, causing warping of the valve or of the cylinder face.

Efficiency of a perfect heat engine.—A perfect heat engine, according to Carnot's theory, is one having the following cycle of operations. The engine is supposed to take in its supply of heat from a source of heat which is kept at a constant absolute temperature τ_1, and to get rid of heat not required to another source which is kept at a constant lower absolute temperature τ_2. Thus, any heat passing into or out of the engine will do so at constant temperature, either τ_1 or τ_2, and will thus give isothermal operations. Any other operations taking place in the engine will be conducted without heat being given to or abstracted from the steam or other working substance, *i.e.* these operations will be adiabatic. Suppose the working substance to be at temperature τ_2; the cycle may be imagined to be conducted as follows: first compress the working substance adiabatically, thus raising its temperature; stop compression when the temperature is τ_1, and then allow the working substance to expand doing work, keeping the temperature constantly

τ_1 by allowing heat to pass in from the source of heat. This operation is stopped at any desired point, and the expansion thereafter is continued adiabatically. The temperature will fall, and the expansion is continued until τ_2 is reached. The working substance is now compressed isothermally, allowing heat to escape to the cold source at constant temperature τ_2 until the initial conditions are reached, when the cycle may be started again.

It may be shown that such an engine is more efficient than any other engine having a different cycle, and that its efficiency is measured by the ratio

$$\text{Efficiency} = \frac{\tau_1 - \tau_2}{\tau_1}.$$

Practically, the cycle cannot be realised and the efficiencies of all engines fall far short of this amount. The expression, however, indicates that theoretically the greater the distance between τ_1 and τ_2 the higher will be the efficiency. Practice confirms this conclusion within limits.

Means of increasing the ratio of expansion.—To obtain a large difference between τ_1 and τ_2, high boiler pressures are used (up to 300 lbs. per square inch, the temperature being $417\frac{1}{2}°$ F.) and the steam is expanded many times, with a correspondingly large fall in temperature. In engines discharging their exhaust into the atmosphere, the terminal pressure may be 20 lbs. per square inch absolute, giving a terminal temperature of 228° F. If a condenser be employed, the terminal pressure and temperature may be lowered to 4 lbs. per square inch and 153° F., thus enabling larger ratios of expansion to be used.[1]

To convert a non-condensing engine into a condensing engine, a condenser and air pump must be added. The condenser may be a **surface** or a **jet** condenser. In the first type, which is shown in diagram form at *D* in Fig. 304, the exhaust steam is brought into contact with the outer surface of a large number of tubes *E*, kept cool by causing water to circulate through them, the water entering at *F* and being discharged at *G*. A pipe *H* leads to the air pump *J*, the function of which is to clear the water of condensation and air from the condenser.

[1] Recently in some motor car engines having steel cylinders 4″ diameter, steam pressures up to 1000 lbs. per square inch have been employed.

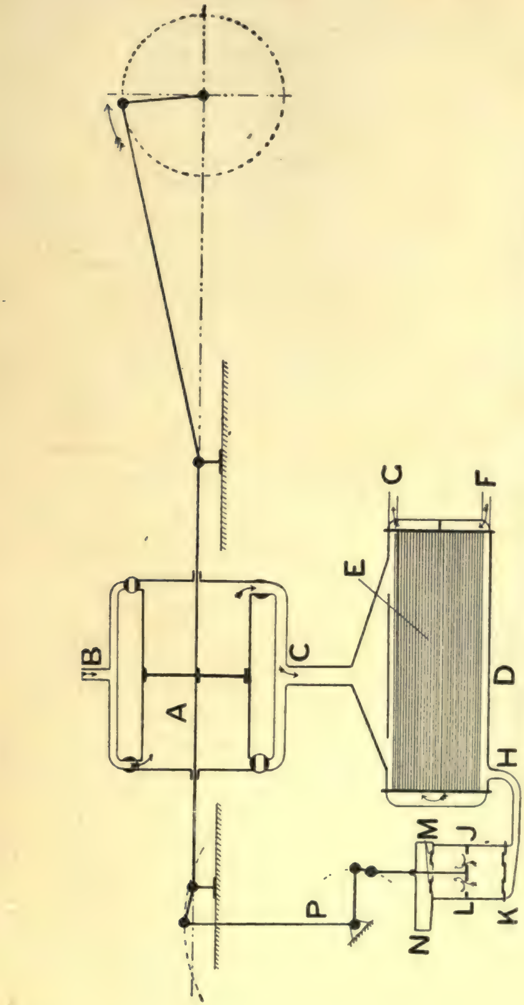


FIG. 304.—Diagram of a steam engine with surface condenser and air pump.

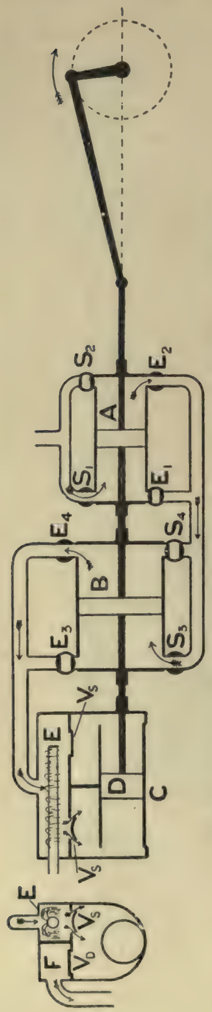


Fig. 305.—Diagram of a tandem compound condensing engine with jet condenser.

The air is contained in the feed water, passes from the boiler with the steam and is discharged finally into the condenser. The air pump has foot valves at *K*, bucket valves at *L* and head valves at *M*. The water is discharged by the air pump into the hot well *N* and is pumped by the feed pump back to the boiler. The air pump shown is driven from the tail end of the piston rod by means of a bent lever *P*.

In the jet condenser (Fig. 305, *E*) a spray of cold water is played on the entering exhaust steam, thus condensing it. It will be noticed that in this form of condenser, the injection water and water of condensation mingle, while in the surface condenser they are kept apart.

Ratios of expansion higher than 2 or 3 cannot be carried out successfully in a single cylinder, as there would be much too great a difference between the piston effort at the beginning and end of the stroke, with correspondingly great differences in the turning moment, and also the range of temperature through which the walls have to pass is much too large. The condensation in such a cylinder, having a ratio of expansion of say 12, would be enormous. It is the custom, therefore, to admit the steam into one cylinder and begin the expansion therein, then to exhaust from this cylinder into another where the expansion is continued. Such engines are called **compound engines**. A third and fourth cylinder may be used,

giving **triple expansion** and **quadruple expansion engines** respectively. The ratio of expansion may thus be increased to 12 or 15.

A common type of horizontal compound engine is shown in outline in Fig. 305. The cylinders are arranged **tandem**, *i.e.* in the same straight line and working on the same crank. Both pistons are mounted on the same rod. The H.P. (high pressure) cylinder is shown at A, the L.P. (low pressure) cylinder at B, and the air pump at C, the bucket of the last being mounted also on the piston rod. In Fig. 305 the piston rod is moving towards the crank on the out stroke; steam is entering the H.P. cylinder through the valve S_1, and is being exhausted from the other side of the piston into the L.P. cylinder through E_2 and S_3. The other side of the L.P. cylinder is in communication with the condenser through the valve E_4. The condenser illustrated is of the jet type, with a double acting air pump. There are no valves in the air pump bucket D; each side of the pump is furnished with suction and discharge valves V_S and V_D respectively. The suction valves allow water and air to pass from the condenser E into the air pump, and the discharge valves communicate with the hot well F.

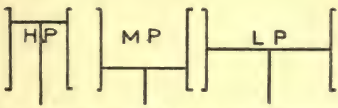


FIG. 306.—Cylinder arrangement of a three crank triple expansion engine.

Compound engines are arranged more usually with the cylinders side by side working on separate cranks. Triple

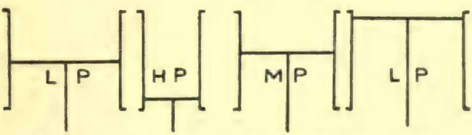


FIG. 307.—Cylinder arrangement of a four crank triple expansion engine.

expansion engines are arranged in many different ways. Figs. 306 and 307 show two such arrangements, in the first of which

there are three cranks and in the second four cranks, there being two L.P. cylinders, which divide between them the steam discharged from the M.P. (intermediate pressure) cylinder.

Too large a ratio of expansion is undesirable, as little benefit will be derived from the expansion in the later stages. The last cylinder is large, and condensation and friction in it will tend to counterbalance any gain which may otherwise be obtained.

Gain from condenser.—Besides the gain which has been noted already due to the effect of the condenser in lowering the terminal pressure and temperature, the condenser gives a means of obtaining a supply of hot feed water. The water produced from the condensation of the exhaust steam in a surface condenser may be at a temperature of 120° F., and, if fed into the boiler, will require less heat to evaporate it than if cold feed water is employed. Further, this water already has been through the boiler and is therefore distilled water, *i.e.* all salts have been got rid of. Thus, in marine engines, a means is provided of feeding fresh water into the boiler instead of sea water, the salts from which would be deposited in the boiler.

Wasted heat in the boiler.—Some of the causes of waste of heat are due to bad stoking, giving **imperfect combustion** of the fuel and the consequent escape of combustible gases up the chimney. The admission of air to the furnace in excess of that required for complete combustion of the fuel is also a source of waste. Each pound of **excess air** is heated in passing through the furnace, where it does no good, and carries away some of the heat which would otherwise be utilised in heating the water in the boiler. The presence of **soot**, etc., on the hot-gases' side of the heating surface, and of **scale** on the water side, **impairs the heating surface** by preventing the free passage of heat through the plates and so lowering the efficiency. **Bad circulation** of the water in the boiler prevents the free formation of steam. To obtain the best results from the heating surface, the current of hot gases should be broken up and **scrubbed** as it were against the plates, in order to extract as much heat from them as possible. Other causes of waste are in unburnt fuel dropping through the spaces in the fire bars into the ash pit, and in conduction from the surfaces of the boiler which are exposed to

the air. To minimise the latter, all such surfaces should be coated as far as possible with non-conducting material.

Some of the heat which otherwise would escape up the chimney may be saved by the use of **economisers** or feed-water-heaters placed in the flues between the boilers and the chimney and heated by the furnace gases. The best known type is **Green's economiser** in which there is a number of vertical tubes through which the feed water is pumped. The furnace gases pass round the exteriors of the tubes and give up some of their heat to the water.

Draught.—Chimney draught is relied upon often for the production of the required current of air through the furnace. The chimney operates by reason of the difference in temperature inside and outside of it. Hot air weighs less per cubic foot than cold air. Thus, the column of gas inside the chimney weighs less than the corresponding column outside, and there will be less pressure inside at the base of the chimney than exists outside. The excess pressure of the atmosphere outside will cause a current to flow through the furnace towards the chimney.

In other cases chimney draught may not be sufficient to cause a high enough rate of combustion per square foot of grate area. Chimney draught may be increased somewhat by extending the height of the chimney, but there is a limit to this gain, set by the additional frictional resistances of the walls in a tall chimney, and also by the cooling of the gases. Artificial means of increasing the draught are employed then. In **forced draught,** the air is driven through the furnace by a fan or blower operated by a steam or other motor. In **induced draught,** the fan is placed at the bottom of the chimney, and operates by drawing the gases from the flues and discharging up the chimney. Artificial draught is beneficial in securing a plentiful supply of air properly distributed in the furnace and so may produce better combustion. It is found also that the proportion of excess air can be reduced considerably by the proper application of artificial draught. Artificial draught is much used for marine purposes, especially naval, and enables maximum power to be obtained out of the minimum dead weight of boilers.

Mechanical stoking.—With certain classes of coal being burned in large quantities in a battery of boilers, there is often

difficulty in obtaining with hand firing that uniformity of conditions which is essential to efficiency. In such cases, mechanical stokers often show a distinct economy over hand firing.

A well-known form of furnace in which the worst kinds of slack or bituminous coal can be burned without the production of black smoke is shown in Fig. 308. The grate consists of a broad endless chain, consisting of many short links, and passes over two drums, one outside the boiler front and one inside the

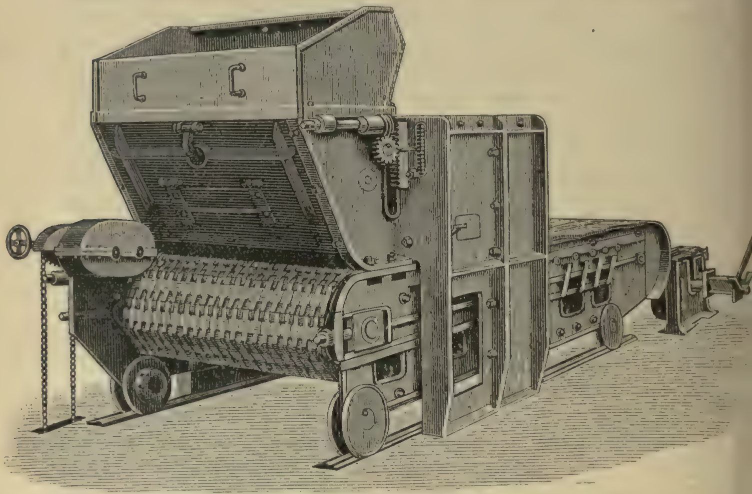


FIG. 308.—Babcock & Wilcox chain grate and coking mechanical stoker.

furnace. The front drum is revolved slowly by a worm and worm wheel, driven by a ratchet gear, having a rod actuated from a line shaft driven by a motor or small engine. The top side of the chain grate thus advances slowly inwards. Coal is supplied to a hopper and descends from it on to the grate to be carried into the furnace. The thickness of coal layer is adjusted by vertical lifting fire doors on the boiler front. Conversion into coke first occurs as the coal becomes heated, then the residual coke becomes incandescent as the hotter part of the furnace is reached. By the time the fuel has reached the inside drum, combustion is complete, and the ashes, etc., drop over the

drum as the chain bends round it, and so fall into the ash pit. It will be observed that clinker (fused ash) and ash are cleared automatically from the furnace and thus there is no necessity for opening the fire doors for cleaning or clinkering. No cold air can enter the furnace otherwise than between the bars of the grate. Further, the bars entering the furnace are cool, and thus there is less chance of clinker forming. The whole arrangement is mounted on a carriage running on rails and can be withdrawn clear from the boiler for examination and repairs.

EXERCISES ON CHAPTER XX.

1. In a certain steam boiler, 11 lbs. of water were evaporated per lb. of coal burned. The steam pressure was 150 lbs. per square inch absolute, and the temperature of the feed supply 120° F. The heating value of the coal was 15,200 B.T.U. per lb. Calculate what percentage of the heat supplied in the coal has been passed into the dry steam leaving the boiler.

2. A certain engine develops 1 I.H.P. for a consumption of steam of 15 lbs. weight per hour. Assuming the steam to be dry and at a pressure of 195 lbs. per square inch absolute in the steam chest, what quantity of heat per I.H.P. per hour is supplied to the engine?

3. In Question 2, the water of condensation is discharged from the condenser at a temperature of 130° F. Calculate what quantity of heat per I.H.P. per hour is available for conversion into work. What percentage of this is actually converted into work?

4. Give a brief account of the action of the cylinder walls on the steam during (*a*) admission, (*b*) expansion, (*c*) exhaust.

5. Explain briefly the effect of the following factors on cylinder condensation, (*a*) separators, (*b*) use of superheated steam, (*c*) drainage, (*d*) size of engine, (*e*) higher speed of revolution, (*f*) steam jackets.

6. Reciprocating engines cannot use economically a very large ratio of expansion. Explain why this is so.

7. What additional parts must be supplied to a simple engine in order to convert it into a condensing engine? Give an outline sketch and name the parts of a compound condensing engine with either jet or surface condenser.

8. Explain clearly why expansion carried out in several cylinders in series is more economical than the same degree of expansion carried out in one cylinder only. State any other practical reasons for the adoption of multiple expansion engines.

9. Enumerate the causes of wasted heat in generating steam in a boiler. Explain how these may be reduced.

10. Explain the action of any type of feed heater or economiser.

11. Give a description of the working of any type of mechanical stoker.

12. What are the objects in using (*a*) moderate forced draught, (*b*) high forced draught?

13. What do you understand by the efficiency of an engine? What would be the efficiency of a good marine engine and boiler which indicates one horse-power for every 2 lbs. of coal consumed in the furnace of the boiler per hour, supposing the coal to have a calorific power of 14,500 Fahrenheit Thermal Units per lb.?

14. Choose any kind of boiler. Explain how by its construction, 1st, the combustion is made as complete as possible; 2nd, as much of the heat as possible is given to the water. You need not speak of careful firing.

CHAPTER XXI.

INTERNAL COMBUSTION ENGINES.

Internal combustion engines.—Engines in which the combustion of the fuel is carried out in the cylinder or in vessels in direct communication with the cylinder are called internal combustion engines. The fuel employed may be solid, liquid, or gaseous. The use of solid fuel, such as coal in the form of dust, is in the experimental stage at present, and there are great difficulties to be overcome before a practical engine can be put on the market. Engines using various kinds of gas and oil are used very largely and are perfectly trustworthy. Almost all of these are operated under the Beau-de-Rochas cycle, laid down in 1862 and first successfully used in the Otto silent gas engine in 1876.

Beau-de-Rochas cycle.—This cycle is carried out on one side of the piston during four consecutive strokes as follows:

1. **Charging stroke.**—During the first out-stroke of the piston, an explosive mixture of gaseous fuel and air, or of oil which has been first gasified and air, is drawn into the cylinder.

2. **Compression stroke.**—This is the in-stroke of the piston immediately following the charging stroke. During this stroke, all the cylinder valves are closed, and the explosive mixture, or charge, is compressed by the returning piston into the clearance space at the rear of the cylinder.

3. **Explosion and expansion stroke.**—At the beginning of this stroke the charge is ignited. A very rapid increase of pressure occurs, due to the heat energy of the charge being liberated by the combustion, and the piston moves forward and completes its out-stroke under the influence of the pressure of the expanding products of combustion.

4. **Exhaust stroke.**—An exhaust valve is opened a little before the end of the expansion stroke is reached, and the products of combustion at once begin to rush out of the cylinder. During the succeeding in-stroke of the piston the waste gases are driven out of the cylinder, and, at the end of this stroke, the engine is ready to take in a fresh charge.

Diagram showing the cycle.—Fig. 309, which has been drawn from an indicator diagram taken from a gas engine, will explain the Beau-de-Rochas cycle. AL is a datum line showing atmospheric pressure.

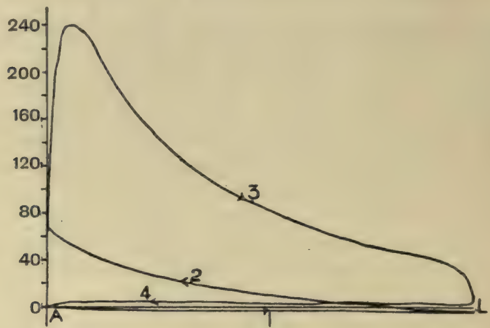


FIG. 309.—Indicator diagram showing the cycle in a gas engine.

1 is the charging stroke, during which the pressure falls slightly below atmospheric.

2 is the compression stroke, at the end of which the pressure will range from 50 to 200 lbs. per square inch above atmospheric pressure, depending on the type of engine.

3 is the explosion and expansion stroke.

4 is the exhaust stroke, during which the pressure usually rises slightly above that of the atmosphere.

General description of the engine.—Reference is made to Fig. 310, showing an internal combustion engine in diagram form. The cylinder A, in which the Beau-de-Rochas cycle is carried out is generally open at one end and only one side of the piston is used, *i.e.* the engine is single acting. The

piston B is connected direct to the crank C by a connecting rod D. The various valves for admitting the charge and for effecting the exhaust are of the mushroom type, held down on their seats by springs; the exhaust valve is shown at E. These valves are opened by levers operated from a side shaft driven from the crank shaft; as each valve is opened but once during every two revolutions of the crank shaft, it is convenient to reduce, by means of toothed wheels, the speed of the side shaft to half that of the crank shaft. As the heat developed during the combustion is large and produces a very high temperature, means must be taken for keeping the cylinder cool. Usually the cooling is effected by causing water to circulate in a water-jacket F, formed round the cylinder. Ignition of

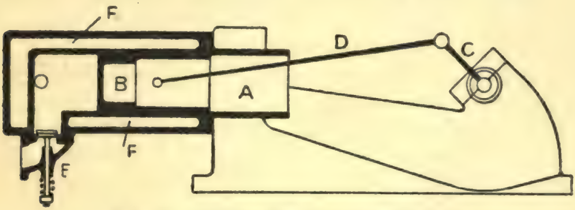


Fig. 310.—Outline diagram of a gas engine.

the charge may be effected by means of a hot tube, or by an electric spark. Governing of the speed of the engine is secured by cutting off or regulating the supply of fuel to the cylinder, the governor by which this is done being generally of the centrifugal type.

Cycle in oil engines.—Most oil engines in use at the present time follow the Beau-de-Rochas cycle of operations, the liquid fuel being vaporised and mixed with air so as to form an **explosive mixture.** The oil is prepared for introduction into the cylinder in several different ways. In many engines the oil is sprayed into a vaporising chamber which is heated externally by a lamp kept burning continually while the engine is running. In the vaporiser, the spray is mixed with air which has been warmed previously, and is converted into a vapour ready to be swept into the cylinder during the charging stroke. In other forms the vaporiser is kept warm by a

jacket through which the exhaust gases pass on their way from the cylinder to the atmosphere. In the Hornsby-Akroyd oil engine, a chamber at the rear of the cylinder, and in constant communication with it, is kept hot by the heat generated during the combustion in the cylinder. The chamber has no water jacket, and, when the engine is loaded so that a large quantity of heat is being generated in the cylinder, becomes red hot. Oil is sprayed direct into it, and the resulting vapour is mixed with the air supply drawn into the cylinder through a separate port. All these devices are suitable for the successful treatment of **heavy oils**, such as Royal Daylight (American) and Russolene (Russian) oils. The oil is supplied under a head obtained either by placing the oil supply tank at an elevation, whence the oil gravitates to the vaporiser, or by the use of a pump driven by the engine.

In engines using **light oils**, such as petrol, the vapour is much easier to obtain. These light oils give off inflammable vapour even at ordinary atmospheric temperatures, and consequently are vaporised very easily. It is sufficient merely to spray the oil into the incoming supply of air. The vaporiser, or carburettor as it is called in such cases, is placed close to the engine cylinder, and so is warmed slightly by conduction of heat from the cylinder. The effect is to produce carburretted air, *i.e.* air charged with petrol vapour, forming an explosive mixture.

Petrol motors.—Small motors using petrol (petroleum spirit) are used extensively for operating motor cars and cycles. The B.H.P. obtainable from one cylinder does not exceed 9 or 10, and usually the engines are smaller. Cycle motors range from 2 to 4 B.H.P. To obtain a larger power the number of cylinders is increased up to as many as six or eight. The arrangement is convenient, for the cranks can be placed relative to one another in such a manner as to produce a balanced or nearly balanced machine. Motor car engine cylinders are generally water-cooled, cycle engine cylinders are air-cooled. The cycle usually followed is the ordinary Beau-de-Rochas. Ignition is secured generally by means of an electric spark.

Description of cycle motor.—In Fig. 311 a small cycle motor is shown in section and in end elevation. A is the cylinder, of

cast-iron, fitted with a cast-iron piston *B*, which is kept gas-tight by means of spring rings. The cylinder is kept cool by the device of exposing as large a surface as possible to the currents of air flowing past the machine when in motion. To

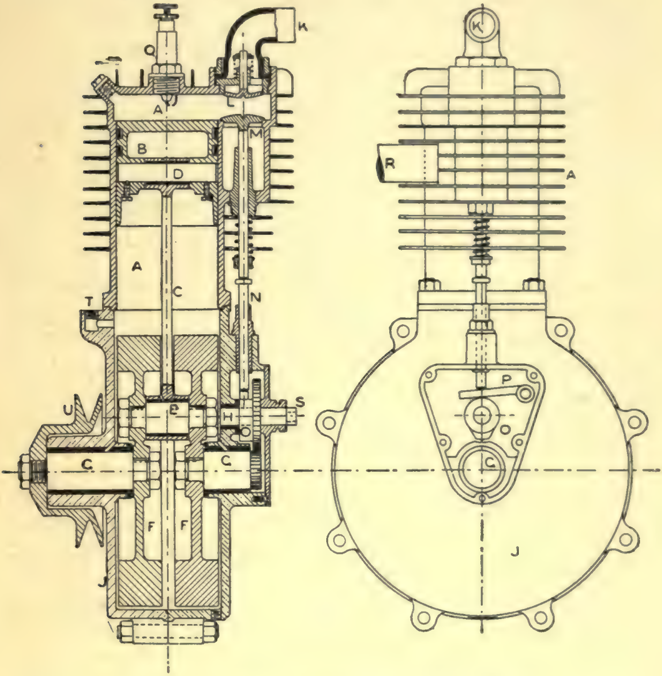


FIG. 311.—Section and side elevation of a cycle motor.

secure this, a large number of thin ribs project from the outer surface of the cylinder. The connecting rod *C* is connected to the piston by a pin *D*, and at the lower end to the crank pin *E*. *F*, *F*, are fly-wheels, secured by nuts and keys to the crank shaft *GG* and to the crank pin *E*. *H* is the side shaft carrying two cams *O* and *S*, used respectively for operating the exhaust valve and the ignition arrangement. The

cams consist of discs fixed to the shaft and having projections on their rims which operate the levers at the proper time. *H* is driven at half the speed of revolution of the crank shaft by means of toothed wheels. The fly-wheels, gearing, etc., are contained in a crank chamber *J*, which, for the sake of lightness, is constructed of aluminium. The cylinder *A* is bolted to the crank chamber. The explosive mixture enters the cylinder during the suction stroke through the pipe *K* and admission valve *L*. In the motor illustrated, the admission valve is not operated mechanically, but is simply drawn open during the suction stroke by the rush of gas entering the cylinder. At other times it is held to its seat by a light spring. In later patterns of small motors the admission valve is operated at the right instant by means of a cam on the shaft *H* and rods leading from it to the valve. *M* is the exhaust valve, held down on its seat by an external spring and operated from the cam *O* through a small lever *P* and a tappet rod *N*. Ignition of the explosive mixture is obtained by use of an electric spark, the current being supplied from a small accumulator, which, of course, must be charged periodically. The accumulator current passes through a coil, and the spark crosses between two platinum points having a small gap between. These points are attached to an ignition plug *Q*, and are situated, as shown, inside the cylinder *A* near to the admission port. The spark will therefore pass in a space containing rich explosive mixture.

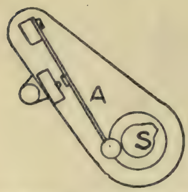


FIG. 312.—Cycle motor contact maker.

The spark is timed by means of the cam *S* (Fig. 312) mounted on the cam shaft. The projection on the cam pushes a spring *A* at the proper instant and causes it to come into contact with a fixed platinum point, thus closing the circuit. *U* is an external pulley having a grooved rim (Fig. 311); a band passes from this to a similar but larger pulley on the rear wheel of the cycle, and so drives the wheel at a speed of rotation lower than that of the motor.

Carburettor.—Fig. 313 shows in section a carburettor suitable for the motor under consideration. The supply of petrol enters

at A from a storage tank, and passes through a strainer B of fine wire gauze with the object of excluding grit from the fine passages of the carburettor. Having passed the strainer, the petrol enters the feed chamber C, which is furnished with an arrangement for controlling the supply. This consists of a pin E, pointed to form a valve at its lower end, and having two collars near its top end engaging with two small pivoted levers F, weighted so as to hold the pin E in the position shown. D is a hollow cylindrical float, which will rise when a sufficient quantity of petrol has entered the chamber, and will press the weighted ends of the levers upwards, thus lowering the pin E and so closing the inlet orifice at the bottom of the chamber.

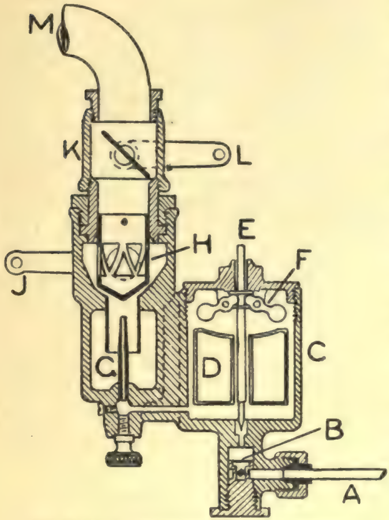


FIG. 313.—Carburettor for a cycle motor.

The pipe K on the engine (Fig. 311) is connected to the pipe M (Fig. 313) on the carburettor. During the suction stroke, petrol will flow from the chamber C and will be discharged as a fine jet from the vertical tube G. This jet is broken up into spray and gasified by the conical baffle plate placed above the tube G, and by the rush of air entering through a hole H situated on the back of the carburettor. A handle J operating a small grating serves to regulate the supply of air. The gasified mixture now passes a butterfly throttle valve K, which may be adjusted by means of a handle L, and so controls the quantity of mixture to be drawn into the motor cylinder ; the mixture finally makes its exit to the cylinder through the pipe M. As the carburettor has to be kept warm in order to gasify the

spray of petrol efficiently, the connecting pipe to the engine cylinder should be short. Sufficient heat will then be conducted from the cylinder to the carburettor.

Motor car engines.—In Fig. 314 is shown in outline a set of petrol engines designed for driving a motor car. There are four cylinders, A, B, C and D, placed vertically over the crank shaft EF; the latter has four cranks, G, H, K and L, to which the pistons are connected by rods M, N, O and P. The cranks are arranged so that the pistons in A and D are at the top of the stroke when those in B and C are at the bottom, as is shown in Fig. 314. The order of firing the charges is as follows: D, B, A, C. As each cylinder has an explosion every two revolutions, there will be an explosion and working stroke for each and every half-revolution of the crank shaft. The arrangement of cranks is also advantageous in producing a well-balanced and smooth-running engine.

The crank shaft runs in three bearings formed in a crank chamber Q, and the cylinders are mounted on the top of this chamber. The lower part of the chamber contains oil, into which the bottom ends of the connecting rods dip and thus pick up oil for lubricating the crank pins.

The inlet and exhaust valves, one of which is shown at R, are all placed on the same side of the engine, and are operated by cams on the side shaft S, which is driven at half the speed of the crank shaft. Ignition is effected by electric sparks, the current for which is generated by a small magneto machine T, which is driven by the engine.

U and V are water jackets, surrounding the upper and hotter parts of the cylinders, and supplied with water by a circulating pump W, which is driven by the engine. The water leaving the engine is conducted by a pipe to a radiator, situated at the front of the car; the radiator has a large number of tubes, furnished with projecting ribs so as to expose a large cooling surface to the currents of air when the car is moving. A set of engines of this type, having cylinders $4\frac{5}{8}$ inch bore $\times$ 5 inch stroke is capable of developing 36 brake-horse-power at 1000 revolutions per minute. Fig. 314 has been drawn from working drawings kindly supplied by The Wolseley Tool and Motor Car Co., Ltd.

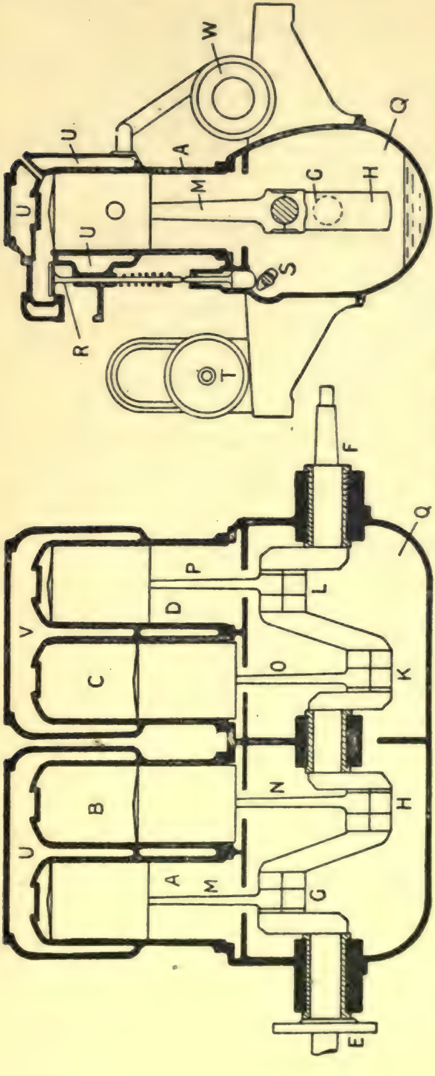


FIG. 314.—Set of petrol engines for a motor car.

EXERCISES ON CHAPTER XXI.

1. Explain the four stroke or Beau-de-Rochas cycle. Sketch an indicator diagram and name each stroke on it.

2. The piston of a gas engine is 6·7 inches in diameter by 1·2 feet stroke. At 200 revolutions per minute there are 80 explosions per minute and the engine gives $6\frac{1}{2}$ I.H.P. Calculate what the mean pressure must have been.

3. The crank shaft of a gas engine is giving out steadily 20 horse-power at an average speed of 150 revolutions per minute. How many foot-pounds is this per cycle (of two revolutions)? About how much of this must be stored and unstored by the fly-wheel if there are 75 explosions per minute?

4. Describe generally the action in any oil or petrol engine you know.

5. Explain with sketches what must be done to the liquid fuel before being mixed with air.

6. Sketch in section an oil engine cylinder showing the valves. Name and explain the use of each part.

7. Explain with sketches how the charge may be ignited in a petrol engine.

8. Why is a single cylinder engine seldom found in a modern motor car?

MATHEMATICAL TABLES.

USEFUL CONSTANTS.

1 inch	=2·54 centimetres=25·4 millimetres.
1 metre	=39·37 inches.
5280 feet	=1 mile.
6 feet	=1 fathom.
1 Gunter's chain	=66 feet.
80 Gunter's chains	=1 mile.
1 kilometre	=0·621 mile.
1 square inch	=6·45 square centimetres.
1 square metre	=1550 square inches.
1 cubic inch	=16·39 cubic centimetres.
1 cubic metre	=61,025 cubic inches=1·308 cubic yards.
1 litre	=1000 cubic centimetres=1·762 pint.
1 gallon	=0·1605 cubic foot=4·541 litres.
1 bushel	=1·284 cubic feet.
1 radian	=57·3 degrees.
π	=3·1416.

1 knot=6080 feet per hour.
60 miles per hour=1 mile per minute=88 feet per second.
The value of g at London=32·182 feet per sec. per sec.

One pound avoirdupois=7000 grains=453·6 grams.
One kilogram=2·205 pounds.
One gallon of pure water at 62° F. weighs 10 lbs.
One cubic foot of pure water at 62° F. weighs 62·3 lbs.

Weight of 1 lb. in London = 445,000 dynes.

One cubic foot of air at 0° C. and 1 atmosphere pressure weighs 0·0807 lb.

One cubic foot of hydrogen at 0° C. and 1 atmosphere pressure weighs 0·00559 lb.

1 atmosphere = 14·7 lbs. per square inch
= 2116 lbs. per square foot
= 10^6 dynes per square centimetre nearly.

1 kilogram per square centimetre = 14·22 lbs. per square inch.

A column of mercury 760 millimetres (= 30 inches) high produces a pressure at its base of 1 atmosphere.

A column of water 2·3 feet high produces a pressure at its base of 1 lb. per sq. inch.

1 foot-pound	$= 1·3562 \times 10^7$ ergs.
1 metre-kilogram	= 7·235 foot-pounds.
1 horse-power	= 33,000 foot-pounds per minute = 746 watts.
1 horse-power-hour	$= 33,000 \times 60$ foot-pounds.
Volts × amperes	= watts.
1 electrical unit	= 1000 watt-hours.
1 B.T.U.	$= \frac{5}{9}$ lb.-degree-Cent. unit.
	= 252 gram-calories.

Absolute temperature $\tau = t°$ C. + 273·7
$= t°$ F. + 461.

Joule's equivalent $= \begin{cases} 778 \text{ ft.-lbs.} = 1 \text{ B.T.U.} \\ 1400 \text{ ft.-lbs.} = 1 \text{ lb.-degree-Cent. unit.} \end{cases}$

Properties of Steam. (Fahrenheit Units.)

Pressure, lbs. per sq. inch.	Temperature, F.	Volume of 1 lb. in cubic feet.	Heat Units (Fahrenheit).			Pressure, lbs. per sq. inch.	Temperature, F.	Volume of 1 lb. in cubic feet.	Heat Units (Fahrenheit).		
			H	h	$(H-h)=L$				H	h	$(H-h)=L$
1	102	334·2	1113·0	70·1	1043	115	337·8	3·821	1185·0	309·5	875·5
2	126·3	173·2	1120·4	94·4	1026	120	341·0	3·671	1185·9	312·8	873·2
3	141·6	118·0	1124·9	109·9	1015	125	344·1	3·534	1186·9	316·0	870·9
4	153·1	89·80	1128·6	121·4	1007	130	347·1	3·406	1187·8	319·0	868·7
5	162·3	72·50	1131·4	130·7	1001	135	349·9	3·287	1188·7	322·1	866·6
6	170·1	61·10	1133·8	138·6	995·2	140	352·8	3·177	1189·5	325·0	864·6
7	176·9	53·00	1135·9	145·4	990·5	145	355·5	3·074	1190·4	327·8	862·6
8	182·9	46·60	1137·7	151·5	986·2	150	358·2	2·978	1191·2	330·6	860·6
9	188·3	41·82	1139·4	156·9	982·4	155	360·7	2·888	1192·0	333·2	858·7
10	193·2	37·80	1140·9	161·9	979·0	160	363·3	2·803	1192·7	335·9	856·9
15	213·0	25·87	1146·9	181·9	965·0	165	365·7	2·724	1193·5	338·4	855·1
20	227·9	19·72	1151·4	197·0	954·4	170	368·2	2·649	1194·2	340·9	853·3
25	240·0	15·99	1155·1	209·3	945·8	175	370·5	2·578	1194·9	343·4	851·6
30	250·2	13·48	1158·3	219·7	938·5	180	372·8	2·510	1195·6	345·8	849·9
35	259·2	11·66	1161·0	228·8	932·1	185	375·1	2·447	1196·3	348·1	848·2
40	267·1	10·29	1163·4	236·9	926·5	190	377·3	2·386	1197·0	350·4	846·6
45	274·3	9·21	1165·6	244·3	921·3	195	379·4	2·328	1197·7	352·7	845·0
50	280·8	8·34	1167·6	251·0	916·6	200	381·6	2·273	1198·3	354·9	843·4
55	286·9	7·63	1169·4	257·1	912·3	205	383·7	2·221	1199·0	357·1	841·9
60	292·5	7·03	1171·2	262·9	908·2	210	385·7	2·171	1199·6	359·2	840·4
65	297·8	6·52	1172·8	268·3	904·5	215	387·7	2·124	1200·2	361·3	838·9
70	302·7	6·09	1174·3	273·4	900·9	220	390·0	2·07	1200·8	—	—
75	307·4	5·70	1175·7	278·2	897·5	230	394	1·99	1201·9	—	—
80	311·8	5·37	1177·0	282·7	894·3	240	397·5	1·91	1203·0	—	—
85	316·0	5·07	1178·3	287·0	891·3	250	401	1·84	1204·0	—	—
90	320·0	4·81	1179·5	291·2	888·4	260	404·5	1·77	1205·0	—	—
95	323·9	4·57	1180·7	295·1	885·6	270	408	1·71	1206·0	—	—
100	327·6	4·356	1181·8	298·9	882·9	280	411	1·65	1207·0	—	—
105	331·1	4·161	1182·9	302·6	880·3	290	414·4	1·60	1207·9	—	—
110	334·5	3·984	1184·0	306·1	877·9	300	417·5	1·55	1208·9	—	—

Properties of Steam. (Centigrade Units.)

Temperature, C.	Pressure.		Volume of 1 lb. in cubic feet.	Heat Units (Cent.).				
	Lbs. per sq. inch.	Lbs. per sq. foot.		H	h	$(H-h) = L$	External work during evaporation.	Intrinsic energy of 1 lb. of steam.
0	0·085	12·27	3398	606·5	0·00	606·5	29·92	576·6
5	0·122	17·62	2412	608·0	5·00	603·0	30·51	577·5
10	0·173	24·92	1736	609·5	10·00	599·5	31·06	578·4
15	0·241	34·77	1268	611·1	15·00	596·0	31·65	579·4
20	0·333	47·87	936·9	612·6	20·01	592·6	32·19	580·4
25	0·452	65·06	700·8	614·1	25·02	589·1	32·73	581·4
30	0·607	87·40	530·7	615·6	30·03	585·6	33·30	582·3
35	0·806	116·1	405·9	617·2	35·04	582·1	33·84	583·4
40	1·06	152·6	313·6	618·7	40·05	578·6	34·34	584·4
45	1·38	198·6	244·6	620·2	45·07	575·1	34·87	585·3
50	1·78	256·0	192·5	621·7	50·09	571·7	35·37	586·3
55	2·27	327·0	152·8	623·3	55·11	568·2	35·85	587·4
60	2·88	414·3	122·3	624·8	60·13	564·7	36·37	588·4
65	3·62	520·6	98·7	626·3	65·17	561·1	36·88	589·4
70	4·51	649·1	80·23	627·8	70·20	557·6	37·39	590·4
75	5·58	803·3	65·64	629·4	75·24	554·1	37·85	591·5
80	6·86	987·6	54·06	630·9	80·28	550·6	38·32	592·6
85	8·38	1206	44·81	632·4	85·33	547·1	38·79	593·6
90	10·16	1463	37·36	633·9	90·38	543·6	39·23	594·7
95	12·26	1765	31·34	635·5	95·44	540·0	39·70	595·8
100	14·70	2116·4	26·43	637·0	100·5	536·5	40·15	596·8
105	17·53	2524	22·40	638·5	105·6	533·0	40·58	597·9
110	20·80	2994	19·08	640·9	110·6	529·4	41·02	599·0
115	24·54	3534	16·32	641·6	115·7	525·8	41·40	600·2
120	28·83	4152	14·04	643·1	120·8	522·3	41·83	601·3
125	33·71	4854	12·12	644·6	125·9	518·7	42·23	602·4
130	39·25	5652	10·51	646·1	131·0	515·1	42·64	603·5
135	45·49	6551	9·147	647·7	136·1	511·6	43·00	604·7
140	52·52	7563	7·995	649·2	141·2	508·0	43·40	605·8
145	60·40	8698	7·009	650·7	146·3	504·4	43·76	606·9
150	69·21	9966	6·168	652·2	151·5	500·8	44·14	608·1
155	79·03	11380	5·446	653·8	156·5	497·2	44·48	609·3
160	89·86	12940	4·827	655·3	161·7	493·5	44·83	610·5
165	101·9	14680	4·290	656·8	166·9	489·9	45·20	611·6
170	115·1	16580	3·823	658·3	172·0	486·3	45·50	612·8
175	129·8	18690	3·419	659·9	177·2	482·7	45·86	614·0
180	145·8	20990	3·065	661·4	182·4	479·0	46·17	615·2
185	163·3	23520	2·756	662·9	187·6	475·3	46·52	616·4
190	182·4	26270	2·482	664·4	192·8	471·7	46·80	617·6
195	203·3	29270	2·242	666·0	198·0	468·0	47·10	618·9
200	225·9	32520	2·031	667·5	203·2	464·3	47·42	620·1
205	250·3	36050	1·843	669·0	208·4	460·6	47·68	621·3
210	276·9	39870	1·676	670·6	213·7	456·9	47·97	622·6
215	305·5	43990	1·529	672·1	218·9	453·2	48·27	623·8

Boiling points of water at pressures near standard atmospheric pressure.

(The pressures are given in mm. of mercury at a temperature of 0° C. situated at the sea level in latitude 45°.)

Pressure mm.	Temp. deg. Cent.	Pressure mm.	Temp. deg. Cent.	Pressure mm.	Temp. deg. Cent.	Pressure mm.	Temp. deg. Cent.
733	98·9939	745	99·4449	757	99·8897	769	100·3286
734	99·0318	746	·4822	758	·9265	770	·3649
735	·0695	747	·5194	759	·9633	771	·4012
736	·1073	748	·5567	760	100·0000	772	·4374
737	·1449	749	·5938	761	·0367	773	·4736
738	·1826	750	·6310	762	·0733	774	·5098
739	·2202	751	·6681	763	·1099	775	·5459
740	·2577	752	·7051	764	·1465	776	·5820
741	·2953	753	·7421	765	·1830	777	·6180
742	·3327	754	·7791	766	·2194	778	·6540
743	·3702	755	·8160	767	·2559	779	·6900
744	·4075	756	·8529	768	·2923	780	·7259

Table of coefficients of linear expansion.

(These are given as the increase in length which a bar of unit length undergoes when heated through one degree Fahrenheit.)

Steel alloyed with 36 % nickel	0·000000483
Wrought-iron and mild steel	0·00000673
Cast-iron	0·0000063
Copper	0·0000096
Zinc	0·0000162
Brass	0·0000105
Phosphor bronze	0·0000107

Table of Specific Heats.

Aluminium	0·2022	Nickel	0·1084
Copper	0·09232	Platinum	0·03147
Iron	0·1098	Tin	0·0559
Lead	0·0315	Zinc	0·0935

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7119	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS.

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·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
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·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
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ANSWERS.

Chapter I., p. 12.

1. 2·908 metres. **2.** 9 feet 7·74 inches. **3.** 5·129 kilometres.
4. 2·114 inches. **5.** 12·58 square inches. **6.** 53 square cms.
7. 7·3 square inches. **8.** (*a*) 44 cms. ; (*b*) 154 square cms.
9. 381·9 cubic inches. **10.** 5·44 square inches. **11.** 14·5 square inches.

Chapter II., p. 20.

1. 6·72 lbs. wt. **2.** 2·21 lbs. wt. **3.** 277 lbs. wt.
4. 17·26 lbs. wt. **5.** 82·9 lbs. wt. **6.** 17·49 lbs. wt.
7. 3·27 lbs. wt. **8.** 3·05 lbs. wt. **9.** 14·4 lbs. wt.
10. 496 lbs. wt. **11.** 8·7″.

Chapter III., p. 34.

2. 5·7 lbs. wt. **3.** 5 lbs. wt. **4.** 29 lbs. wt.
5. 21 lbs. wt., acting towards the left.
6. (*a*) 11·7 lbs. wt. ; (*b*) 8·7 lbs. wt. ; (*c*) 14 lbs. wt.
7. 19·5 lbs. wt. **8.** 14·5 lbs. wt. **10.** 8·96 lbs. wt. ; 7·32 lbs. wt.
11. 9·65 cwts. **13.** 108·6 lbs. wt. ; 51 lbs. wt.
14. 160 lbs. wt. ; 81 lbs. wt. **15.** 10 lbs. wt. ; 17·32 lbs. wt.
16. Push in AB = 0·16 ton wt. ; push in AC = 0·89 ton wt.
17. Push in jib = 7·12 lbs. wt. ; pull in tie = 5·75 lbs. wt.
18. Push in jib = 5·28 tons wt. ; pull in tie = 2·36 tons wt.
19. Push in jib = 7·78 tons wt. ; pull in tie = 4·86 tons wt.
20. Pull in AC = 0·16 ton wt. ; push in BC = 0·83 ton wt.
21. Push in AC = 1·16 tons wt. ; pull in BC = 0·83 ton wt.
22. (*a*) 4·66 lbs. wt. ; (*b*) 4·23 lbs. wt. ; (*c*) 4·88 lbs. wt.

Chapter IV., p. 49.

1. At 25″ from C, on the other side of the pivot from W.
2. 69·2 lbs. wt. **3.** $P = 64·1$ lbs. wt.
4. At 2·25 feet from the 3 lb. load.
6. 0·833 ton wt.; 0·417 ton wt. **7.** 3·55 tons wt.; 2·45 tons wt.
8. 1·33 tons wt. at A; 4·66 tons wt. at C.
9. (a) 2400 lbs. wt.; (b) 5376 lbs. wt.
10. 345 lbs. wt.; 245 lbs. wt. **11.** 5·417 tons wt.; 4·583 tons wt.
12. 200 lbs. wt.; 100 lbs. wt. **13.** 663 lbs. wt. **14.** 67 lbs. wt.
15. 1600 lb.-feet; 707 lb.-feet; wall will fall.
17. On the shorter portion of the beam, 2·24 ft. from the pivot.
18. 1·77 ft. from the heavier end of the ladder.

Chapter V., p. 67.

1. 2·8 tons per sq. inch. **2.** 12·5 tons.
3. Strain = 0·00083; $E = 30{,}000{,}000$ lbs. per sq. inch.
4. 0·504″. **5.** 0·0556″. **6.** 0·076 lb. **7.** 9062 lbs.
8. $E = 15{,}700{,}000$ lbs. per sq. inch. **9.** 0·747 sq. inch.

Chapter VI., p. 81.

1. $M = 1500$ lb.-ft. **2.** $M = 3000$ lb.-ft. **3.** $M = 1600$ lb.-ft.
4. $M = 3600$ lb.-ft. **5.** (a) 0·5 ton; (b) 1 ton.
6. 5·21 cwts. **7.** 1·8 tons; 3·6 tons.
8. 30,190,000 lbs. per sq. inch. **11.** 40,500 lb.-inches.

Chapter VII., p. 98.

1. 600 ft.-tons. **2.** 135,000 ft.-lbs. **3.** 697,000 ft.-lbs.
4. 375 ft.-tons. **5.** 3400 ft.-lbs. **6.** (a) 140 ft.-lbs.; (b) 220 ft.-lbs.
7. 2,250,000 ft.-lbs. **8.** 324 ft.-lbs. **9.** 24,550,000 ft.-lbs.
10. 7500 ft.-lbs. **11.** 463,200 ft.-lbs. **12.** 15,440,000 ft.-lbs.
13. 0·64 H.P. **14.** 318 H.P. **15.** 101 H.P.
16. 19·8 per cent. **17.** 0·091 H.P.; 0·68 H.P.
18. 10·95 H.P.; 48 amperes. **19.** 0·265. **20.** 0·433. **21.** 0·288.
22. 1800 lbs.; 144 H.P. **23.** 423·4 ft.-lbs. **24.** 192 H.P.; 480 H.P.

Chapter VIII., p. 119.

1. 212; 318. **2.** 14·5 H.P. **4.** 15.
5. 238 lbs. **6.** 48. **7.** 940 degrees.
8. 71·6 lbs.; 301·6 lbs. **12.** 85; 112·6.

Chapter IX., p. 136.

1. 440 feet. **2.** 32·7 miles per hour. **3.** 17·4 feet per sec.
4. 47 miles per hour. **5.** 0·084 feet per sec. per sec. **6.** 12 feet.
7. 1·708 tons. **8.** 1500 ft.-lbs. **9.** 49·7 ft.-lbs.
10. 265,600 ft.-tons. **12.** 10 feet per sec. **13.** 43 feet per sec.
14. 150 lbs.; 160 lbs. **15.** 5 tons wt. **16.** 62,100 lbs. wt.
17. 3882 lbs. wt. **18.** 5·24 tons wt. **19.** 12·9 lbs. wt.
20. 1325 lbs. **21.** 15 lbs. **22.** 39·12 inches.
23. 1·7 tons. **24.** Tangent = 0·2; 11°.

Chapter X., p. 156.

1. 30,720 lbs.; 9600 lbs.; 6400 lbs. **2.** 900 lbs.; 300 lbs.
3. 22,500 lbs.; 45,000 lb.-feet. **4.** 3·25 feet.
5. 138·1 feet. **6.** 16,900 lbs.; 3·8 feet from the bottom.
7. 358,000 cu. feet; 350,000 cu. feet. **8.** 18·25 lbs.
9. 18·6 lbs. **10.** 287 square feet.
11. 1166 cubic inches; 28·12 lbs. **12.** 9463 lbs.
15. 7. **16.** 1·807 lbs.

Chapter XI., p. 174.

1. 2765 foot-lbs. **2.** 71·6 gallons. **4.** 1414 lbs.; 90,500 lbs.
5. 0·41 ton per square inch; 1950 foot-tons.
6. 201,600 foot-lbs.; 107,520 foot-lbs.
7. 108,000 foot-lbs.; 17,280 foot-lbs.; 960,000 foot-lbs.
8. 470; 425; 90·4 per cent. **10.** 3·21 H.P.; 67 per cent.
11. 11·8 H.P. **14.** 382,000 foot-lbs. per sec.; 555 H.P.
15. 48,830 foot-lbs.

Chapter XII., p. 191.

2. 284° F. **3.** −40° C. **4.** −459·4° F.
5. 34·05″. **6.** 432° F. **8.** 1·505″.

Chapter XIII., p. 206.

2. 23·55 lb.-degree-Cent. units; 10,680 gram-calories.
3. 33·12 B.T.U. **5.** 77·1° F. **6.** 64·07° F.
7. 62·9° F. **9.** 140,040 foot-lbs. **10.** 391·5° C.
11. 11,670,000 foot-lbs.; 15,950,000 foot-lbs.; 466,900 foot-lbs.
12. 7,795,000 B.T.U. **16.** 0·6.

Chapter XIV., p. 226.

4. 78 lbs. per square foot.
7. 0·513 cubic foot.
8. 0·533 cubic inch.
9. 27·2 inches.
11. 173·5 cubic feet.
12. 929·2° C.
14. 32·15 B.T.U.
15. 20·75 lbs. per square inch.

Chapter XV., p. 245.

4. 66·64 lbs. per square inch.
5. 4·368 cubic feet.
6. $L = 905{\cdot}5$ B.T.U.; $H = 1173{\cdot}5$ B.T.U.
9. 102,660 B.T.U.
10. 34·6 lbs.
11. 1125·5 B.T.U.; 12·78 lbs.
12. 618·3 lb.-degree-Cent. units.

Chapter XVI., p. 280.

1. 33·9 feet.
2. 44,000 foot-lbs.; 2·2 H.P.
3. 166·6 lbs.
4. 0·033 H.P.
5. 405·1 cubic inches.
6. 80 cubic feet.
7. 26·6 inches of mercury.
15. 3900 lbs.

Chapter XVII., p. 299.

2. 28 lbs.; 228·3 lbs.
3. 50 cubic feet.
4. 126 lbs.; 1027 lbs.
5. 32,480,000 B.T.U.
7. 13,925 B.T.U.
8. 10·95 lbs.; 22 lbs.
11. 14,505 B.T.U.
12. 19,040,000 lb.-deg.-C.; £7. 10*s.* 3·6*d.*; £252.
13. 3·47 lbs.; 15·09 lbs.

Chapter XVIII., p. 318.

8. 4600 lb.-feet.
9. 1000 lbs.; 14·3 sq. inches.
11. 15,081 lbs.
12. 30°.
14. 367·8 foot-tons.
15. 116 revs. per min.
17. 3·43.

Chapter XIX., p. 337.

11. 214·5 I.H.P.
13. 7·174 B.H.P.
14. 8·69 I.H.P.; 50,160 ft.-lbs.
16. 94,690 foot-lbs.

Chapter XX., p. 351.

1. 80 per cent.
2. 17,960 B.T.U.
3. 16,490 B.T.U.; 15·4 per cent.
13. 8·76 per cent.

Chapter XXI., p. 362.

2. 63·4 lbs. per sq. inch.
3. 8800 foot-lbs.; about three quarters, viz. 6600 foot-lbs.

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